SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2007
Year 11
Half Yearly Exam

Mathematics Extension 1 - Continuers

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- All necessary working should be shown in every question.
- Hand in your answers in 2 separate bundles. Section A and Section B as indicated in the paper.

Total Marks – 80

- Attempt questions 1-6
- All questions are **NOT** of equal value.

Examiner: *F. Nesbitt*

SECTION A

Question 1 (13 marks)

(a) Simplify $3-4(x-5)$ 1

(b) Write 7 546 423 in scientific notation correct to 3 significant figures. 1

(c) (i) Show that the function $f(x)=2x^6-3x^4+x^2$ is an even function. 2

(ii) Find $f(\sqrt{x})$ 2

(d) Solve $|x-2|=3$ 2

(e) Find the value of $\dfrac{3.24}{\sqrt{5.13-1.89}}$

Give your answer correct to 2 decimal places. 1

(f) Find the exact value of

(i) $\sin 600^0$ (ii) $\tan\dfrac{2\pi}{3}$ 2

(g) A triangle has sides 4.8 cm, 6.6 cm and 3.2 cm. Use the cosine rule to find the size of the largest angle to the nearest minute. 2

Question 2 (10 marks)

(a) For the function $f(x) = x^2 - 5x + 6$

(i) Find its x and y intercepts. 2

(ii) Sketch the function. 1

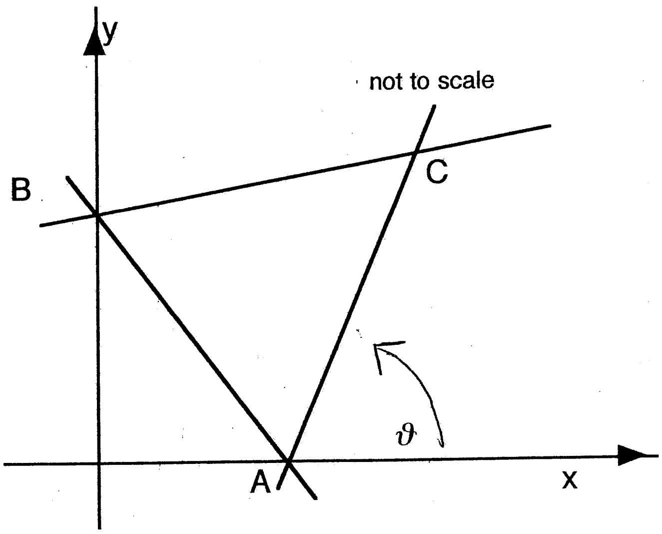
(iii) Find its Domain and range. 2

(b) (i) Solve $\dfrac{2}{x} < 3$ 2

(ii) Graph the solution on the number line. 1

(c) Find the exact value of $\sin\dfrac{2\pi}{3} - \cos\dfrac{\pi}{4}$. Give your answer as a single fraction. 2

Question 3 (14 marks)



The points A, B, and C have coordinates (4 ,0), (0,5) and (5,9). The angle between AC and the x axis is ϑ.

(a) Copy this diagram into your answer booklet. **1**

(b) Find the gradient of the line AC. **1**

(c) Find the size of the angle ϑ to the nearest degree. **1**

(d) Find, in general form, the equation of the line AC. **2**

(e) Find the coordinates of D, the midpoint of AC. **1**

(f) Show that BD is perpendicular to AC and state what this shows about ΔABC. **2**

(g) Find the area of ΔABC. **2**

(h) Find a point E on BD extended such that BD = DE. **2**

(i) What is the shape of the quadrilateral ABCE? Give a reason. **2**

SECTION B (Start a new booklet)

Question 4 (14 marks)

4. (a) Write with a rational denominator $\dfrac{1}{\sqrt{5}-2}$ **2**

(b) Simplify fully $\dfrac{m^2-m}{m^2-1}$ **2**

(c) Write as a single simplified fraction $\dfrac{1}{1-x^2}+\dfrac{1}{1+x}$ **2**

(d) Write $0.\dot{3}\dot{1}$ in the form $\dfrac{m}{n}$ where m and n are integers. **1**

(e) Each interior angle in a regular polygon is 135^0. How many sides does the polygon have? **2**

(f) A function is defined as:

f(x) = 6 for $x < 3$

f(x) = 2x for $x \geq 3$

Find (i) f(4) **1**

(ii) f(0) **1**

(iii) f(- 2) **1**

(g) If the point (2, k) lies on the line $2y-3x+5=0$

find the value of k. **2**

Question 5 (15 marks)

5. (a) By completing the square, find the centre and radius of the circle

$$x^2 + y^2 - 4x + 10y - 7 = 0$$ **3**

(b) An interval PQ has gradient - 3. A second interval passes through A (-2, 4) and B (1, k). Find the value of k if AB is parallel to PQ. **2**

(c) Factorise completely

(i) $u^2w + vw - u^2x - vx$ **2**

(ii) $27 - x^3$ **1**

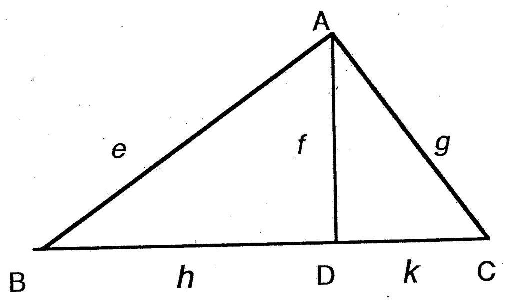
(d) Expand and simplify $2\sqrt{2}\left(\sqrt{5} - \sqrt{2}\right)$ **1**

(e) Write $\sqrt{500}$ in the form $a\sqrt{b}$ **1**

(f) Find the point of intersection of the lines

$x + y = 2$ and $4x - y = 13$ **2**

(g)



In the triangle ABC above, the angle BAC is a right angle. AD is perpendicular to BC. Use Pythagoras' theorem to show that

$f^2 = hk$ **3**

QUESTION 6 (14 marks)

6. (a) Find the equation of the locus of the point P (x, y) so that it is always the same distance from the point (2, 1) as from the point (1, 5) **2**

(b) Find the equation of a line passing through the intersection of the lines $x+2y-6=0$ and $3x-2y-6=0$ and passing through the point (2, - 1) **3**

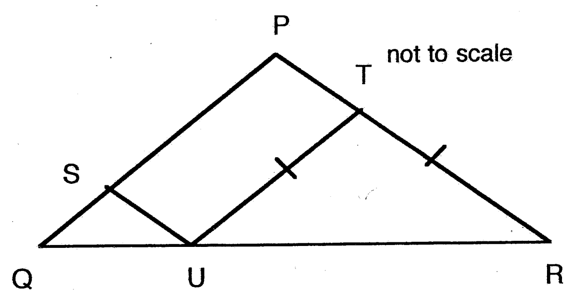
(c) (i) On the same set of axes, draw graphs of the functions

$y=|2x-5|$ and $y=x+2$ **2**

(ii) Use the graphs to solve the inequality

$|2x-5| \geq x+2$ **2**

(d)



In the triangle PQR above, TU = TR, SP is parallel to UT and PT is parallel to SU.

(i) Prove that the triangle PQR is isosceles **3**

(ii) If angle QPR is three times the size of the angle PRQ, prove that the angle PRQ = 36° **2**

END OF PAPER

Section A.

Question 1:

a) $3-4(x-5)$
$=3-4x+20$
$=23-4x$ (1)

b) 7.55×10^{6} (1)

c) i) $f(x)=2x^{6}-3x^{4}+x^{2}$

$f(-x)=2(-x)^{6}-3(-x)^{4}+(-x)^{2}$
$=2x^{6}-3x^{4}+x^{2}$

$\therefore f(x)=f(-x)\therefore$ even function (2)

c ii) $f(\sqrt{x})=2(\sqrt{x})^{6}-3(\sqrt{x})^{4}+(\sqrt{x})^{2}$
$=2(x^{1/2})^{6}-3(x^{1/2})^{4}+(x^{1/2})^{2}$
$=2x^{3}-3x^{2}+x$ (2)

d) $|x-2|=3$

+ve case
$x-2=3$
$x=5$ (1)

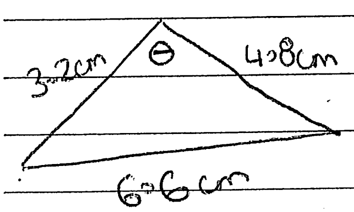
-ve case
$-x+2=3$
$-x=1$
$x=-1$ (1)

e) $\dfrac{3.24}{\sqrt{5.13-1.89}}=\dfrac{3.24}{\sqrt{3.24}}=\dfrac{3.24}{1.8}=1.80$ (1) (2dp).

f) i) $\sin600^\circ=-\sqrt{3}/2$ (1)

ii) $\tan\frac{2\pi}{3}=-\sqrt{3}$ (1)

g) 4.8cm, 6.6cm, 3.2cm



(not to scale)

$\cos A=\dfrac{b^{2}+c^{2}-a^{2}}{2bc}$

$\cos A=\dfrac{(4.8)^{2}+(3.2)^{2}-(6.6)^{2}}{2\times4.8\times3.2}$
$=-0.334635416$

$\therefore A=109^\circ 33'$ (2)

Question 2:

a) i) x - intercepts: $0 = x^2 - 5x + 6$

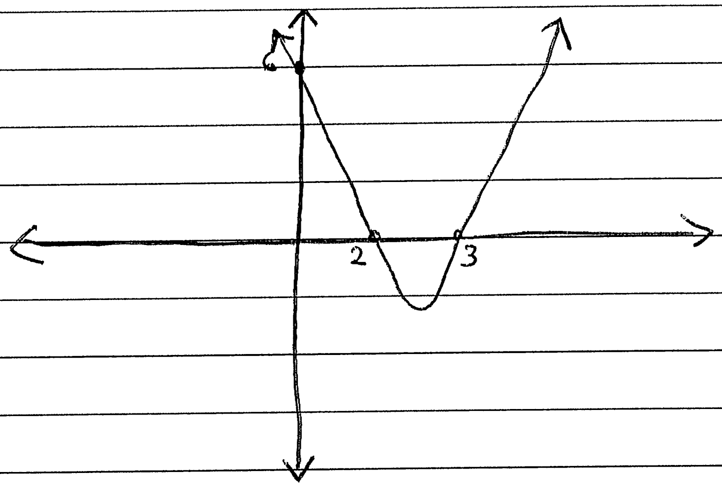
$0 = (x-3)(x-2)$

$\therefore x = 3, \ x = 2$ (1)

y-intercepts: $y = 0 - 0 + 6$

$y = 6$ (1)

ii)



(1)

iii) Domain: all real x. (1)

Range: $y \geq -1/4$ (1) axis of symmetry is half way between 2 + 3 $\therefore$ 2·5

$f(2{\cdot}5) = 2{\cdot}5^2 - 5 \times 2{\cdot}5 + 6$

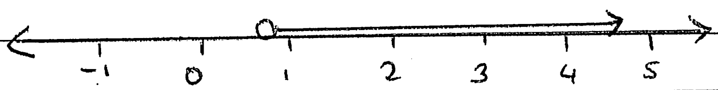
$= -1/4$

B) i solve $\frac{2}{x} < 3$

$2 < 3x$ 1 for $x < 2/3$

$x > 2/3$ (2)

ii



(1)

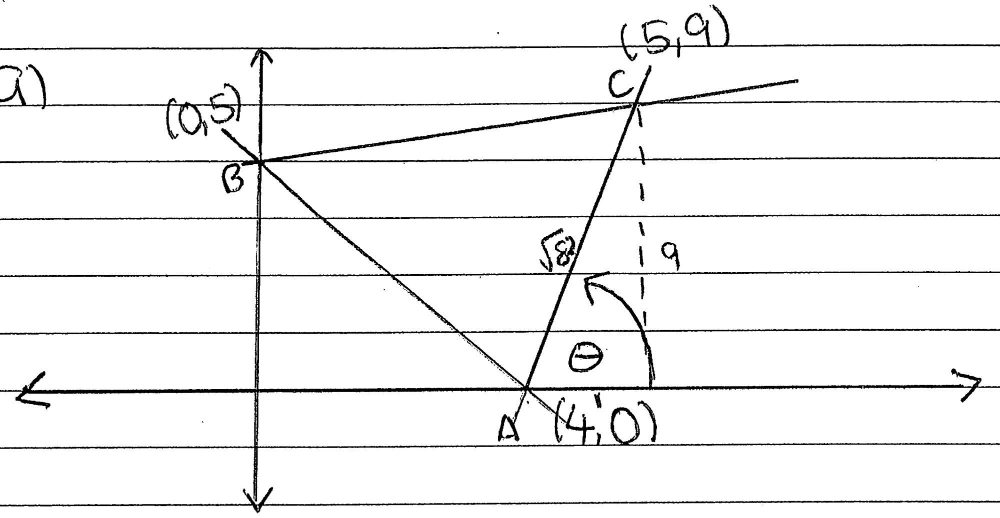
c) $\sin\frac{2\pi}{3} - \cos\frac{\pi}{4} = \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} = \frac{\sqrt{6} - 2}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{4\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$

$\sqrt{12} - 2\sqrt{2} = 2\sqrt{3} - 2\sqrt{2}$

Question 3:

a)



(1)

b) $m \text{ of } AC = \frac{9-0}{5-4}$

$= \frac{9}{1}$

$= 9$ (1)

c) Distance of $AC = \sqrt{(5-4)^2 + (9-0)^2}$

$= \sqrt{1 + 81}$

$= \sqrt{82}$

$\cos\theta = \frac{1^2 + \sqrt{82}^2 - 9^2}{2 \times 1 \times \sqrt{82}}$

$= 0.110431526$

$\therefore \theta = 83.659808 25$

$= 84°$ (nearest degree) (1)

d) AC: $y - 9 = 9(x-5)$

$y - 9 = 9x - 45$

$9x - y - 36 = 0$ (2)

e) $D = \left(\frac{5+4}{2}, \frac{9+0}{2}\right)$

$= \left(\frac{9}{2}, \frac{9}{2}\right)$ (1)

f) m of BD $= \frac{9/2 - 5}{9/2 - 0}$

$= -1/9$

$m_1 \times m_2 = 9 \times -1/9$

$= -1$

$\therefore$ BD ⊥ to AC (1)

$\therefore \triangle ABC$ is isosceles as $\triangle ABD \equiv \triangle BCD$

$\therefore BC = BA$. (1)

g) Distance of BD $= \sqrt{(0 - 9/2)^2 + (5 - 9/2)^2}$

$= \sqrt{81/4 + 1/4}$

$= \sqrt{82/4}$

$= \sqrt{82}/2$

$\therefore$ Area of $\triangle ABC = \frac{1}{2} \times \sqrt{82} \times \frac{\sqrt{82}}{2}$

$= \frac{82}{4}$

$= 20\frac{1}{2}\ u^2$ (2)

h) Find E such that BD = DE

$\therefore$ D is the midpoint of BE.

$\therefore \left(\frac{9}{2}, \frac{9}{2}\right) = \left(\frac{0 + x}{2}, \frac{5 + y}{2}\right)$

$\therefore x = 9$ $\quad 5 + y = 9$

$y = 4$

$\therefore$ the coordinates of E are (9, 4) (2)

i) ABCE is a rhombus.

$\triangle ABC \equiv \triangle ACE$

$\therefore BC = BA = EC = EA$ (2)

& diagonals bisect at ⊥.

4a) $\frac{1}{\sqrt{5}-2} = \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$ (1 for rationalising ✓)

$= \frac{\sqrt{5}+2}{1}$

$= \sqrt{5}+2$ (2)

b) $\frac{m^2-m}{m^2-1} = \frac{m(m-1)}{(m-1)(m+1)}$ ½ each, 1 for factors (both)

$= \frac{m}{m+1}$ (2)

c) $\frac{1}{1-x^2} + \frac{1}{1+x} = \frac{1}{(1-x)(1+x)} + \frac{1}{1+x}$

$= \frac{1+1-x}{(1-x)(1+x)}$

$= \frac{2-x}{(1-x)(1+x)}$ (2)

d) $x = 0.313131\ldots\ldots$

$100x = 31.313131\ldots\ldots$

$99x = 31$

$x = \frac{31}{99}$ (1)

e) $\frac{180(n-2)}{n} = 135$

$180(n-2) = 135n$

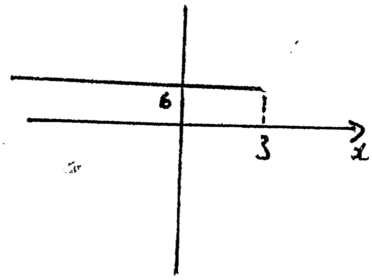
$180n - 360 = 135n$

$45n = 360$

$n = 8$ (2)

f) $f(x) = 6 \quad x < 3$

$f(x) = 2x \quad x \geq 3$



$f(4) = 8$ (1)

$f(0) = 6$ (1)

$f(-2) = 6$ (1)

g) $2y - 3x + 5 = 0$

Since $(2, k)$ lies on line

$2k - 6 + 5 = 0$

$2k = 1$

$k = 1/2$ (2)

Must show working

QUESTION FIVE (15)

a) $x^2 + y^2 - 4x + 10y - 7 = 0$

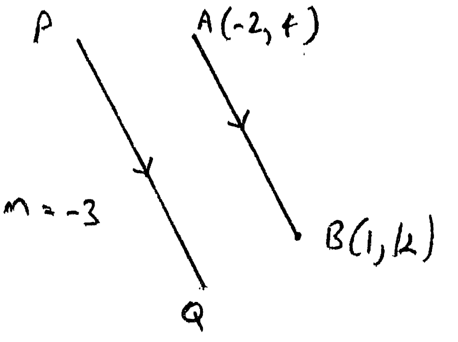
$x^2 - 4x + y^2 + 10y = 7$

$x^2 - 4x + 4 + y^2 + 10y + 25 = 36$ (1)

$(x-2)^2 + (y+5)^2 = 36$ (1)

Circle centre $(2, -5)$ radius 6 (3) (1)

b)



Eqn of AB

$y - 4 = -3(x+2)$

$y - 4 = -3x - 6$

$3x + y + 2 = 0$

Since $(1, k)$ lies on this line

$3 + k + 2 = 0$

$k = -5$ (2)

b) $\frac{4-k}{-2-1} = -3$

$4 - k = 9$

$k = -5$

c) $u^2w + vw - u^2x - vx$

$= w(u^2+v) - x(u^2+v)$

$= (w-x)(u^2+v)$ (2)

ii) $27 - x^3 = 3^3 - x^3$

$= (3-x)(9+3x+x^2)$ (1) (½)

d) $2\sqrt{2}(\sqrt{5} - \sqrt{2}) = 2\sqrt{10} - 4$ (1)

e) $\sqrt{500} = \sqrt{100} \times \sqrt{5}$

$= 10\sqrt{5}$ (1)

f) $x + y = 2$

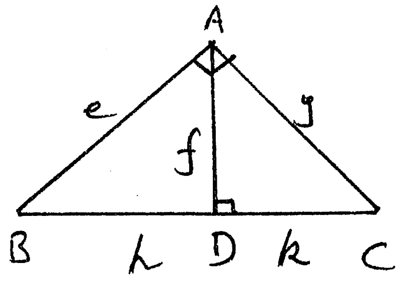
$4x - y = 13$

$5x = 15$

$x = 3$

$y = -1$ (2)

g)



$e^2 + g^2 = (h+k)^2$

$e^2 + g^2 = h^2 + 2hk + k^2$ — ①

Then

$e^2 = f^2 + h^2$

$g^2 = f^2 + k^2$

In ①

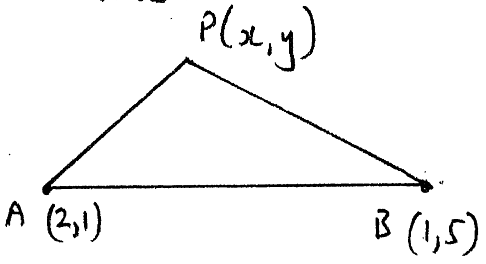
$f^2 + h^2 + f^2 + k^2 = h^2 + 2hk + k^2$

$2f^2 = 2hk$

$f^2 = hk$ (3)

QUESTION SIX.

14



PA = PB

$$\sqrt{(x-2)^2+(y-1)^2} = \sqrt{(x-1)^2+(y-5)^2} \qquad \tfrac{1}{2}$$

$$x^2-4x+4+y^2-2y+1 = x^2-2x+1+y^2-10y+25$$

$$-4x-2y+5 = -2x-10y+26$$

$$-2x+8y-21=0$$

$$2x-8y+21=0 \qquad 2$$

OR Locus of P is the perp. bisector of AB

Mid point of AB $(3/2, 3)$

Gradient of AB -4

Locus of P

$$y-3 = +\tfrac{1}{4}(x-\tfrac{3}{2})$$

$$4y-12 = x-\tfrac{3}{2}$$

$$8y-24 = 2x-3$$

$$2x-8y+21=0$$

b) Required line has eqn.

$$x+2y-6+k(3x-2y-6)=0$$

Since $(2,-1)$ lies on line.

$$2-2-6+k(6+2-6)=0$$

$$2k=6$$

$$k=3$$

Required line is

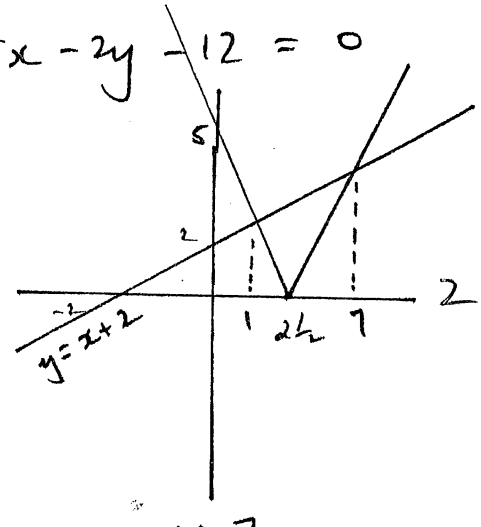
$$x+2y-6+3(3x-2y-6)=0$$

$$x+2y-6+9x-6y-18=0$$

2

ie $5x-2y-12=0$

c)



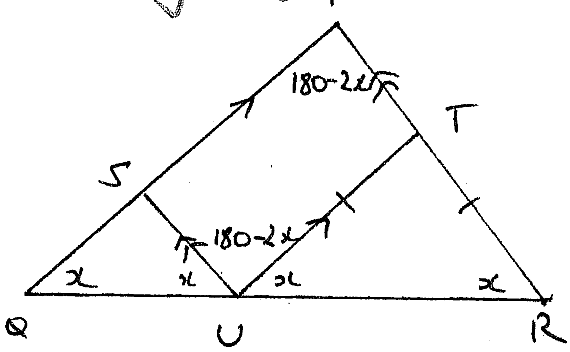
2

$x \geq 7$

and $x \leq 1$ 2

Need to use graphs (only if graphs not used.)

d)



Let $P\hat{R}Q = x$

Then $S\hat{U}Q = x$ (corresponding)

$T\hat{U}R = x$ (isos Δ)

$S\hat{U}T = 180-2x$

$Q\hat{P}R = 180-2x$ (opp angles of ‖'gram)

$P\hat{Q}R = x$ Angle Sum of Δ PRQ 3

Given

$Q\hat{P}R = 3 \times P\hat{R}Q$

$$180-2x = 3x$$

$$5x = 180°$$

$$x = 36° \qquad 2$$