SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

May 2008

Half Yearly Examination

Mathematics
Extension

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators may be used.
- All necessary working should be shown in every question

Total Marks – 60

- All Questions may be attempted
- Each Section should be handed up in a separate examination Booklet.
- Full Marks may not be awarded for careless or poorly set out work.

Examiner – *A.M.Gainford*

Section A
(Start a new booklet)

Question 1. (10 Marks)

Marks

(a) Find the exact value (as a fraction in lowest terms) of: **2**

$$\frac{\frac{3}{4}-\frac{7}{24}}{1+\frac{3}{4}\times\frac{7}{24}}$$

(b) Evaluate $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$, correct to two decimal places. **2**

(c) Find the point of intersection of the lines $3x+2y=10$ and $4x+3y=13$. **3**

(d) Simplify $x(2-y)-y(2-x)$. **1**

(e) Find x and y if $x+y+\sqrt{x-y}=10+2\sqrt{2}$. **2**

Question 2. (10 Marks)

Marks

(a) Express $3\cdot57\times10^{7}\times7\cdot64\times10^{6}$ in scientific notation, correct to three significant figures. **1**

(b) Find all the solutions of $|5x-2|=|3x+4|$. **2**

(c) Factorise: (i) $4x^2+4x-3$ **3**

(ii) $x^2-3x+2ax-6a$

(iii) $1+27a^3$

(d) Solve for x: $3x(x-3)=0$ **2**

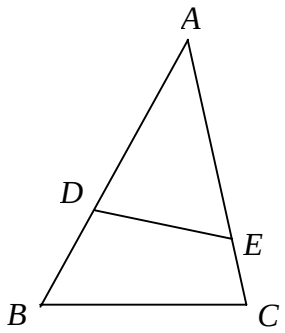
(e) Solve $2-3x\geq 8$, and graph the solution on the number line. **2**

Section B
(**Start a new booklet**.)

Question 3. (10 Marks)

Marks

(a) In the diagram at right AD = 8cm, DB = 7cm, AE = 10cm, EC = 2cm, BC = 21cm. **3**



(i) Copy the diagram to your answer sheet.

(ii) Prove that ΔADE is similar to ΔACB, and hence find the length of DE (giving full reasons).

(b) **2**

Figure A

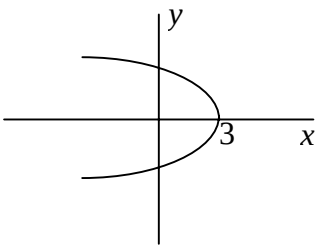
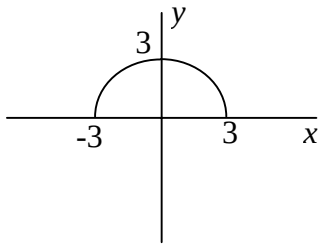


Figure B



Indicate which of these graphs represents a function, and state its domain.

(c) For the points $A(-1, 2)$ and $B(2, 4)$: **5**

(i) Find the coordinates of M, the midpoint of AB.

(ii) Write the equation of the line AB.

(iii) Find the equation of the line through M perpendicular to AB.

Section continued overleaf.

Question 4. (10 Marks)

		Marks
(a)	Express $\frac{5\pi}{9}$ radians in degrees.	**1**
(b)	Express $0\cdot1\dot{3}\dot{7}$ as a common fraction in lowest terms.	**1**
(c)	State the exact value of:	**2**
	(i) $\cos 135^\circ$	
	(ii) $\text{cosec}\, 330^\circ$	
(d)	Solve for x:	**6**
	(i) $x^4 - 25x^2 + 144 = 0$	
	(ii) $\frac{3}{x-1} > 3$	
	(iii) $\frac{1}{\lvert 2-x \rvert} < 2$	

Section C
(Start a new booklet)

Question 5. (10 Marks)

		Marks
(a)	Show that $\tan\theta + \cot\theta = \text{cosec}\,\theta \sec\theta$.	**1**
(b)	Find the perpendicular distance of the point $(1,-2)$ from the line $x-2y=-3$.	**1**
(c)	Find the equation of the line through the intersection of the lines $x-2y+1=0$ and $3x-y-2=0$, and passing through $(3,1)$.	**2**
(d)	Find the equation of the locus of all points equidistant from the fixed points $A(2,-1)$ and $B(-3,2)$.	**2**
(e)	On separate diagrams, sketch the graphs of the following:	**4**

(i) $y = 1 - 2^x$

(ii) $y = \dfrac{|x|}{x}$

Section continued overleaf.

Question 6. (10 Marks)

Marks

(a) Show that the function $f(x)=\dfrac{2^x+2^{-x}}{1-x^2}$ is an even function. **2**

(b) Solve the simultaneous equations: **3**

$$5a-2b+6c=3$$
$$6a+4b-4c=0$$
$$3a-4b+8c=3$$

(d) **5**

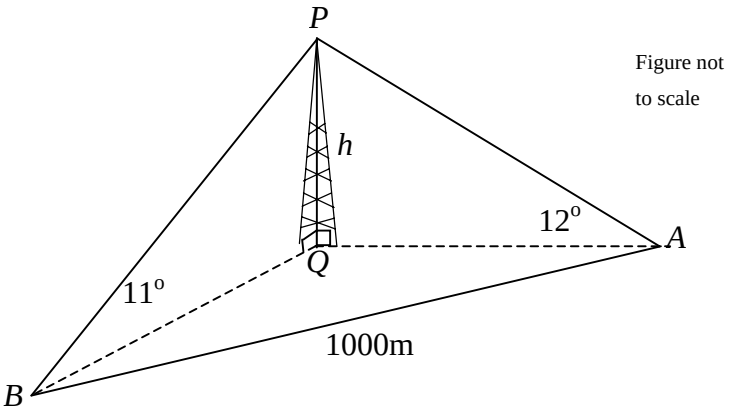


Figure not to scale

The angle of elevation of a tower PQ of height h metres at a point A due east of it is 12^o. From another point B, the bearing of the tower is 051^oT and the angle of elevation is 11^o. The points A and B are 1000 metres apart and on he same level as the base Q of the tower.

(i) Show that $\angle AQB = 141^O$.

(ii) Consider the triangle APQ and show that $AQ = h\tan 78^O$.

(iii) Find a similar expression for BQ.

(iv) Use the cosine rule in the triangle AQB to calculate h to the nearest metre.

This is the end of the paper.

QUESTION 1

(a) $\frac{11}{24} \div \frac{39}{32}$

$= \frac{44}{117}$

(b) 3.73

(c) ① $3x + 2y = 10$

② $4x + 3y = 13$

① $9x + 6y = 30$

② $8x + 6y = 26$

$x = 4$

① $12 + 2y = 10$

$y = -1$

$(4, -1)$

(d) $2x - xy - 2y + xy$

$2x - 2y$

(e) $x + y = 10$

$\sqrt{x-y} = 2\sqrt{2} = \sqrt{8}$

$x - y = 8$

$2x = 18$

$x = 9$

$y = 1$

QUESTION 2

(a) 2.73×10^{14}

(b) $5x - 2 = 3x + 4$

$2x = 6$

$x = 3$

or

$5x - 2 = -3x - 4$

$8x = -2$

$x = -\frac{1}{4}$

$x = 3, -\frac{1}{4}$

(c) (i) $(2x - 1)(2x + 3)$

(ii) $x(x-3) + 2a(x-3)$

$(x + 2a)(x - 3)$

(iii) $(1 + 3a)(1 - 6a + 9a^2)$

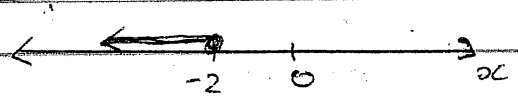
(d) $3x(x - 3) = 0$

$x = 0, \quad x = 3$

(e) $2 - 3x \geq 8$

$-3x \geq 6$

$x \leq -2$



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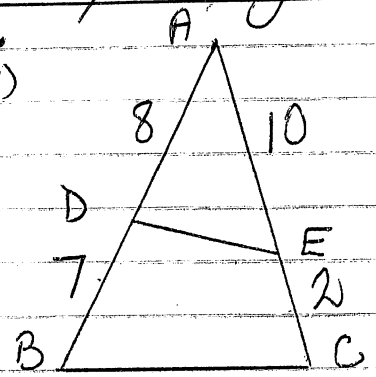
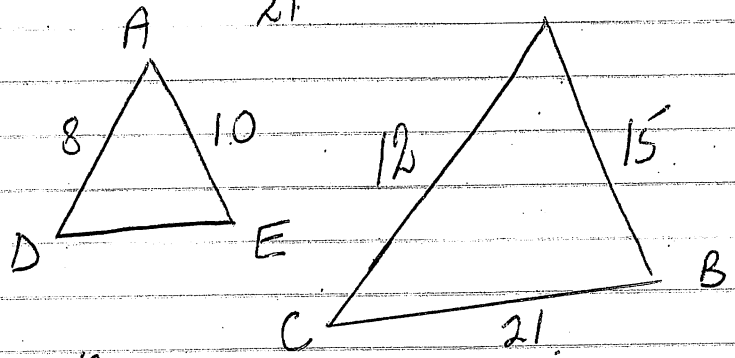
(3) (a) (i) /10



(1½)

(ii)



$\frac{AC}{AD} = \frac{12}{8} = 1.5$

$\frac{AB}{AE} = \frac{15}{10} = 1.5$

} sides in same ratio

(1½)

$D\hat{A}E = C\hat{A}B$ equal angle, same angle etc

So $\triangle ADE \,|||\, \triangle ACB$ sides in common ratio and included angle test.

$\frac{BC}{DE} = 1.5$ (sides in same ratio)

$\therefore$ so $\frac{21}{DE} = 1.5$ So $DE = 14$ cm (1)

(b) Figure B. Domain $-3 \le x \le 3$ (2)

(c) (i) $M\left(\frac{-1+2}{2}, \frac{2+4}{2}\right) = \left(\frac{1}{2}, 3\right)$ (1)

(ii) $m = \frac{4-2}{2+1} = \frac{2}{3}$ using $(y-y_1) = m(x-x_1)$

$y - 2 = \frac{2}{3}(x+1)$

$y - 2 = \frac{2}{3}x + \frac{2}{3}$

So $y = \frac{2}{3}x + 2\frac{2}{3}$.

(2)

or $3y - 6 = 2x + 2$

$0 = 2x - 3y + 8$

(iii) $m = -\frac{3}{2}$ $m(\frac{1}{2}, 3)$.

$(y-3) = -\frac{3}{2}(x - \frac{1}{2})$

$y - 3 = -\frac{3x}{2} + \frac{3}{4}$

$y = -\frac{3x}{2} + 3\frac{3}{4}$ or // $2y - 6 = -3x + \frac{3}{2}$.

$4y - 12 = -6x + 3$

$6x + 4y - 15 = 0$ (2)

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(4) (a) $\frac{5 \times 180}{9} = 100^\circ$ (1) /10

(b) let $x = 0.13737\ldots$

$100x = 13.73737$

$99x = 13.6$

$x = \frac{13.6}{99} = \frac{136}{990} = \frac{68}{495}$ (1)

(c) (i) $\cos 135^\circ = -\cos 45^\circ = -\frac{1}{\sqrt{2}}$ (1)

(ii) $\operatorname{cosec} 330^\circ = -\operatorname{cosec} 30^\circ = -\frac{1}{\sin 30^\circ} = -\frac{1}{\frac{1}{2}} = -2$ (1)

(d) (i) $x^4 - 25x^2 + 144 = 0$

let $w = x^2$

$w^2 - 25w + 144 = 0$

$(w - 9)(w - 16) = 0$

$w = 9$ and $w = 16$

So $x^2 = 9$, $x^2 = 16$.

$x = \pm 3, \pm 4$ (2)

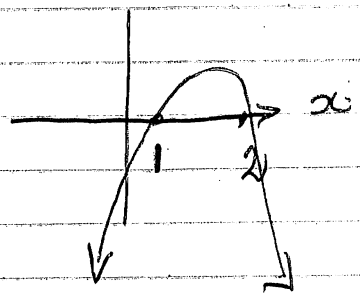
(4) (d) (ii) $\dfrac{3}{(x-1)} > 3 \qquad x \neq 1$

$$(x-1)^2 \times \frac{3}{(x-1)} > 3(x-1)^2$$

$$3(x-1) - 3(x-1)^2 > 0$$

$$3(x-1)\left[1-(x-1)\right] > 0$$

$$3(x-1)\left[2-x\right] > 0$$



soln $1 < x < 2$. (2)

(iii) $\dfrac{1}{|2-x|} < 2$

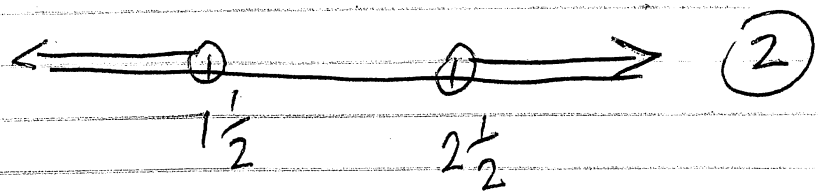
$x \neq 2$. take reciprocals.

$$|2-x| > \frac{1}{2}$$

$2-x > \frac{1}{2}$ $\qquad$ $2-x < -\frac{1}{2}$

$-x > -1\frac{1}{2}$ $\qquad$ $-x < -2\frac{1}{2}$

$x < 1\frac{1}{2}$ $\qquad$ $x > 2\frac{1}{2}$.



(2)

SECTION C

Question 5

(a) LHS $= \tan\theta + \cot\theta$ 1

$= \frac{s}{c} + \frac{c}{s}$

$= \frac{c^2 + s^2}{cs}$

$= \frac{1}{sc}$

$= \frac{1}{s} \cdot \frac{1}{c}$

$= \operatorname{cosec}\theta . \sec\theta$

$=$ RHS

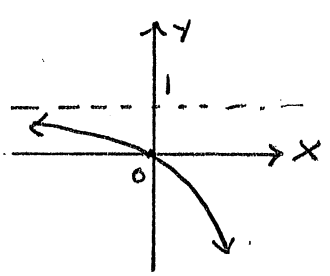
(b) $\frac{|1(1) - 2(-2) + 3|}{\sqrt{(1)^2 + (-2)^2}} = \frac{8}{\sqrt{5}}$ 1

(d) $PA = PB$

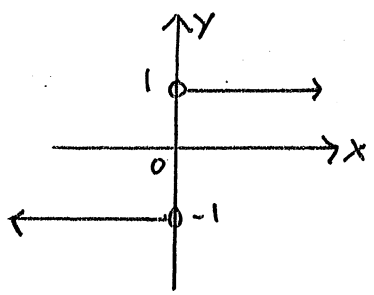
$\sqrt{(x-2)^2 + (y+1)^2} = \sqrt{(x+3)^2 + (y-2)^2}$ 2

$5x - 3y + 4 = 0$

(e) (i)



(ii)



QUESTION 6

(a) $f(x) = \frac{2^x + 2^{-x}}{1 - x^2}$

$f(-x) = \frac{2^{-x} + 2^{-(-x)}}{1 - (-x)^2}$ 2

$= \frac{2^{-x} + 2^x}{1 - x^2} = f(x)$

$\Rightarrow f(x)$ even

(b) $10a - 4b + 12c = 6$ — (A)

$6a + 4b - 4c = 0$ — (B)

$3a - 4b + 8c = 3$ — (C) 3

(A) + (B) $\Rightarrow 16a + 8c = 6$ — (D)

(B) + (C) $\Rightarrow 9a + 4c = 3$ — (E)

$\therefore a = 0, \; b = c = \frac{3}{4}$

(c) Required line

$(x - 2y + 1) + k(3x - y - 2) = 0$

$(3 - 2 + 1) + k(9 - 1 - 2) = 0$

$\Rightarrow k = -1/3$

$\therefore (x - 2y + 1) - \frac{1}{3}(3x - y - 2) = 0$

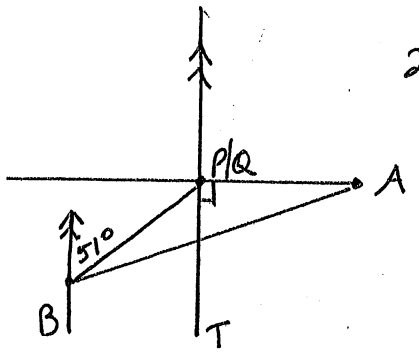
$3(x - 2y + 1) - (3x - y - 2) = 0$

ie $-5y + 5 = 0$

$y = 1$

(c)

(i)



2D DIAGRAM (TOP VIEW)

$A\hat{Q}B = 90° + B\hat{Q}T$

$= 90° + 51°$

$AQB = 141°$ 1

(ii) $\tan Q\hat{P}A = \frac{AQ}{h}$

$\tan 78° = \frac{AQ}{h}$ 1

$\Rightarrow AQ = h\tan 78°$

(iii) In ΔBPQ

$\tan 11° = \frac{h}{BQ}$ 1

$BQ = \frac{h}{\tan 11°} = h\cot 11°$

or $BQ = h\tan 79°$

(iv) $1000^2 = h^2\tan^2 79° + h^2\tan^2 78° - 2h\tan 79° h\tan 78°$ 1

$= h^2\left[\tan^2 79° + \tan^2 78° - 2\tan 79°\tan 78°\right]$

$\Rightarrow h = \frac{1000}{[\quad]^{\frac{1}{2}}}$

$h = 108$ metres 1