SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2010

YEAR 11
Half Yearly Examination

Mathematics – Continuers

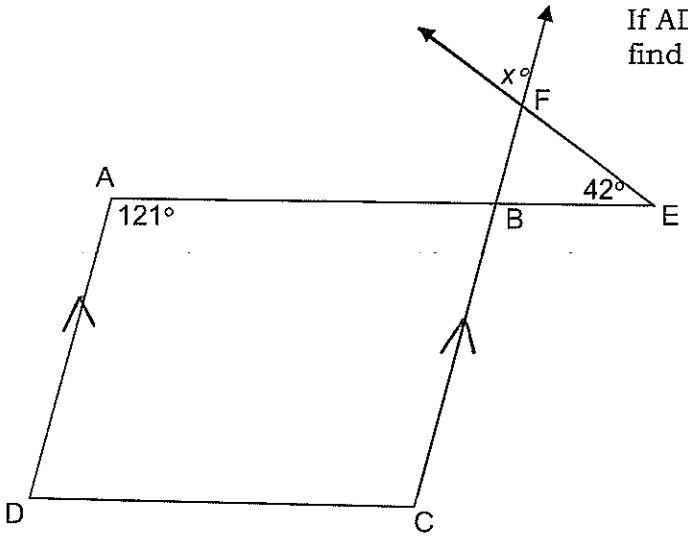
General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Marks may **NOT** be awarded for messy or badly arranged work.
- All necessary working should be shown in every question.
- Answer in simplest exact form unless otherwise instructed

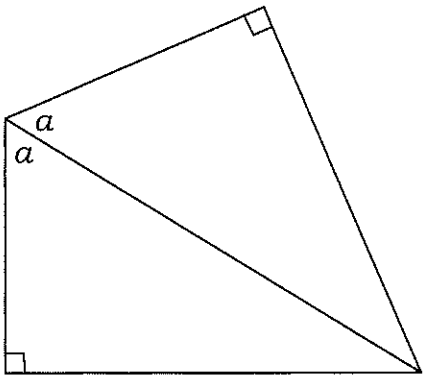
Total Marks – 100

- Attempt all questions
- All questions are NOT of equal value

Examiner: *R. Boros & T.Evans*

Number	Question	Marks
1.	Solve $\frac{1}{3}(x-2)-\frac{1}{2}(2-x)=1$	2
2.	Find the value of x if $\sqrt{x}=\sqrt{50}-\sqrt{18}$	2
3.	Solve the following quadratic equation leaving your answer in surd form $(2x-1)^2=6$	3
4.	Express $\frac{1}{\sqrt{3}-2}$ with a rational denominator	2
5.	Expand and simplify $\sqrt{(a-5)(a+5)+25}$	2
6.	The 3 legs of a triangular sailing course for the London Olympics have lengths 8km, 10km and 16km. a) Draw a sketch showing this information. b) Mark in angle α where the smallest angle should be. c) Calculate this angle α correct to the nearest minute.	 1 1 2
7.	Express $1.0\dot{2}\dot{6}$ as a rational number	2
8.	 If AD is parallel to CF, find x giving reasons.	2
9.	Find the exact value of $\tan 120^\circ \times \sin(-30^\circ)$	2
10.	Evaluate $\frac{5.3}{9.6-3.7}$ correct to 2 significant figures	2
11.	What is 0.0000309 written in scientific notation?	1
12.	How many zeros are significant in the number 0.0050309?	1

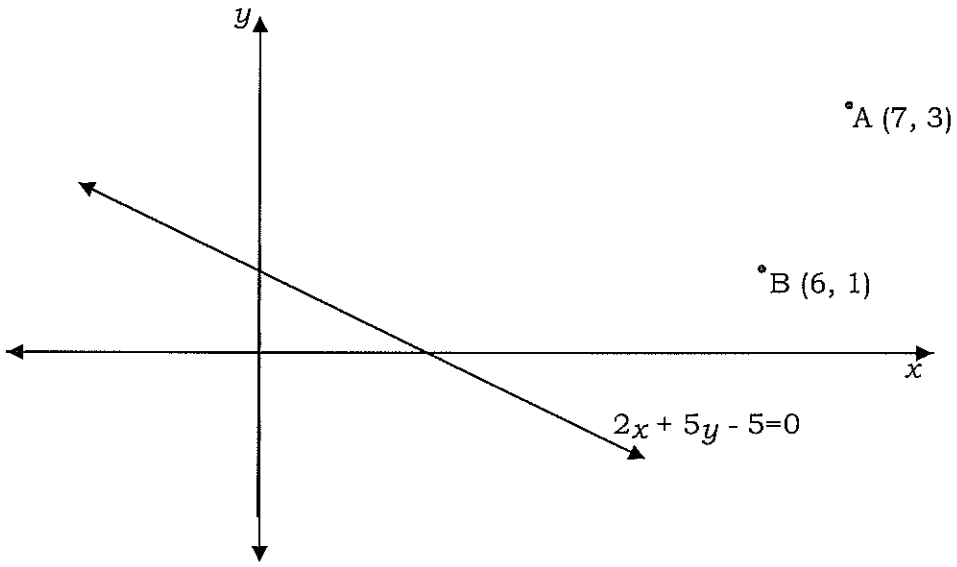
13. Which congruency test would be used to establish the congruency of these 2 triangles? 1



14. Which elements in the set are rational numbers? 2

$$\left\{\sin 30°, \pi, \sqrt{10}, 3.\dot{4}, 2^{\frac{1}{2}}\right\}$$

15.



NOT TO SCALE

2
1

1

The diagram above shows the line $2x + 5y - 5 = 0$ and the points A (7, 3) and B (6. 1).

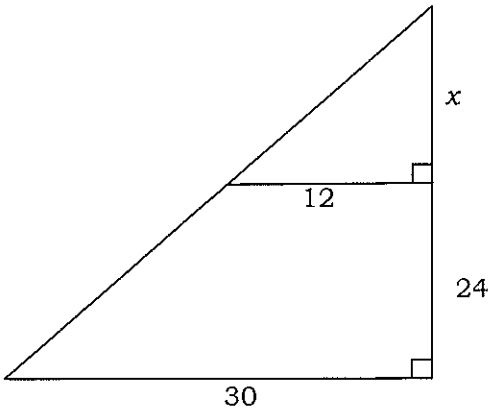
Copy the diagram onto your worksheet.

a) Find the equation of the line AB.
b) Find the coordinates of the point of intersection, P, of the line $2x + 5y - 5 = 0$ and the line AB. 5
c) Find the shortest distance from P to the line $y + 2 = 0$.

16. Fully factorise:

a) $3a^2 - 13a + 12$ 2
b) $64 - a^3$ 2
c) $ay - ax - cx + cy$ 2

17. Find x, giving reasons/working 2



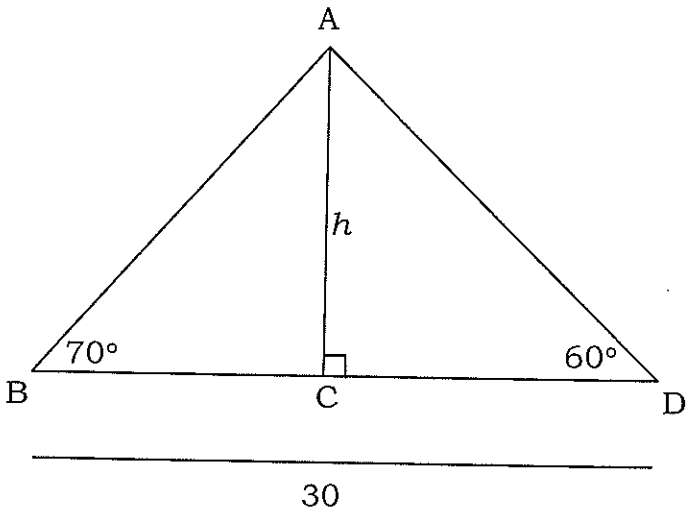
18. Solve these equations for x:

a) $x^2 = 6x$ 2
b) $3 - 2x \geq 7$ 2

19. If $f(x) = |2x - 5|$, solve $f(x) = f(4)$ 2

20. Solve for x, $|3 + 2x| \geq |x - 1|$ 3

21. Find h, correct to 2 decimal places. 3



22. Solve these simultaneous equations: 3

$$5(2x-y)=7x+1$$
$$3(3x+y)=5(x-y+12)$$

23. Each interior angle of a regular polygon is 140°. How many sides does the polygon have? 2

24. Simplify $\dfrac{5}{x-3}\div\dfrac{x^2+3x}{x^2-9}$ 3

25. Simplify $\dfrac{\cos(180^\circ-A)}{\sin(90^\circ-A)}$ 2

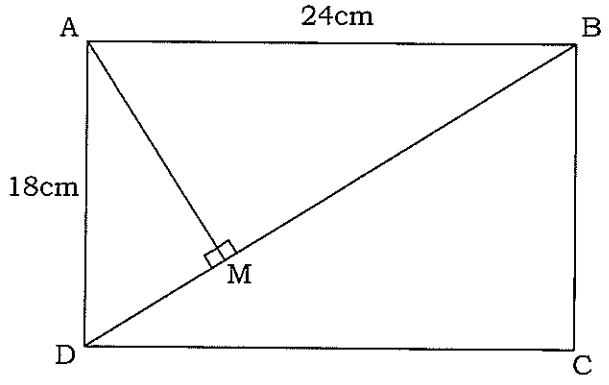
26. Given that A = $\left[\dfrac{9}{5}\right]^3$ 4

B = $\left[\dfrac{1}{25}\right]$

C = 81

Find the value of x and y if $\dfrac{A^2}{B^5C^3}=3^x\times5^y$

27.



ABCD is a rectangle with dimensions as shown.

a) Find the length of BD 2
b) Find the length of BM 2

28. A teacher is employed in 1980 at a initial salary of $27750 p.a. After each year of service she recieves an increment of $1050 until she reaches the maximum salary of $37200.

a) What is her salary after 8 years of service? 2
b) How long does she have to work until she receieves the maximum salary? 2
c) What are her total earnings for the first 10 years of service? 2

29. By considering $0.\dot{2}\dot{9}$ as a recurring decimal which is a sum on an infinite geometric series, find the equivalent fraction to $0.\dot{2}\dot{9}$. Show all working. 2

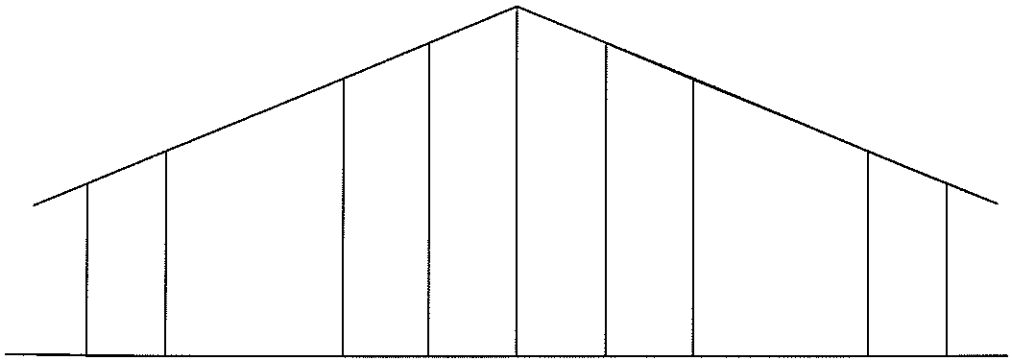
30. Calculate the interest earned on an investment of \$11750 at 9%p.a. compounded quarterly for 5 years. 2

31. An employee invests \$950 at the beginning of each year in a superannuation scheme. Assuming interest is paid at $7\frac{1}{2}\%$ p.a. on the investment, how much to the nearest \$ will this investment grow to after 40 years? 4

32. Loukia borrowed \$60000 at 18%p.a. where the interest is compounded monthly on the balance owing. If she pays off this loan in equal monthly instalments over 25 years, calculate (to the nearest cent):

a) the amount of each monthly repayment. 3
b) the total amount paid for the loan. 2
c) the total interest paid 2
d) the rate of simple interest (to 2 d.p.) equivalent to this compound interest. 2

33.



A <u>*symmetrical*</u> roof is to be supported at regular intervals by vertical posts.
The shortest posts are 'a' metres long and consecutive posts differ in length by 'd' metres. The total length of all posts is 'S' metres.
Let the number of posts be $(2n+1)$.

a) Prove that $S=dn^2+2an+a$ 3
b) If $S=64.4,\ d=0.1,\ a=2$
find:
i) The number of posts 2
ii) The length of the longest post. 2

END OF EXAMINATION

Mathematics Continuers

1. $\frac{1}{3}(x-2) - \frac{1}{2}(2-x) = 1$

$2(x-2) - 3(2-x) = 6$

$2x - 4 - 6 + 3x = 6$

$5x - 10 = 6$

$5x = 16$

$x = 16/5$ (2)

2. $\sqrt{x} = \sqrt{50} - \sqrt{18}$

$= \sqrt{25}\cdot\sqrt{2} - \sqrt{9}\cdot\sqrt{2}$

$= 5\sqrt{2} - 3\sqrt{2}$

$= 2\sqrt{2}$

$= \sqrt{8}$

$x = 8$ (2)

3. $(2x-1)^2 = 6$

$2x - 1 = \pm\sqrt{6}$

$2x = 1 \pm \sqrt{6}$

$x = \frac{1 \pm \sqrt{6}}{2}$ (3)

4. $\frac{1}{\sqrt{3}-2} \times \frac{\sqrt{3}+2}{\sqrt{3}+2}$

$= \frac{\sqrt{3}+2}{3-4}$

$= \frac{\sqrt{3}+2}{-1}$

$= -1(\sqrt{3}+2)$ (2)

5. $\sqrt{(a-5)(a+5)+25}$

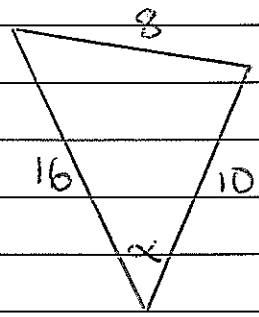
$= \sqrt{a^2 - 25 + 25}$

$= \sqrt{a^2}$

$= a$ (2)

6.

a.



(1)

b. (1)

c. $\cos x = \frac{16^2 + 10^2 - 8^2}{2 \times 16 \times 10}$

$= 0.9125$

$x = 24° 9'$ (2)

7. $x = 1.0\dot{2}\dot{6}$

$10x = 10.\dot{2}\dot{6}$

$1000x = 1026.\dot{2}\dot{6}$

$990x = 1016$

$x = \frac{1016}{990} = 1\,13/495$ (2)

8. $\angle ABC = 59°$ (coint $\angle$'s)

$\angle FBE = 59°$ (vert opp $\angle$'s)

$\therefore \angle BFE = 79°$ ($\angle$ sum $\triangle$)

$\therefore x° = 79°$ (vert opp) (2)

9. $\tan 120 \times \sin(-30°)$

$= -\sqrt{3} \times -1/2$

$= \frac{\sqrt{3}}{2}$ (2)

10. $\frac{5.3}{9.6 - 3.7}$

$= 0.898305084$

$= 0.90$ (2 sf) (2)

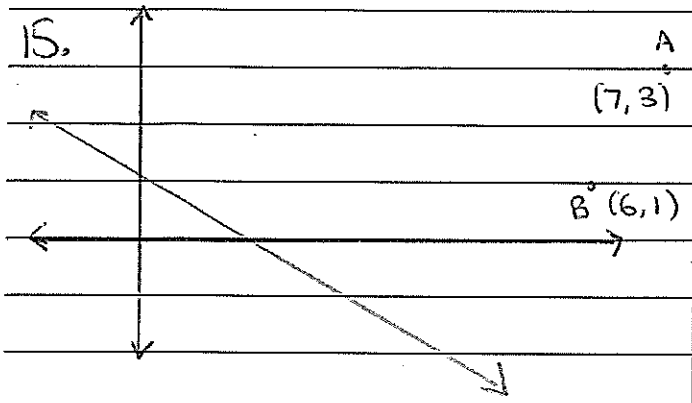
11. 3.09×10^{-5} (1)

12. 2 (1)

13. AAS (1)

14. $\sin 30$, 3.4 (2)

15.



a) $m = \frac{3-1}{7-6} = \frac{2}{1} = 2$

$y - 3 = 2(x - 7)$

$y - 3 = 2x - 14$

$y = 2x - 11$

or $2x - y - 11 = 0$. (2)

b) $2x - y - 11 = 0$ (1)

$2x + 5y - 5 = 0$ (2)

(2) − (1)

$6y + 6 = 0$

$6y = -6$

$\boxed{y = -1}$

sub into (1)

$2x + 1 - 11 = 0$

$2x = 10$

$\boxed{x = 5}$

c) $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$(x_1, y_1) = (5, -1)$

$ax + by + c = 0$ $y + 2$

$d = \frac{|0 \times 5 + 1 \times -1 + 2|}{\sqrt{0^2 + 1^2}}$

$= \frac{|0 - 1 + 2|}{\sqrt{1}}$

$= \frac{1}{1} = 1$ (1)

16 a) $3a^2 - 13a + 12$ (+36)

$\frac{(3a - 9)(3a - 4)}{3}$

$= (a - 3)(3a - 4)$ (2)

b) $64 - a^3$

$= (4^3 - a^3)$

$= (4 - a)(16 + 4a + a^2)$ (2)

c) $ay - ax - cx + cy$

$= a(y - x) + c(-x + y)$

$= (a + c)(y - x)$ (2)

17. Triangles are similar AAA ∴ sides in the same ratio.

$\frac{12}{30} = \frac{x}{24 + x}$

$12(24 + x) = 30x$

$288 + 12x = 30x$

$288 = 18x$

$\boxed{x = 16}$ (2)

18 a) $x^2 - 6x = 0$

$x(x - 6) = 0$

$x = 0, \quad x = 6$ (2)

b. $3 - 2x \geq 7$

$-2x \geq 4$

$x \leq -2$ (2)

19. $f(x) = |2x - 5|$

$f(4) = |2 \cdot 4 - 5|$

$= |8 - 5|$

$= 3$ (2)

20. $|3 + 2x| \geq |x - 1|$

$3 + 2x \geq x - 1$

$x \geq -4$

a

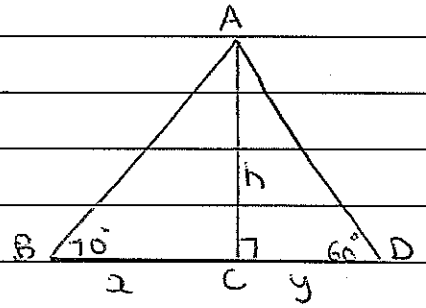
$3 + 2x \leq -x + 1$

$3x \leq -2$

$x \leq \frac{-2}{3}$

$-4 \leq x \leq -2/3$ (3)

21



$\tan 70° = h/x$

$x = \frac{h}{\tan 70}$

$\tan 60° = h/y$

$y = \frac{h}{\tan 60°}$

$x + y = 30$

$\therefore \frac{h}{\tan 70} + \frac{h}{\tan 60} = 30$

$h \tan 60 + h \tan 70 = 30 \tan 70 \tan 60$

$h(\tan 60 + \tan 70) = 30 \tan 70 \tan 60$

$h = \frac{30 \tan 70 \tan 6}{\tan 60 + \tan 70}$

$= 31.87$ (2dp) (3)

22. $10x - 5y = 7x + 1$

$3x - 5y = 1$ (1)

$9x + 3y = 5x - 5y + 60$

$4x + 8y = 60$ (2)

(1) × 4 $12x - 20y = 4$ (3)

(2) × 3 $12x + 24y = 180$ (4)

(4) − (3) $44y = 176$

$\boxed{y = 4}$

sub into (1)

$3x - 20 = 1$

$3x = 21$

$\boxed{x = 7}$

(3)

check in (2)

$4 \times 7 + 8 \times 4 = 60$ ✓

23. $140^\circ = \frac{(n-2)\times 180}{n}$

$140n = 180n - 360$

$360 = 40n$

$n = 9$ sides. (2)

24. $\frac{5}{x-3} \times \frac{(x+3)(x-3)}{x(x+3)}$

$= \frac{5}{x}$ (3)

25. $\frac{\cos(180-A)}{\sin(90-A)} = \frac{-\cos A}{\cos A}$

$= -1$ (2)

26. $\frac{\left(\left[\frac{9}{5}\right]^3\right)^2}{\left(\frac{1}{25}\right)^5 \cdot (81)^3}$

$= \frac{531\,441}{15625} \div \frac{531\,441}{9765625}$

$= \frac{531441}{15625} \times \frac{9765625}{531441}$

$= 625$

$= 3^0 \times 5^4$

$\therefore x = 0, \quad y = 4$ (4)

27a $(BD)^2 = 18^2 + 24^2$

$BD = 30$ (2)

b. Area of $\triangle ABD$

$= \frac{1}{2} \times 24 \times 18$

$= 216$

So $216 = \frac{1}{2} \times 30 \times AM$

$AM = 14.4$

$\therefore 24^2 = 14.4^2 + BM^2$

$BM = 19.2$ (2)

28.a. $a = 27750$

$d = 1050$

$n = 8$

$S_n = a + (n-1)d$

$= 27750 + (7) \times 1050$

$= \$35100$ (2)

b. $37200 = 27750 + (n-1) \times 1050$

$9450 = (n-1)1050$

$n - 1 = 9$

$\therefore n = 10$ years (2)

c. $S_n = \frac{n(a+l)}{2}$

$= \frac{10(27750 + 37200)}{2}$

$= \$324\,750$ (2)

29. $0.\dot{2}\dot{9} = 0.29 + 0.0029 + 0.000029 + \ldots$

$r = \frac{1}{100}, \quad a = 0.29$

$S_\infty = \frac{a}{1-r}$

$= \frac{0.29}{1 - 1/100}$

$= \frac{0.29}{0.99}$

$= \frac{29}{99}$ (2)

30. $A = P(1 + r\%)^n$

$= 11750\left(1 + \frac{2.25}{100}\right)^{20}$

$= \$18335.98$

∴ Interest is $\$6585.98$ (2)

31. Year 1 $A = 950(1 + 7.5\%)^{40}$

Year 2 $A = 950(1 + 7.5\%)^{39}$

⋮

Year 40 $A = 950(1 + 7.5\%)^{1}$

total $= 950(1.075)^1 + 950(1.075)^2 + \ldots\ldots + 950(1.075)^{40}$

$= 950(1.075)\left[1 + 1.075^1 + \ldots + 1.075^{39}\right]$ 33 a $S_n = \frac{n}{2}(2a + (n-1)d) \times 2 \div n$

Super $= 950(1.075) \times \frac{1.075^{40} - 1}{0.075}$

$= \$232,086$ (nearest $) (4)

$= n(2a + nd - d) \div n$

$= 2a + nd - d + a + nd$

$= dn^2 + 2a - d$ (3)

32. $60000

18% pa = 1.5% p month

300 months.

a.

$A_1 = 60000(1 + 0.015) - M$

$= 60000(1.015) - M$

$A_2 = 60000(1.015)^2 - 1.015M - M$

$= 60000(1.015)^2 - M(1 + 1.015)$

$A_{300} = 60000(1.015)^{300} - M(1 + 1.015 + \ldots\ldots + 1.015^{299})$

$M = \frac{60000(1.015)^{300}}{1 + 1.015 + \ldots\ldots + 1.015^{299}}$

$= \frac{60000(1.015)^{300}}{\frac{1.015^{300} - 1}{0.015}}$

$= \$910.46$ (3)

b. $\$910.46 \times 300 = \$273138$ (2)

c. $\$273138 - \$60000 = \$213138$ (2)

d. $\$213138 = 60000 \times \frac{r}{100} \times 25$

$r = 14.21\%$ p.a flat. (2)

b) $64.4 = 0.1n^2 + 2 \times 2n + 2$

$62.4 = 0.1n^2 + 4n$

$n^2 + 40n - 624 = 0$

$n = \frac{-40 \pm \sqrt{1600 - 4 \times 1 \times -624}}{2}$

$= \frac{-40 \pm 64}{2}$, +ve case

$n = 12$ (2)

c) longest post $a + nd = 2 \times 12 \times 0.1$

$= 3.2\text{m}$ (2)