SYDNEY GRAMMAR SCHOOL

2013 Half-Yearly Examination

# FORM V
# MATHEMATICS EXTENSION 1

**Wednesday 15th May 2013**

**General Instructions**

- Writing time — 2 hours
- Write using black or blue pen.
- Board-approved calculators and templates may be used.

**Total — 100 Marks**

- All questions may be attempted.

**Section I – 9 Marks**

- Questions 1–9 are of equal value.
- Record your solutions to the multiple choice on the sheet provided.

**Section II – 91 Marks**

- Questions 10–16 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

**Collection**

- Write your name, class and master on each booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Place your multiple choice answer sheet inside the answer booklet for Question Ten.
- Write your name and master on this question paper and submit it with your answers.

| | | | |
|---|---|---|---|
| 5A: DNW | 5B: PKH | 5C: RCF | 5D: BDD |
| 5E: KWM | 5F: FMW | 5G: LRP | 5H: TCW |

**Checklist**

- SGS booklets — 7 per boy
- Multiple choice answer sheet
- Candidature — 152 boys

**Examiner**

RCF

**SECTION I - Multiple Choice**

**Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.**

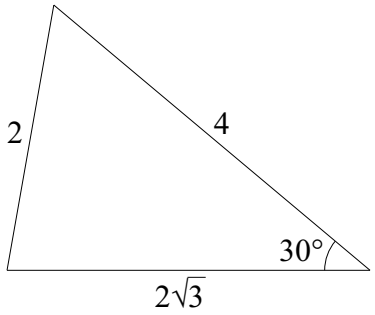
---

**QUESTION ONE**

$(2\sqrt{3}+3\sqrt{2})^2$ simplifies to:

(A) 30 (B) $30+6\sqrt{6}$ (C) $4\sqrt{3}+9\sqrt{2}$ (D) $30+12\sqrt{6}$

**QUESTION TWO**




The area of the triangle drawn above is:

(A) $4\,u^2$ (B) $2\sqrt{3}\,u^2$ (C) $4\sqrt{3}\,u^2$ (D) $2\sqrt{6}\,u^2$

**QUESTION THREE**

The expression $\cot x \times \sec x$ simplifies to:

(A) $\sin x$ (B) $\operatorname{cosec} x$ (C) $\cos x$ (D) $\sec x$

**QUESTION FOUR**

Which of the following is NOT an even function?

(A) $f(x)=\cos x$ (B) $f(x)=\dfrac{1}{x}$

(C) $f(x)=x^2+3$ (D) $f(x)=-\sqrt{4-x^2}$

**QUESTION FIVE**

When $2\log a+\log b+\log b$ is simplified, the result is:

(A) $2\log(ab)$ (B) $\log(2a^2b)$ (C) $\log(a^2+2b)$ (D) $\log(2a+2b)$

**Exam continues next page ...**

**QUESTION SIX**

The derivative of $\sqrt[3]{x}$ is:

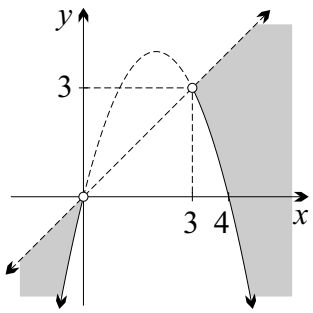
(A) $3x^{-\frac{2}{3}}$ (B) $\frac{1}{3}x^{-\frac{1}{3}}$ (C) $\frac{1}{3}x^{-\frac{2}{3}}$ (D) $\frac{1}{\sqrt[3]{x}}$

**QUESTION SEVEN**

The perpendicular distance from the point $(-1, 2)$ to the line $2x - 3y + 4 = 0$ is:

(A) $\frac{4\sqrt{13}}{13}$ (B) $0$ (C) $\frac{8\sqrt{13}}{13}$ (D) $-\frac{4\sqrt{13}}{13}$

**QUESTION EIGHT**




Which of the following pairs of inequalities best describes the shaded region above?

(A) $y - x \leq 0$ and $y \geq 4x - x^2$ (B) $y - x < 0$ and $y < 4x - x^2$

(C) $y - x > 0$ and $y \leq 4x - x^2$ (D) $y - x < 0$ and $y \geq 4x - x^2$

**QUESTION NINE**

The sum of the first $n$ terms of the geometric sequence 3, $-9$, 27 ... is given by:

(A) $\frac{3}{4}(1 - 3^n)$ (B) $\frac{3}{4}\left(1 - (-3)^n\right)$

(C) $\frac{3}{4}\left((-3)^n - 1\right)$ (D) $\frac{3}{4}(3^n - 1)$

End of Section I

**Exam continues overleaf ...**

**SECTION II - Written Response**

**Answers for this section should be recorded in the booklets provided.**

**Show all necessary working.**

**Start a new booklet for each question.**

---

**QUESTION TEN** (13 marks) Use a separate writing booklet. **Marks**

(a) Evaluate:

(i) $|-4|-|6|$ **1**

(ii) $\log_3 81$ **1**

(b) Determine the exact value of $\cos 150^\circ$. **2**

(c) Simplify fully:

(i) $\dfrac{2x+4}{6x+12}$ **1**

(ii) $\sqrt{x}\left(\sqrt{x}-\dfrac{1}{\sqrt{x}}\right)$ **1**

(d) Rationalise the denominator of $\dfrac{4}{3-\sqrt{2}}$. **2**

(e) Consider the points $A(-3,5)$ and $B(7,-2)$ in the number plane.

(i) Determine the co-ordinates of the midpoint of $AB$. **1**

(ii) Find the exact length of the interval $AB$. **1**

(iii) Calculate the gradient of the interval $AB$. **1**

(f) (i) Given an arithmetic sequence with first term 4 and common difference 3, what is the tenth term? **1**

(ii) Given a geometric sequence with first term 3 and common ratio $\frac{1}{2}$, what is the limiting sum? **1**

**Exam continues next page ...**

**QUESTION ELEVEN** (13 marks) Use a separate writing booklet. **Marks**

(a) (i) State the co-ordinates of the centre of the circle with equation $(x-3)^2+(y+2)^2=16$. **1**

(ii) Find the co-ordinates of the vertex of the parabola with equation $y=(x-4)(x+2)$. **2**

(b) Differentiate:

(i) $3x^2+4x$ **1**

(ii) $x^{-3}$ **1**

(iii) $3\sqrt{x}$ **1**

(c) Solve:

(i) $3-x>5$ **1**

(ii) $|x+2|=7$ **2**

(iii) $x-2x^2=0$ **2**

(iv) $x^2-4x-12\geq 0$ **2**

**QUESTION TWELVE** (13 marks) Use a separate writing booklet. **Marks**

(a) Given $\sin\theta=\frac{2}{3}$ and $\cos\theta<0$, find the exact value of $\tan\theta$. **2**

(b) The fourth term of an arithmetic progression is 5 and the seventh term is 23.

(i) Find the first term and the common difference. **2**

(ii) Find the twentieth term. **1**

(iii) Find the sum of the first twenty terms. **1**

(iv) If the last term is 179, how many terms are there in the AP? **1**

(c) Solve the following trigonometric equations, for $0^\circ\leq\theta\leq 360^\circ$:

(i) $\cos\theta=\frac{1}{2}$ **1**

(ii) $\sqrt{2}\sin\theta\cos\theta=\sin\theta$ **2**

(d) In $\triangle ABC$, side $AB$ is 6 metres, $BC$ is 11 metres and $\angle BCA$ is $25^\circ$. Find the possible values for $\angle BAC$. Give your answers correct to the nearest degree. **3**

**Exam continues overleaf ...**

**QUESTION THIRTEEN** (13 marks) Use a separate writing booklet. **Marks**

(a) Let $f(x) = 2^x$.

(i) Write down the equation of the function obtained by translating $y = f(x)$ two units to the right. **1**

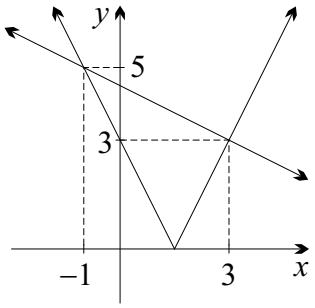
(ii) Write down the equation of the function obtained by reflecting $y = f(x)$ in the $x$-axis. **1**

(b) State the natural domain of:

(i) $y = \dfrac{1}{2x-3}$ **1**

(ii) $y = -\sqrt{16-x^2}$ **1**

(c) **1**




The graph above shows the absolute value function $y = |2x-3|$ and the straight line $y = \dfrac{9-x}{2}$. Use the graph to solve the inequation $|2x-3| \geq \dfrac{9-x}{2}$.

(d) Consider the cubic function $y = 3x - x^3$.

(i) Determine the gradient of the tangent to the curve at the point where $x = 2$. **2**

(ii) Hence find the equation of the normal to the curve at the point where $x = 2$. **2**

(e) Differentiate:

(i) $y = \left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)\left(\sqrt{x} - \dfrac{1}{\sqrt{x}}\right)$ **2**

(ii) $y = \dfrac{x^4 + 2x^2 + 4}{3x^3}$ **2**

**Exam continues next page ...**

**QUESTION FOURTEEN** (13 marks) Use a separate writing booklet. **Marks**

(a) State the range of $y = \sec x$. **1**

(b) Let the points $P$ and $Q$ be $(-2, 5)$ and $(7, -13)$ respectively. Find the co-ordinates of the point $R$ which divides the interval $PQ$ in the ratio $4 : 5$. **2**

(c) Solve the following trigonometric equations, for $0^\circ \leq \theta \leq 360^\circ$. Approximate your answers to the nearest degree where necessary.

(i) $3\cos^2\theta = 2(1 - \sin\theta)$ **3**

(ii) $\cos(2\theta - 50^\circ) = \dfrac{1}{\sqrt{2}}$ **3**

(d) Solve the inequation $\dfrac{x}{x-3} \leq 2$. **4**

**QUESTION FIFTEEN** (13 marks) Use a separate writing booklet. **Marks**

(a) Consider the function $f(x) = x^2 - 3x$.

(i) Find $f(x + h)$. **1**

(ii) Hence differentiate $f(x)$ from first principles. **2**

(b) Consider the circle $(x-1)^2 + (y-3)^2 = 4$ and the straight line $2x - y + k = 0$, where $k$ is a constant.

(i) Use the perpendicular distance formula to get an expression for the distance from the line to the centre of the circle. **1**

(ii) Determine the range of values of $k$ for which the line intersects the circle. **2**

(c) Prove the following trigonometric identities:

(i) $\cot\theta + \tan\theta = \sec\theta \operatorname{cosec}\theta$ **2**

(ii) $\dfrac{\operatorname{cosec}\alpha}{\operatorname{cosec}\alpha - \sin\alpha} = \sec^2\alpha$ **2**

(d) In a certain geometric sequence, the sum of the first two terms is 9 and the sum of the first three terms is 21. Find the possible values of the common ratio. **3**

**Exam continues overleaf ...**

**QUESTION SIXTEEN** (13 marks) Use a separate writing booklet. **Marks**

(a) Consider the rational function $f(x) = \dfrac{x^2 + 2x - 3}{2x^2 - x - 3}$.

(i) Find the $x$-intercepts and $y$-intercept of the graph of $y = f(x)$. **2**

(ii) Find the equations of any vertical asymptotes, showing clear working. **1**

(iii) Find the equation of the horizontal asymptote, showing clear working. **1**

(iv) Find the $x$ value where the graph crosses the horizontal asymptote. **1**

(v) Carefully sketch the function, showing these features. **1**

(b) Find $x$ given $\displaystyle\sum_{n=1}^{2x}\left(\sqrt[4]{n} + \sqrt[4]{n+1}\right)\left(\sqrt[4]{n+1} - \sqrt[4]{n}\right) = 2$. **3**

(c) Inside a large semicircle with centre $O$ and diameter $AOB$ of length $2R$, a smaller semicircle is drawn with diameter $AO$ and centre $P$. A small circle is drawn with centre $Q$ and radius $r$ which is tangent to the large semicircle at $T$ and the small semicircle at $S$. Let $\angle POQ = \theta$.

(i) Carefully draw a diagram of the situation described. **1**

(ii) What can be said about the points $O$, $Q$ and $T$? Justify your answer. **1**

(iii) Use the cosine rule to show that **2**

$$\frac{r}{R} = \frac{1 - \cos\theta}{3 - \cos\theta}.$$

End of Section II

## END OF EXAMINATION

SYDNEY GRAMMAR SCHOOL

NAME: .....

CLASS: .....MASTER: .....

**2013**
**Half-Yearly Examination**
**FORM V**
**MATHEMATICS EXTENSION 1**
**Wednesday 15th May 2013**

- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

**Question One**

A ○ B ○ C ○ D ○

**Question Two**

A ○ B ○ C ○ D ○

**Question Three**

A ○ B ○ C ○ D ○

**Question Four**

A ○ B ○ C ○ D ○

**Question Five**

A ○ B ○ C ○ D ○

**Question Six**

A ○ B ○ C ○ D ○

**Question Seven**

A ○ B ○ C ○ D ○

**Question Eight**

A ○ B ○ C ○ D ○

**Question Nine**

A ○ B ○ C ○ D ○

# Extension One 2013 Half Yearly

**Question One**

$(2\sqrt{3}+3\sqrt{2})(2\sqrt{3}+3\sqrt{2}) = 12+18+12\sqrt{6}$
$= 30+12\sqrt{6}$ (D) ✓

**Question Two**

$A = \frac{1}{2}ab\sin C = \frac{1}{2}\times 4\times 2\sqrt{3}\times \sin 30° = 4\sqrt{3}\times\frac{1}{2} = 2\sqrt{3}\ u^2$ (B) ✓

**Question Three**

$\cot x\sec x = \frac{\cos x}{\sin x}\times\frac{1}{\cos x}$
$= \frac{1}{\sin x}$
$= \operatorname{cosec} x$ (B) ✓

**Question Four**

$f(x) = \frac{1}{x}$ has ODD symmetry (B) ✓

**Question Five**

$2\log a + \log b + \log b = \log a^2 + \log b^2$
$= \log a^2b^2$
$= 2\log ab$ (A) ✓

**Question Six**

$y = x^{\frac{1}{3}}$
$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$ (C) ✓

**Question Seven**

$2x-3y+4=0$
$(-1,2)$

$p = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$
$= \frac{|(-2)+(-6)+4|}{\sqrt{4+9}}$
$= \frac{4}{\sqrt{13}}$
$= \frac{4\sqrt{13}}{13}$ (A) ✓

**Question 8**

$y<x$ ie $y-x<0$ and $y \geq 4x-x^2$ (D) ✓

**Question 9**

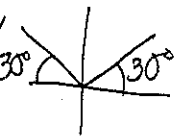
GP $a=3$ $r=(-3)$

$S_n = \frac{a(1-r^n)}{1-r} = \frac{3(1-(-3)^n)}{1-(-3)}$
$= \frac{3}{4}(1-(-3)^n)$ (B) ✓

**Question 10**

(a) (i) $|-4|-|-16| = 4-6$
$= (-2)$ ✓ (ii) $\log_3 81 = 4$ ✓

(b) $\cos 150$
$= -\cos 30°$ ✓
$= -\frac{\sqrt{3}}{2}$ ✓




(c) (i) $\frac{2(x+2)}{6(x+2)} = \frac{1}{3}$ ✓ (ii) $\sqrt{x}\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right) = x-1$ ✓

(d) $\frac{4}{3-\sqrt{2}}\times\frac{(3+\sqrt{2})}{(3+\sqrt{2})}$ ✓ $= \frac{12+4\sqrt{3}}{9-2}$
$= \frac{12+4\sqrt{3}}{7}$ ✓ or $\frac{4(3+\sqrt{2})}{7}$

(e) $A(-3,5)$ $B(7,-2)$

(i) $M\left(\frac{-3+7}{2}, \frac{5+(-2)}{2}\right) = \left(2, \frac{3}{2}\right)$ ✓

(ii) $AB = \sqrt{10^2+7^2}$
$= \sqrt{149}\ u$ ✓

(iii) $m_{AB} = \frac{5-(-2)}{(-3)-7}$
$= \frac{7}{-10}$
$= \left(-\frac{7}{10}\right)$ ✓

(c) (i) AP $a=7$ $d=-3$ $t_{10}=a+9d$ $=4+27$ $=31$ ✓ tenth term is 31.

(ii) GP $a=3$ $r=\frac{1}{2}$ $S_\infty=\frac{a}{1-r}=\frac{3}{1-\frac{1}{2}}=6$ ✓ limiting sum is 6.

## Question Eleven

(a) (i) $(x-3)^2+(y+2)^2=16$ ✓
Circle
Centre $(3,-2)$ ✓

(ii) $y=(x-4)(x+2)$
x-intercepts are $(4,0)$ and $(-2,0)$
x co-ord of vertex is $x=1$ ✓
y co-ord $y=-3\times3=-9$ ✓
Vertex $(1,-9)$

b) (i) $y=3x^2+4x$
$\frac{dy}{dx}=6x+4$ ✓

(ii) $y=x^{-3}$
$\frac{dy}{dx}=-3x^{-4}$ ✓

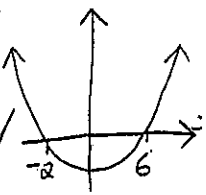
(iii) $y=3x^{\frac{1}{2}}$
$\frac{dy}{dx}=\frac{3}{2}x^{-\frac{1}{2}}$ ✓

c) (i) $3-x>5$
$(+x)\quad(+x)$
$3>x+5$
$(-5)\quad(-5)$
$(-2)>x$ ✓

(ii) $|x+2|=7$
$x+2=7$ ✓ or $x+2=(-7)$
$x=5$ ✓ or $x=(-9)$ ✓

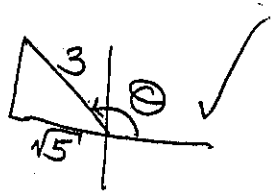
(iii) $x-2x^2=0$
$x(1-2x)=0$
$x=0$ ✓ or $x=\frac{1}{2}$ ✓

(iv) $x^2-4x-12\geq0$
$(x-6)(x+2)\geq0$ ✓
$x\geq6$ or $x\leq(-2)$ ✓




## Question Twelve

a) $\sin\theta=\frac{2}{3}$ $\cos\theta<0$ $\Rightarrow$ 2nd quad ✓




$\therefore\tan\theta=\left(\frac{-2}{\sqrt{5}}\right)$ ✓

b) AP
$a+3d=5$ ①
$a+6d=23$ ②
② − ① $3d=18$
$d=6$ ✓
Sub into ① $a+18=5$
$a=(-13)$ ✓

First term is $(-13)$
Common difference is 6.

(ii) $t_{20}=a+19d$
$=(-13)+19\times6$
$=(-13)+114$ ✓
$=101$

(iii) $S_n=\frac{n}{2}(2a+(n-1)d)$ or $S_n=\frac{n}{2}(a+l)$
$=10(-26+19\times6)$ $=10(-13+101)$
$=10(114-26)$ $=880$
$=880$ ✓

(iv) $t_n=179=a+(n-1)d$
$(-13)+6(n-1)=179$
$6n-19=179$
$6n=198$
$n=33$ ✓
There are 33 terms in AP.

12(i) $\cos\theta = \frac{1}{2}$

$\theta = 60°, 300°$ ✓

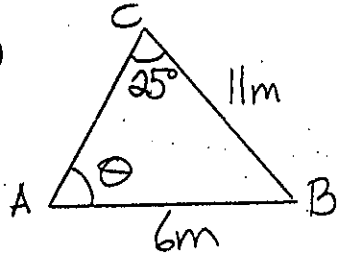
(ii) $\sqrt{2}\sin\theta\cos\theta = \sin\theta$ ✓

$\sin\theta(\sqrt{2}\cos\theta - 1) = 0$

$\sin\theta = 0$ or $\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = 0°, 180°, 360°$ or $\theta = 45°, 315°$ ✓

b)




Let $\angle BAC = \theta$

$\frac{\sin\theta}{11} = \frac{\sin 25}{6}$ ✓

$\sin\theta = \frac{11\sin 25}{6}$

$\div 0.7748\ldots$

$\theta \div 51°$ ✓ or $129°$ ✓

Note: Ambiguous Case

## Question Thirteen

a)(i) $f(x) = 2^{x-2}$ ✓ (ii) $g(x) = -2^x$ ✓

b)(i) $y = \frac{1}{2x-3}$ $\therefore 2x-3 \neq 0$

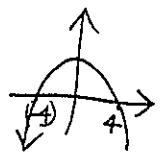
$x \neq \frac{3}{2}$ ✓

Domain $\{x : x \in \mathbb{R}, x \neq \frac{3}{2}\}$

(ii) $y = -\sqrt{16-x^2}$ $\therefore 16-x^2 \geq 0$

(or recognise lower semicircle) $(4-x)(4+x) \geq 0$ ✓

$(-4) \leq x \leq 4$ ✓




Domain $\{x : x \in \mathbb{R}, -4 \leq x \leq 4\}$

(c) Absolute Value above straight line

$x \geq 3$ or $x \leq (-1)$ ✓

(d)(i) $y = 3x - x^3$

$\frac{dy}{dx} = 3 - 3x^2$ ✓

$\left(\frac{dy}{dx}\right)_{x=2} = 3 - 3(2)^2 = (-9)$ ✓ Gradient of tangent is $(-9)$

(ii) $m_{Norm} = \frac{1}{9}$

$y = 3(2) - 2^3$ ✓

$= 6 - 8$

$= (-2)$

Eqn of normal

$y - (-2) = \frac{1}{9}(x-2)$

$y + 2 = \frac{1}{9}(x-2)$

$9y + 18 = x - 2$

$0 = x - 9y - 20$ ✓

(e)(i) $y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$

$= x - \frac{1}{x}$

$= x - x^{-1}$ ✓

$\frac{dy}{dx} = 1 + x^{-2}$

$= 1 + \frac{1}{x^2}$ ✓

$= \frac{x^2+1}{x^2}$

(ii) $y = \frac{x^4 + 2x^2 + 4}{3x^3}$

$= \frac{1}{3}x + \frac{2}{3}x^{-1} + \frac{4}{3}x^{-3}$ ✓

$\frac{dy}{dx} = \frac{1}{3} - \frac{2}{3}x^{-2} - 4x^{-4}$

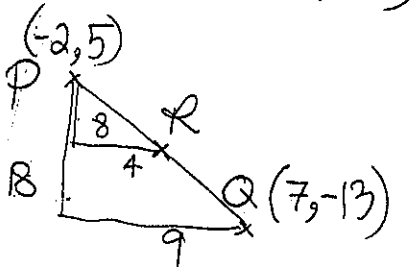
$= \frac{1}{3} - \frac{2}{3x^2} - \frac{4}{x^4}$ ✓

$= \frac{x^4 - 2x^2 - 4}{3x^4}$

## Question Fourteen

a) $y = \sec x$ $y \geq 1$ or $y \leq (-1)$ ✓

b) $P(-2,5)$ $Q(7,-13)$ in ratio 4:5




Point R.

$x = \frac{5(-2)+4\times 7}{4+5}$ $\quad y = \frac{5\times 5+4\times(-1}{4+5}$

$= \frac{18}{9}$ $\quad = \frac{25-52}{9}$

$= 2$ ✓ $\quad = -3$ ✓

$R(2,-3)$

(d) $\frac{x}{x-3} \leq 2$, $(x \neq 3)$

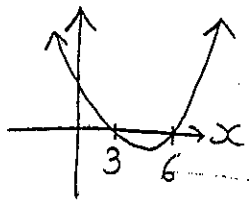
$(x-3)^2$ ✓

$x(x-3) \leq 2(x-3)^2$

$0 \leq 2(x-3)^2 - x(x-3)$

$0 \leq (x-3)[2x-6-x]$ ✓

$0 \leq (x-3)(x-6)$ ✓




∴ Solns $x < 3$ or $x \geq 6$ ✓

(NB $x \neq 3$)

c) (i) $3\cos^2\theta = 2(1-\sin\theta)$ $\quad 0° \leq \theta \leq 360°$

$3\cos^2\theta + 2\sin\theta - 2 = 0$

$3(1-\sin^2\theta) + 2\sin\theta - 2 = 0$

$-3\sin^2\theta + 2\sin\theta + 1 = 0$ ✓

$3\sin^2\theta - 2\sin\theta - 1 = 0$

$(3\sin\theta + 1)(\sin\theta - 1) = 0$

$\therefore \sin\theta = (-\frac{1}{3})$ or $\sin\theta = 1$ ✓

$\theta = 199°, 341°$ $\quad \theta = 90°$ ✓

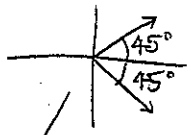
(ii) $\cos(2\theta - 50°) = \frac{1}{\sqrt{2}}$ $\quad 0° \leq \theta \leq 360°$

$0° \leq 2\theta \leq 720°$

$2\theta - 50° = -45°, 45°, 315°$ ✓ $\quad -50° \leq 2\theta - 50° \leq 670°$ ✓

$405°$, ~~675°~~

$2\theta = 5°, 95°, 365°, 455°$




$\theta = 2\frac{1}{2}°, 47\frac{1}{2}°, 182\frac{1}{2}°, 227\frac{1}{2}°$ ✓

## Question Fifteen

(a) $f(x) = x^2 - 3x$

(i) $f(x+h) = (x+h)^2 - 3(x+h)$

$= x^2 + 2xh + h^2 - 3x - 3h$ ✓

(ii) $f'(x) = \lim_{h\to 0}\left\{\frac{f(x+h)-f(x)}{(x+h)-x}\right\}$

$= \lim_{h\to 0}\left\{\frac{(x^2+2xh+h^2-3x-3h)-(x^2-3x)}{h}\right\}$ ✓

$= \lim_{h\to 0}\left\{\frac{2xh+h^2-3h}{h}\right\}$

$= \lim_{h\to 0}\{2x-3+h\}$ ✓

$= 2x-3$

2) $(x-1)+(y-3)=1$

Centre $(1,3)$ $r=2$

$y=2x+k$

$\therefore 2x-y+k=0$

$a=2, b=(-1), c=k$

$$p=\frac{|2\times 1+3\times(-1)+k|}{\sqrt{4+1}}$$

$$=\frac{|k-1|}{\sqrt{5}} \quad \checkmark$$

For intersection or tangency $p \le r$ ✓

$$\frac{|k-1|}{\sqrt{5}} \le 2 \quad \checkmark$$

$$|k-1| \le 2\sqrt{5}$$

$$-2\sqrt{5} \le k-1 \le 2\sqrt{5}$$

$$1-2\sqrt{5} \le k \le 2\sqrt{5}+1 \quad \checkmark$$

3)(i) $\cot\theta+\tan\theta \equiv \sec\theta\operatorname{cosec}\theta$

$$LHS=\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}$$

$$=\frac{\cos^2\theta}{\sin\theta\cos\theta}+\frac{\sin^2\theta}{\sin\theta\cos\theta} \quad \checkmark$$

$$=\frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta}$$

$$=\frac{1}{\sin\theta\cos\theta} \quad \checkmark$$

$$=\operatorname{cosec}\theta\sec\theta$$

$$=RHS$$

(ii) $\dfrac{\operatorname{cosec}\alpha}{\operatorname{cosec}\alpha-\sin\alpha} \equiv \sec^2\alpha$

$$LHS=\frac{\frac{1}{\sin\alpha}}{\frac{1}{\sin\alpha}-\sin\alpha}\ \frac{(\times\sin\alpha)}{(\times\sin\alpha)} \quad \checkmark$$

$$=\frac{1}{1-\sin^2\alpha}$$

$$=\frac{1}{\cos^2\alpha} \quad \checkmark$$

$$=\sec^2\alpha$$

$$=RHS$$

(4) $a+ar=9$ ①

$a+ar+ar^2=21$ ②

②−① $ar^2=12$ ✓ ③

③÷① $\dfrac{ar^2}{a+ar}=\dfrac{12}{9}$

$$\frac{r^2}{1+r}=\frac{4}{3}$$

$3r^2=4r+4$ ✓

$3r^2-4r-4=0$

$(3r+2)(r-2)=0$

$r=(-\frac{2}{3})$ or $r=2$ ✓

Check

if $r=(-\frac{2}{3})$

① $a-\frac{2a}{3}=9$

$\frac{a}{3}=9$

$a=27$

GP $27, -18, 12\ldots$ ✓(works)

if $r=2$

③ $4a=12$

$a=3$

GP $=3, 6, 12\ldots$ ✓(works)

(a) $f(x) = \dfrac{x^2+2x-3}{2x^2-x-3}$

$= \dfrac{(x+3)(x-1)}{(2x-3)(x+1)}$

(i) $x$-ints $f(x)=0$ $x=-3$ or $x=1$ ie $(-3,0), (1,0)$ ✓

$y$-ints $x=0$ $f(0) = \frac{-3}{-3} = 1$ ie $(0,1)$ ✓

(ii) $f(x) = \dfrac{x^2+2x-3}{2x^2-x-3}$ $(\div x^2)$ $(\div x^2)$

$= \dfrac{1+\frac{2}{x}-\frac{3}{x^2}}{2-\frac{1}{x}-\frac{3}{x^2}}$

as $x \to \pm\infty$ $f(x) \to \frac{1}{2}$ Horizontal Asymptote $y = \frac{1}{2}$ ✓

(iii) Vertical Asymptotes where denominator is zero and numerator non-zero

ie $x = (-1)$ or $x = \frac{3}{2}$ ✓ Both

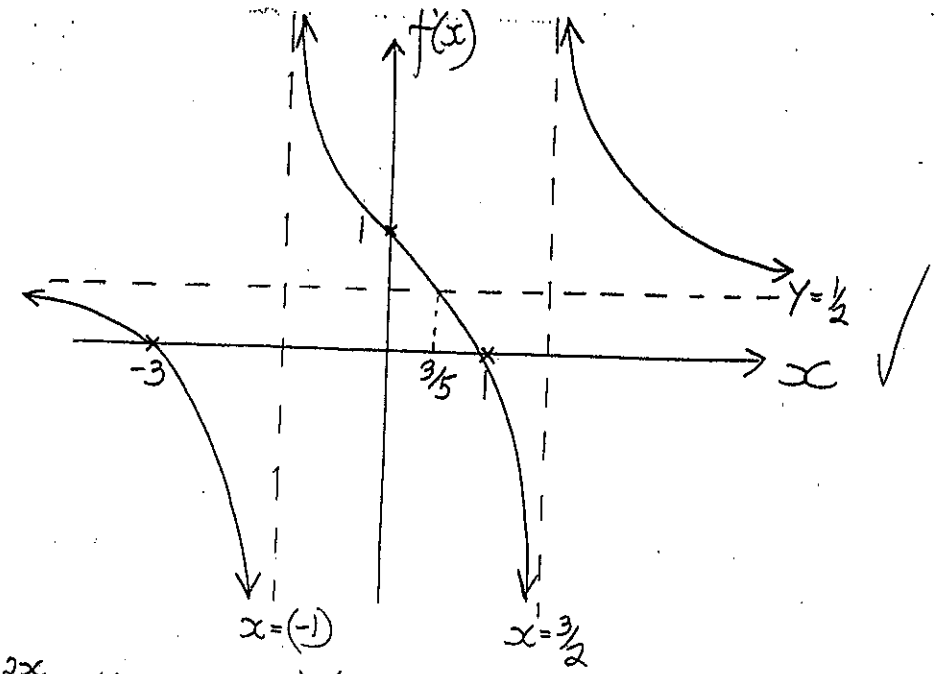
(iv) Graph crosses horizontal asymptote where $y = \frac{1}{2}$

$\therefore \frac{1}{2} = \dfrac{x^2+2x-3}{2x^2-x-3}$

$2x^2 - x - 3 = 2x^2 + 4x - 6$

$3 = 5x$

$\frac{3}{5} = x$ ✓ ie $(\frac{3}{5}, \frac{1}{2})$



✓

(b) $\sum_{n=1}^{2x} \left(\sqrt[4]{n+1} + \sqrt[4]{n}\right)\left(\sqrt[4]{n+1} - \sqrt[4]{n}\right) = 2$ Diff of 2 sqs

$\sum_{n=1}^{2x} \left(\sqrt{n+1} - \sqrt{n}\right) = 2$ ✓

$\left(\sqrt{2} - \sqrt{1}\right) + \left(\sqrt{3} - \sqrt{2}\right) + \ldots + \left(\sqrt{2x} - \sqrt{2x-1}\right) + \left(\sqrt{2x+1} - \sqrt{2x}\right) = 2$

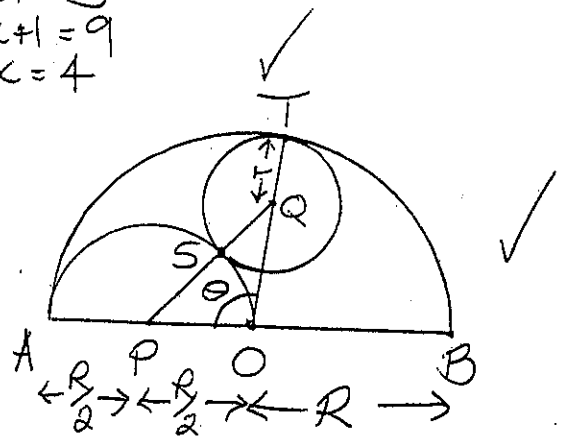
$\sqrt{2x+1} - \sqrt{1} = 2$ ✓

$\sqrt{2x+1} = 3$

$2x+1 = 9$

$x = 4$

(c) (i)



✓

(ii) Points O, Q and T are collinear since T is point of contact of common tangent of circles centred at O and Q. Similarly P, S and Q are collinear with common tangent at S. ✓

(iii) Cosine Rule in $\triangle POQ$. $PO = \frac{R}{2}$

$PQ^2 = OP^2 + OQ^2 - 2 \times OP \times OQ \times \cos\theta$ $OQ = (R-r)$

$\left(\frac{R}{2}+r\right)^2 = \left(\frac{R}{2}\right)^2 + (R-r)^2 - 2\left(\frac{R}{2}\right)(R-r)\cos\theta$ ✓ $PQ = \left(\frac{R}{2}+r\right)$

$\cancel{\frac{R^2}{4}} + Rr + \cancel{r^2} = \cancel{\frac{R^2}{4}} + R^2 - 2Rr + \cancel{r^2} - R(R-r)\cos\theta$

$R(R-r)\cos\theta = R^2 - 3Rr$

$(\div r^2)$ $(\div r^2)$

$\left(\frac{R^2}{r^2} - \frac{R}{r}\right)\cos\theta = \frac{R^2}{r^2} - \frac{3R}{r}$

Let $\frac{R}{r} = k$. $\therefore (k^2 - k)\cos\theta = k^2 - 3k$

$(\div k)$ $(\div k)$

$(k-1)\cos\theta = k - 3$

$k\cos\theta - k = \cos\theta - 3$ ✓

$k(\cos\theta - 1) = \cos\theta - 3$

$k = \frac{\cos\theta - 3}{\cos\theta - 1}$

$\frac{R}{r} = \frac{\cos\theta - 3}{\cos\theta - 1}$ $(\times -1)$ $(\times -1)$

$\frac{R}{r} = \frac{3 - \cos\theta}{1 - \cos\theta}$

$\frac{r}{R} = \frac{1 - \cos\theta}{3 - \cos\theta}$