NAME MASTER

SYDNEY GRAMMAR SCHOOL

2017 Half-Yearly Examination

# FORM V

# MATHEMATICS EXTENSION 1

**Tuesday 23rd May 2017**

**General Instructions**

- Writing time — 1 hour 30 minutes
- Write using black pen.
- Board-approved calculators and templates may be used.

**Total — 80 Marks**

- All questions may be attempted.

**Section I – 8 Marks**

- Questions 1–8 are of equal value.
- Record your answers to the multiple choice on the sheet provided.

**Section II – 72 Marks**

- Questions 9–14 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

**Collection**

- Write your name, class and Master on each answer booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Write your name, class and Master on this question paper and hand it in with your answers.
- Place everything inside the answer booklet for Question Nine.

| | | | |
|---|---|---|---|
| 5A: RCF | 5B: SO | 5C: BR | 5D: REJ |
| 5E: LYL | 5F: LJF | 5G: SDP | 5H: CMDB |

**Checklist**

- SGS booklets — 6 per boy
- Multiple choice answer sheet
- Candidature — 152 boys

**Examiner**

LYL

**SECTION I - Multiple Choice**

**Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.**

---

**QUESTION ONE**

Given $\alpha = \dfrac{-1+\sqrt{7}}{2}$ and $\beta = \dfrac{-1-\sqrt{7}}{2}$, what is the value of $\alpha^2 - \beta^2$?

(A) $1$

(B) $0$

(C) $-\sqrt{7}$

(D) $-1$

**QUESTION TWO**

Which of the following is a geometric sequence?

(A) $2,\ 5,\ 11,\ 20, \ldots$

(B) $8, -2, \frac{1}{2}, -\frac{1}{8}, \ldots$

(C) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$

(D) $22,\ 19,\ 16,\ 13, \ldots$

**QUESTION THREE**

Given that $g(x) = x^2$, what does $g(5-a)$ equal?

(A) $5-a$

(B) $25-a^2$

(C) $25+10a+a^2$

(D) $25-10a+a^2$

**QUESTION FOUR**

What is the largest possible domain for the function $f(x) = \dfrac{x}{\sqrt{9-x^2}}$?

(A) $0 \le x < 3$

(B) $-3 < x < 3$

(C) $0 \le x < 9$

(D) $-9 < x < 9$

**Examination continues next page ...**

**QUESTION FIVE**

The point $P$ divides the interval from $A(-10, 8)$ to $B(4, 1)$ internally in the ratio $3 : 1$. What are the coordinates of $P$?

(A) $\left(\frac{1}{2}, \frac{11}{4}\right)$

(B) $\left(-\frac{13}{2}, \frac{25}{4}\right)$

(C) $\left(\frac{11}{4}, \frac{1}{2}\right)$

(D) $\left(\frac{25}{4}, -\frac{13}{2}\right)$

**QUESTION SIX**

The derivative of $f(x) = \dfrac{4}{x^3}$ is:

(A) $\dfrac{4}{3x^2}$

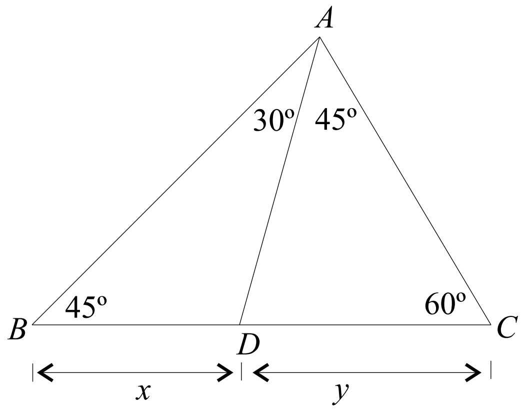
(B) $-\dfrac{12}{x^4}$

(C) $-\dfrac{12}{x^2}$

(D) $-\dfrac{4}{x^4}$

**Multiple choice continues on the next page**

**QUESTION SEVEN**




What is the value of $\dfrac{x}{y}$ in the diagram above?

(A) $\dfrac{1}{\sqrt{3}}$

(B) $\sqrt{3}$

(C) $\dfrac{\sqrt{3}}{2}$

(D) $\dfrac{2}{\sqrt{3}}$

**QUESTION EIGHT**

Consider the equation $\tan(2x - 45) = -1$, for $0^\circ \le x \le 360^\circ$. How many solutions does this equation have in this domain?

(A) 2

(B) 3

(C) 4

(D) 5

End of Section I

**SECTION II - Written Response**

**Answers for this section should be recorded in the booklets provided.**

**Show all necessary working.**

**Start a new booklet for each question.**

---

**QUESTION NINE** (12 marks) Use a separate writing booklet. **Marks**

(a) Simplify $\left(4\sqrt{5}-1\right)^2$. **1**

(b) Differentiate:

(i) $y = 2x^3 - 3x$ **1**

(ii) $y = x(x+3)$ **1**

(iii) $y = \dfrac{x^3+x}{x}$ **1**

(c) Solve $|2x-7| > 3$. **2**

(d) Consider the sequence 40, 38, 36, ... .

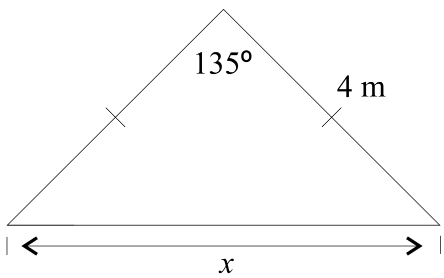
(i) State the common difference. **1**

(ii) Find the thirteenth term. **1**

(e) Factorise $27 - x^3$. **1**

(f) Evaluate $\displaystyle\sum_{n=1}^{5}(n^3-1)$. **1**

(g)




The diagram above shows the gable of a roof. The gable forms an isosceles triangle with two rafters of 4 metres and an angle of 135° at the apex.

(i) Find the value of $x$. Give your answer correct to two decimal places. **1**

(ii) Find the exact area of the gable. **1**

**Examination continues overleaf ...**

**QUESTION TEN** (12 marks) Use a separate writing booklet. **Marks**

(a) Find the perpendicular distance between the parallel lines $3x + 2y - 1 = 0$ and $3x + 2y + 4 = 0$. **2**

(b) Solve $4\cos^2\theta = 3$, for $0^\circ \leq \theta \leq 360^\circ$. **2**

(c) Sketch $y = \dfrac{1}{x+4}$. Show all asymptotes and any intercepts with the axes. **2**

(d) Find the equation of the tangent to the curve $y = 3x^2 + x - 6$ at the point $(-1, -4)$. Give your answer in gradient–intercept form. **2**

(e) Solve the inequation $\dfrac{4}{x-1} \leq 2$. **4**

**QUESTION ELEVEN** (12 marks) Use a separate writing booklet. **Marks**

(a) Differentiate:

(i) $y = (4x - 1)^6$ **2**

(ii) $y = x(3x + 2)^4$ **2**

(b) Given $f(x) = \sqrt{4 - x^2}$, sketch $y = f\left(\frac{1}{2}x\right)$ and clearly mark any significant features. **2**

(c) Prove $\dfrac{\sin\theta + \cos\theta}{\cot\theta + 1} = \sin\theta$. **2**

(d) Consider the graphs of $y = 6x - x^2$ and $y = 2x$.

(i) Find any points of intersection. **1**

(ii) Shade the intersection of the regions $y \leq 6x - x^2$ and $y > 2x$. Indicate the nature of the boundaries and corners clearly. **3**

**Examination continues next page ...**

**QUESTION TWELVE** (12 marks) Use a separate writing booklet. **Marks**

(a) If $\cos\theta = -\frac{4}{7}$ and $\tan\theta > 0$, find the exact value of $\sin\theta$. **2**

(b) The third term of a geometric sequence is 54 and the sixth term is 2. Find the common ratio. **1**

(c) Determine whether $f(x) = \dfrac{x^2 - 3}{x}$ is even, odd or neither. Show your working clearly. **2**

(d) Find the angle of inclination of the line $2x + 3y - 1 = 0$. Give your answer correct to the nearest minute. **2**

(e) Given $\log_b z = \log_b(x + 2) - 2\log_b y$, write an expression for $z$ without logarithms. **2**

(f) Consider the function $f(x) = x^2 - 5x$.

(i) Simplify the expression $f(x + h) - f(x)$. **2**

(ii) Hence use the formula **1**

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

to differentiate $f(x) = x^2 - 5x$ from first principles.

**QUESTION THIRTEEN** (12 marks) Use a separate writing booklet. **Marks**

(a) Without finding the point of intersection, find the equation of the line that passes through $2x - y + 1 = 0$ and $x + 3y - 5 = 0$ and is parallel to the line $y = -2x + 3$. Give your answer in general form. **3**

(b) Solve $\tan^2 x - 2\sec^2 x + 3 = 0$, for $0^\circ \le x \le 180^\circ$. **3**

(c) Consider the graph $y = \dfrac{x - 1}{x^2 - 2x - 3}$.

(i) Find any intercepts with the coordinate axes. **1**

(ii) Write down the equation(s) of any vertical asymptote(s). **1**

(iii) Find the equation of the horizontal asymptote. **1**

(iv) Copy and complete the following table. **1**

| $x$ | $-2$ | $0$ | $2$ | $4$ |
|---|---|---|---|---|
| $y$ | | | | |

(v) Draw a neat sketch of the graph. Show all the features from parts (i) to (iv). **2**

**QUESTION FOURTEEN** (12 marks) Use a separate writing booklet. **Marks**

(a) Factorise fully $x^4 - 3x^2y^2 - 4y^4$. **1**

(b) A function is defined by the rule: **2**

$$f(x) = \begin{cases} (x-1)^3 & \text{for } x \geq 0 \\ (1-x)^3 & \text{for } x < 0 \end{cases}$$

Find a simplified expression for $f\left(\frac{a^2}{2}\right) + f\left(\frac{-a^2}{2}\right)$.

(c) (i) Differentiate $y = \dfrac{2x+1}{\sqrt{2x-1}}$. **2**

(ii) Find the $x$-coordinate of any point(s) where the tangent is horizontal. **1**

(d) It is known that $x, y, z$ are the first three terms of a geometric progression and $x, z, y$ are the first three terms of an arithmetic progression.

(i) Show that $2\left(\dfrac{y}{x}\right)^2 - \left(\dfrac{y}{x}\right) - 1 = 0$. **2**

(ii) Given the geometric sequence has a limiting sum, find $S_\infty$ in terms of $x$. **2**

(e) Consider the function $f(x) = \big||x+3| - 1\big|$. For what values of the constant $k$ does the equation $f(x) = k$ have exactly four distinct solutions? **2**

End of Section II

**END OF EXAMINATION**

NAME: ..............................

CLASS: .........MASTER: .........

SYDNEY GRAMMAR SCHOOL

**2017**
**Half-Yearly Examination**
**FORM V**
**MATHEMATICS EXTENSION 1**
**Tuesday 23rd May 2017**

- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

**Question One**

A ◯ B ◯ C ◯ D ◯

**Question Two**

A ◯ B ◯ C ◯ D ◯

**Question Three**

A ◯ B ◯ C ◯ D ◯

**Question Four**

A ◯ B ◯ C ◯ D ◯

**Question Five**

A ◯ B ◯ C ◯ D ◯

**Question Six**

A ◯ B ◯ C ◯ D ◯

**Question Seven**

A ◯ B ◯ C ◯ D ◯

**Question Eight**

A ◯ B ◯ C ◯ D ◯

# Form V Extension 1 Half Yearly Solutions

## Ext 1 Half Yearly 2017

### Multiple choice

Q1 (C)

$\alpha = \dfrac{-1+\sqrt{7}}{2} = \dfrac{\sqrt{7}-1}{2}$

$\alpha^2 = \dfrac{(\sqrt{7}-1)^2}{4} = \dfrac{7-2\sqrt{7}+1}{4} = \dfrac{8-2\sqrt{7}}{4} = \dfrac{\cancel{2}(4-\sqrt{7})}{\cancel{4}\,2} = \dfrac{4-\sqrt{7}}{2}$

$\beta = \dfrac{-1-\sqrt{7}}{2} = \dfrac{-(\sqrt{7}+1)}{2}$

$\beta^2 = (\sqrt{7}+1)^2 = \dfrac{7+2\sqrt{7}+1}{4} = \dfrac{\cancel{2}(4+\sqrt{7})}{\cancel{4}\,2} = \dfrac{4+\sqrt{7}}{2}$

$\alpha^2 - \beta^2 = \dfrac{4-\sqrt{7}-(4+\sqrt{7})}{2} = \dfrac{-2\sqrt{7}}{2} = -\sqrt{7}$

Q2 (B)

$\left.\begin{array}{l} \text{If } \dfrac{T_2}{T_1} = \dfrac{T_3}{T_2} \\ \text{or } T_2^2 = T_1 \times T_3 \end{array}\right\}$ then GP

Sequence $8, -2, \frac{1}{2}, -\frac{1}{8}, \ldots$
$\qquad\;\; T_1 \quad T_2 \quad T_3$

$T_1 \times T_3 = 8 \times \frac{1}{2} = 4$ $\qquad T_2^2 = (-2)^2 = 4$

$\therefore 8, -2, \frac{1}{2}, -\frac{1}{8}, \ldots$ is a GP

Q3 $g(x) = x^2$

$g(5-a) = (5-a)^2$

$= 25 - 10a + a^2$

D

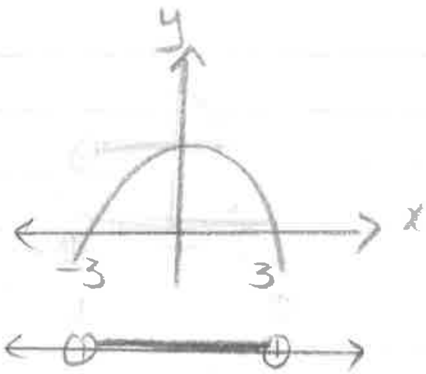
Q4 $f(x) = \dfrac{x}{\sqrt{9-x^2}}$

Domain $\sqrt{9-x^2} > 0$

$9 - x^2 > 0$

$(3-x)(3+x) > 0$

$-3 < x < 3$




Q5 $A(-10, 8)$ $B(4, 1)$ $3 : 1$

$x_1 \; y_1$ $x_2 \; y_2$ $k : l$

$x = \dfrac{kx_2 + lx_1}{k+l}$ $y = \dfrac{ky_2 + ly_1}{k+l}$

$= \dfrac{3 \times 4 + 1 \times -10}{4}$ $= \dfrac{3 \times 1 + 1 \times 8}{4}$

$= \dfrac{12 - 10}{4}$ $= \dfrac{11}{4}$

$= \dfrac{1}{2}$

A

$M\left(\dfrac{1}{2}, \dfrac{11}{4}\right)$

Q6 $f(x) = 4x^{-3}$

$f'(x) = -12x^{-4}$

$= -\dfrac{12}{x^4}$

B

Q7 $\tan(2x - 45) = -1$ $\qquad 0^\circ \le x \le 360^\circ$

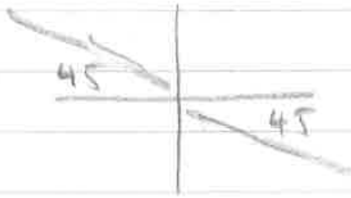
let $\theta = 2x - 45$ $\qquad 0^\circ \le 2x \le 720$

$\qquad -45 \quad -45 \quad -45$

$-45 \le \theta \le 675$

$\tan\theta = -1$

related angle $= 45^\circ$




$\theta = -45^\circ, 135^\circ, 315^\circ, 495^\circ, 675^\circ$

Since $\theta = 2x - 45$

$x = \dfrac{\theta + 45}{2}$

(D)

$x = 0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$

$\therefore$ There are 5 solutions.

Q8 In $\Delta ADB$ $\qquad$ In $\Delta ADC$

$\dfrac{x}{\sin 30^\circ} = \dfrac{AD}{\sin 45^\circ}$ $\qquad \dfrac{y}{\sin 45^\circ} = \dfrac{AD}{\sin 60^\circ}$

$x = \dfrac{AD \sin 30^\circ}{\sin 45^\circ}$ $\qquad y = \dfrac{AD \sin 45^\circ}{\sin 60^\circ}$

$\dfrac{x}{y} = \dfrac{AD \sin 30^\circ}{\sin 45^\circ} \div \dfrac{AD \sin 45^\circ}{\sin 60^\circ}$

$= \dfrac{\cancel{AD} \sin 30^\circ}{\sin 45^\circ} \times \dfrac{\sin 60^\circ}{\cancel{AD} \sin 45^\circ}$

$= \dfrac{1}{2} \div \dfrac{1}{\sqrt{2}} \times \dfrac{\sqrt{3}}{2} \div \dfrac{1}{\sqrt{2}}$

$= \dfrac{\cancel{\sqrt{2}}}{\cancel{2}} \times \dfrac{\sqrt{3}}{2} \times \dfrac{\cancel{\sqrt{2}}}{1}$

$= \dfrac{\sqrt{3}}{2}$

(C)

Q9 a) $(4\sqrt{5}-1)^2$

$= (4\sqrt{5})^2 - 2 \cdot 4\sqrt{5} + 1$

$= 16 \times 5 - 8\sqrt{5} + 1$

$= \underline{81 - 8\sqrt{5}}$ ✓

b) i) $y = 2x^3 - 3x$

$\underline{y' = 6x^2 - 3}$ ✓

ii) $y = x(x+3)$

$= x^2 + 3x$

$\underline{y' = 2x + 3}$ ✓

iii) $y = \frac{x^3 + x}{x}$

$= x^2 + 1$

$\underline{y' = 2x}$ ✓

c) $|2x - 7| > 3$

$2x - 7 > 3$

$2x > 10$

$\underline{x > 5}$ ✓

$2x - 7 < -3$

$2x < 4$

$\underline{x < 2}$ ✓

d) i) $\underline{d = -2}$ ✓

$a = 40$

ii) $T_n = a + (n-1)d$

$T_{13} = 40 + 12 \times -2$

$= 40 - 24$

$\underline{T_{13} = 16}$ ✓

e) $27 - x^3 = 3^3 - x^3$

$= \underline{(3 - x)(9 + 3x + x^2)}$ ✓

Q9 continued

f) $\sum_{n=1}^{5}(n^3-1) = 0 + (2^3-1) + (3^3-1) + (4^3-1) + (5^3-1)$

$= 8 + 27 + 64 + 125 - 4$

$= 224 - 4$

$= \underline{220}$ ✓

g) i) $x^2 = 4^2 + 4^2 - 2.4.4.\cos 135^\circ$

$= 32 - 2.4.4 \times \frac{1}{\sqrt{2}}$

$= 32 - \cancel{2}.4.4 \times \frac{\sqrt{2}}{\cancel{2}}$

$= 32 - 16\sqrt{2}$

$x = \sqrt{32 - 16\sqrt{2}}$ ✓

$\underline{x \doteqdot 7.39\text{m} \ (2\text{dp})}$

ii) Area $= \frac{1}{2} ab \sin 135^\circ$

$= \frac{1}{2} \times 16 \times \frac{1}{\sqrt{2}}$

$= \frac{8}{\sqrt{2}} \frac{\times \sqrt{2}}{\times \sqrt{2}}$

$= \underline{4\sqrt{2}\ u^2}$ ✓

Q10 a) $3x + 2y - 1 = 0$

Sub $x = 1$ $\quad 3 + 2y - 1 = 0$

$2y + 2 = 0$

$y = -1$ $\quad (1, -1)$ ✓

accept any point on either line

Perpendicular distance

$(1, -1)$ to $3x + 2y + 4 = 0$

$a = 3$

$b = 2$

$c = 4$

$$P = \frac{|3 \times 1 + 2 \times -1 + 4|}{\sqrt{3^2 + 2^2}}$$

$$= \frac{5}{\sqrt{13}} \quad ✓$$

$$= \frac{5\sqrt{13}}{13} \text{ units}$$

b) $4\cos^2\theta = 3$

$\cos^2\theta = \frac{3}{4}$

$\cos\theta = \pm\frac{\sqrt{3}}{2}$ ✓

$\cos\theta = \frac{\sqrt{3}}{2}$ $\quad\quad \cos\theta = -\frac{\sqrt{3}}{2}$

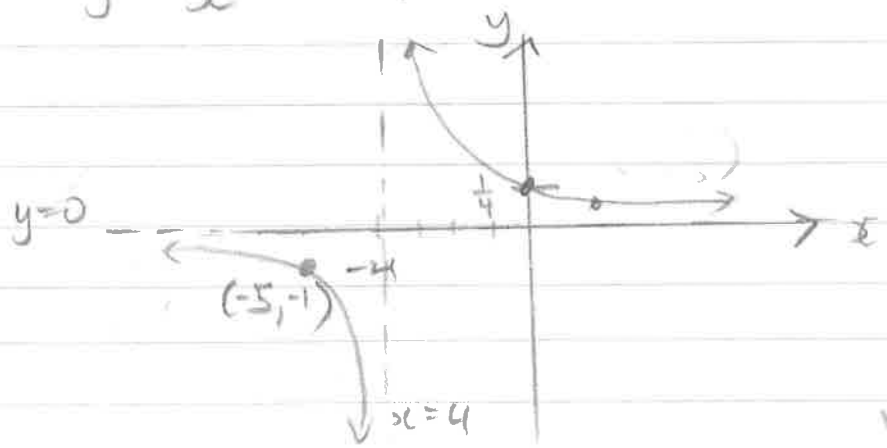
$\underline{\theta = 30^\circ, 330^\circ}$ $\quad\quad \underline{\theta = 150^\circ, 210^\circ}$

✓

Q10 continued

c) $y = \dfrac{1}{x+4}$

$y = \dfrac{1}{x}$ shifted left 4 units.




y-intercept
$x=0 \quad y=\frac{1}{4}$

asymptotes
vertical $x=-4$
horizontal
$y=0$

| $x$ | $-6$ | $-5$ | $0$ | $1$ |
|---|---|---|---|---|
| $y$ | $-\frac{1}{2}$ | $-1$ | $\frac{1}{4}$ | $\frac{1}{5}$ |

✓ shape

d) $y = 3x^2 + x - 6$
$y' = 6x + 1$

At $(-1,-4)$ $\quad y' = -6+1$
$= -5$ ✓

$y - y_1 = m(x - x_1)$
$y + 4 = -5(x+1)$
$\underline{y = -5x - 9}$ ✓

e) $\dfrac{4}{x-1} \leq 2$ $\quad$ * $x \neq 1$

$(x-1)^{2} \times \dfrac{4}{\cancel{x-1}} \leq 2(x-1)^2$ ✓

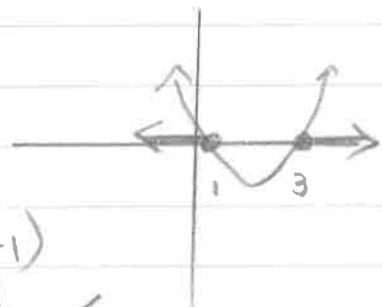
$4(x-1) \leq 2(x-1)^2$

$0 \leq 2(x-1)^2 - 4(x-1)$

$0 \leq 2(x-1)\left[(x-1)-2\right]$ ✓

$2(x-1)(x-3) \geq 0$

$x < 1$ ✓ or $x \geq 3$ ✓



Q11 a) i) $y = (4x-1)^6$ ✓

$y' = 6(4x-1)^5 \cdot 4$ ✓ Chain rule

$= 24(4x-1)^5$

ii) $y = x(3x+2)^5$

$y = uv$

$u = x \qquad v = (3x+2)^4$

$u' = 1 \qquad v' = 4(3x+2)^3 \cdot 3$

$= 12(3x+2)^3$ ✓

$y' = vu' + uv'$

$= (3x+2)^4 + 12x(3x+2)^3$ ✓

$= (3x+2)^3[(3x+2) + 12x]$

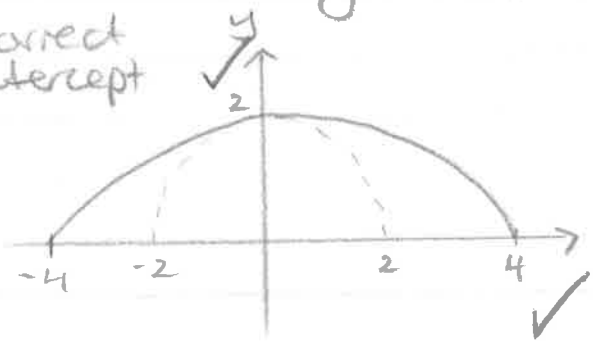
$= (3x+2)^3(15x+2)$

b) $f(x) = \sqrt{4-x^2}$

positive semi-circle radius 2

$f(\frac{1}{2}x)$ stretched horizontally by a factor of 2

correct y-intercept ✓




let $y = f(x)$

--- $y = \sqrt{4-x^2}$

✓ correct $x$ intercepts

c) $LHS = \dfrac{\sin\theta + \cos\theta}{\cot\theta + 1}$

$= \dfrac{\sin\theta + \cos\theta}{\frac{\cos\theta}{\sin\theta} + 1}$ ✓

$= \dfrac{(\sin\theta + \cos\theta)}{1} \div \dfrac{\cos\theta + \sin\theta}{\sin\theta}$

$= \dfrac{\cancel{\sin\theta + \cos\theta}}{1} \times \dfrac{\sin\theta}{\cancel{\cos\theta + \sin\theta}}$ ✓

$= \sin\theta$

$= RHS$ as required.

d) i) $y = 6x - x^2$ $\quad y = 2x$

$6x - x^2 = 2x$

$4x - x^2 = 0$

$x(4 - x) = 0$

$x = 0$ or $x = 4$

Points of intersection

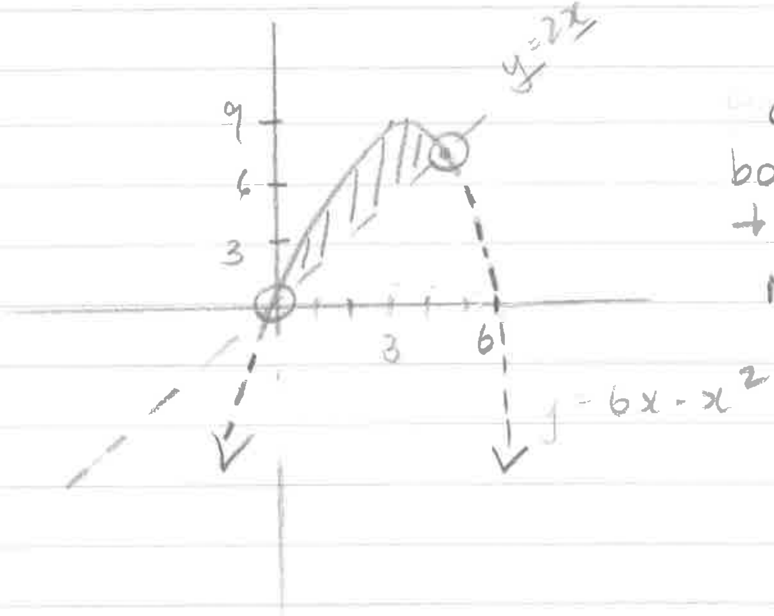
$(0,0)$ and $(4,8)$ ✓

ii)

$y \le 6x - x^2$

and

$y > 2x$

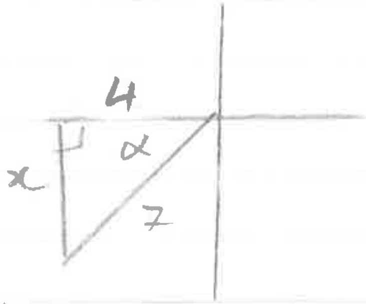



curves ✓

boundaries + corners ✓

region ✓

Q12 a) $\cos\theta = -\frac{4}{7}$ $\tan\theta > 0$




$x^2 + 4^2 = 7^2$

$x^2 = 49 - 16$

$= 33$

$x = \sqrt{33}$ $(x > 0)$ ✓

$\sin\theta = -\frac{\sqrt{33}}{7}$ ✓

b) GP $T_n = ar^{n-1}$

$T_3 = 54$ $T_6 = 2$

$ar^2 = 54$ ① $ar^5 = 2$ ②

② ÷ ①

$\frac{ar^5}{ar^2} = \frac{2}{54}$

$r^3 = \frac{1}{27}$

$r = \frac{1}{3}$ ✓

c) $f(x) = \frac{x^2 - 3}{x}$

$f(-x) = \frac{(-x)^2 - 3}{-x}$ ✓

$= \frac{x^2 - 3}{-x}$

$= -f(x)$ ✓

$\therefore f(x)$ is odd

Q 12 continued

d) $2x + 3y - 1 = 0$

$3y = -2x + 1$

$y = \frac{-2}{3}x + \frac{1}{3}$


$m = -\frac{2}{3}$ ✓

$\tan\theta = \frac{-2}{3}$

$\tan\alpha = \frac{2}{3}$

$\alpha \doteqdot 33^\circ 41'$

$\theta \doteqdot 180 - 33^\circ 41'$ ✓

$\doteqdot 146^\circ 19'$ (nearest minute)

e) $\log_b z = \log_b\left(\frac{x+z}{y^2}\right)$ ✓

$z = \frac{x+z}{y^2}$ ✓

f) $f(x) = x^2 - 5x$

$f(x+h) = (x+h)^2 - 5(x+h)$ ✓

$= x^2 + 2hx + h^2 - 5x - 5h$

$f(x+h) - f(x) = x^2 + 2hx + h^2 - 5x - 5h$
$\quad -x^2 \quad + 5x$

$= 2hx + h^2 - 5h$

$= h(2x + h - 5)$ ✓

$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(2x + h - 5)}{\cancel{h}}$

$= 2x - 5$ ✓

Q13

a) $2x - y + 1 + k(x + 3y - 5) = 0$ *

$(2x + kx)(-y + 3ky) + (1 - 5k) = 0$

$(2+k)x + (3k-1)y + (1-5k) = 0$

$(3k-1)y = -(2+k)x - (1-5k)$ ✓

$y = \frac{-(2+k)}{(3k-1)}x - \frac{(1-5k)}{(3k-1)}$

$\frac{2+k}{1-3k} = -2$

$2+k = -2(1-3k)$

$2+k = -2+6k$

$4 = 5k$

$k = \frac{4}{5}$ ✓

Sub $k = \frac{4}{5}$ into *

$2x - y + 1 + \frac{4}{5}(x + 3y - 5) = 0$

$5(2x - y + 1) + 4x + 12y - 20 = 0$

$\underline{10x} - 5y + 5 + \underline{4x} + 12y - 20 = 0$

$14x + 7y - 15 = 0$ ✓

asked for general form

Q13 continued

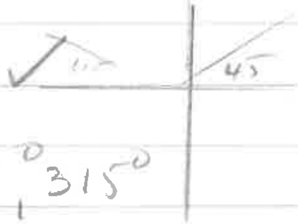
b) $\tan^2 x - 2\sec^2 x + 3 = 0 \qquad \underline{0^\circ \le x \le 180^\circ}$

$$\tan^2 x - 2(1+\tan^2 x) + 3 = 0$$
$$\tan^2 x - 2 - 2\tan^2 x + 3 = 0$$
$$-\tan^2 x + 1 = 0 \qquad \checkmark$$
$$\tan^2 x - 1 = 0$$
$$(\tan x - 1)(\tan x + 1) = 0$$

$\tan x - 1 = 0 \qquad\qquad \tan x + 1 = 0 \quad \checkmark$

$\tan x = 1 \qquad\qquad \tan x = -1$

$x = 45^\circ, 225^\circ \qquad\qquad x = 135^\circ, 315^\circ$




but domain

$0^\circ < x \le 180^\circ$

$\therefore \quad x = 45^\circ$ or $135^\circ$. $\checkmark$

Q13 continued

c) $y = \dfrac{x-1}{x^2-2x-3}$

i) $x = 0 \quad y = \frac{1}{3}$ $\quad (0, \frac{1}{3})$ y intercept

accept $y = \frac{1}{3}$ ✓

$y = 0 \quad x - 1 = 0$

$x = 1$ $\quad (1, 0)$ x-intercept

ii) vertical asymptotes

$x = -1$ or $x = 3$ ✓

iii) $y = \dfrac{\frac{x}{x^2} - \frac{1}{x}}{\frac{x^2}{x^2} - \frac{2x}{x^2} - \frac{3}{x^2}}$

$= \dfrac{0}{1 - \frac{2}{x} - \frac{3}{x^2}}$

$x \to \infty \qquad y \to 0$

$x \to -\infty \qquad y \to 0$

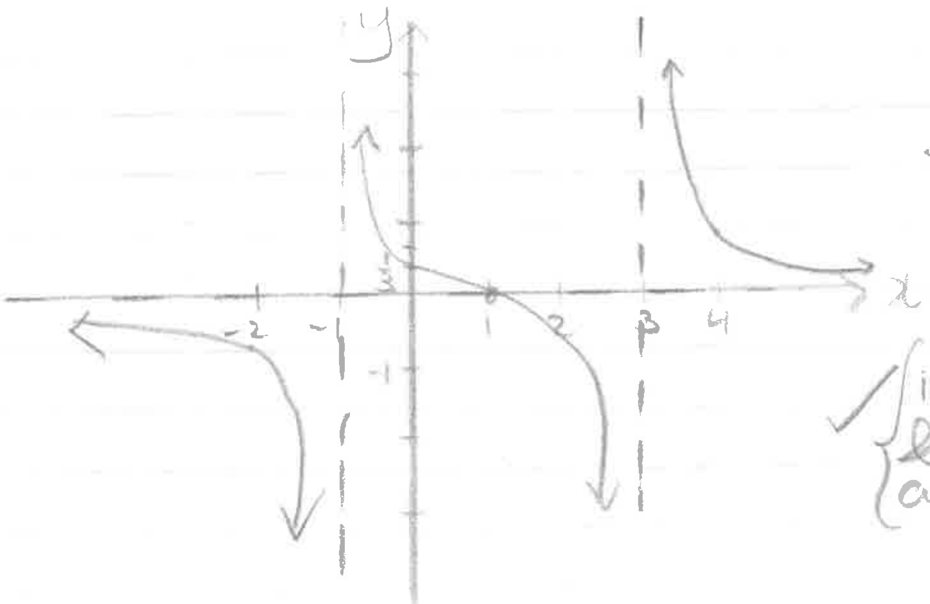
$y = 0$ horizontal asymptote ✓

iv)

| $x$ | $-2$ | $0$ | $2$ | $4$ |
|---|---|---|---|---|
| $y$ | $-\frac{3}{5}$ | $\frac{1}{3}$ | $-\frac{1}{3}$ | $\frac{3}{5}$ |

✓

v)




Shape ✓

✓ intercepts & asymptotes

Q14 a) $x^4-3x^2y^2-4y^4=(x^2-4y^2)(x^2+y^2)$
$=(x-2y)(x+2y)(x^2+y^2)$ ✓

b) $f(x)=\begin{cases}(x-1)^3 & \text{for } x\geq 0\\ (1-x)^3 & \text{for } x<0\end{cases}$

$f\left(\frac{a^2}{2}\right)=\left(\frac{a^2}{2}-1\right)^3$

$f\left(-\frac{a^2}{2}\right)=\left(1-\left(-\frac{a^2}{2}\right)\right)^3$

$=\left(\frac{a^2}{2}+1\right)^3$

Special case when $a=0$
$f\left(\frac{a^2}{2}\right)=-1$
$f\left(-\frac{a^2}{2}\right)=-1$
$\therefore f\left(\frac{a^2}{2}\right)+f\left(-\frac{a^2}{2}\right)=-2$

$f\left(\frac{a^2}{2}\right)+f\left(-\frac{a^2}{2}\right)$

$=\left(\frac{a^2}{2}-1\right)^3+\left(\frac{a^2}{2}+1\right)^3$ ✓ *

$=\left(\frac{a^2}{2}-1+\frac{a^2}{2}+1\right)\left[\left(\frac{a^2}{2}-1\right)^2-\left(\frac{a^2}{2}-1\right)\left(\frac{a^2}{2}+1\right)+\left(\frac{a^2}{2}+1\right)^2\right]$

$=a^2\left[\left(\frac{a^4}{4}-\frac{2a^2}{2}+1\right)-\left(\frac{a^4}{4}-1\right)+\left(\frac{a^4}{4}+\frac{2a^2}{2}+1\right)\right]$

$=a^2\left(3+\frac{a^4}{4}\right)$ ✓

Accept expansion of $f\left(\frac{a^2}{2}\right)$ and $f\left(-\frac{a^2}{2}\right)$

$f\left(\frac{a^2}{2}\right)=\left(\frac{a^2}{2}-1\right)^3$

$=\frac{a^6}{8}-\frac{3a^4}{4}+\frac{3a^2}{2}+1$

$f\left(-\frac{a^2}{2}\right)=\left(\frac{a^2}{2}+1\right)^3$

$=\frac{a^6}{8}+\frac{3a^4}{4}+\frac{3a^2}{2}+1$

* Alternative method 1 mark for correct expansion

Q 14 continued

i) $y = \dfrac{2x+1}{\sqrt{2x-1}}$

let $y = \dfrac{u}{v}$ $\quad u = 2x+1 \quad v = (2x-1)^{\frac{1}{2}}$

$u' = 2 \quad v' = \frac{1}{2}(2x-1)^{-\frac{1}{2}}.2$

$= (2x-1)^{-\frac{1}{2}}$

$= \dfrac{1}{\sqrt{2x-1}}$

$$y' = \frac{vu' - uv'}{v^2}$$

$$= \frac{2\sqrt{2x-1} - \dfrac{(2x+1)}{\sqrt{2x-1}}}{2x-1}$$ ✓

$$= \frac{2(2x-1) - (2x+1)}{\sqrt{2x-1}} \times \frac{1}{2x-1}$$

$$= \frac{4x-2-2x-1}{(2x-1)^{\frac{3}{2}}}$$

$$= \frac{2x-3}{(2x-1)^{\frac{3}{2}}}$$ ✓

ii) when $y' = 0$

$2x - 3 = 0$

$2x = 3$

$x = \dfrac{3}{2}$ ✓

Q14 d) GP $x, y, z$

so $\frac{y}{x} = \frac{z}{y}$ $\qquad r = \frac{y}{x}$

$z = \frac{y^2}{x}$ ①

AP $x, z, y$

$z - x = y - z$ ✓ two equations

$2z = x + y$ ②

Sub ① into ②

$2\frac{y^2}{x} = x + y$

$2y^2 = x^2 + xy$ ✓

$2y^2 - xy - x^2 = 0$

$\div x^2$ $\quad 2\frac{y^2}{x^2} - \frac{xy}{x^2} - \frac{x^2}{x^2} = 0$

$2\left(\frac{y}{x}\right)^2 - \left(\frac{y}{x}\right) - 1 = 0$ as required

GP has a limiting sum

$\therefore |r| < 1 \qquad \left|\frac{y}{x}\right| < 1$

let $r = \frac{y}{x}$

$2r^2 - r - 1 = 0$

$(2r+1)(r-1) = 0$

$2r + 1 = 0 \qquad r - 1 = 0$

$2r = -1$ ✓ $\qquad r = 1$ ✓

$r = -\frac{1}{2}$

$|r| < 1 \therefore r = -\frac{1}{2} \qquad S_\infty = \frac{a}{1-r} = \frac{x}{\frac{3}{2}} = \frac{2x}{3}$

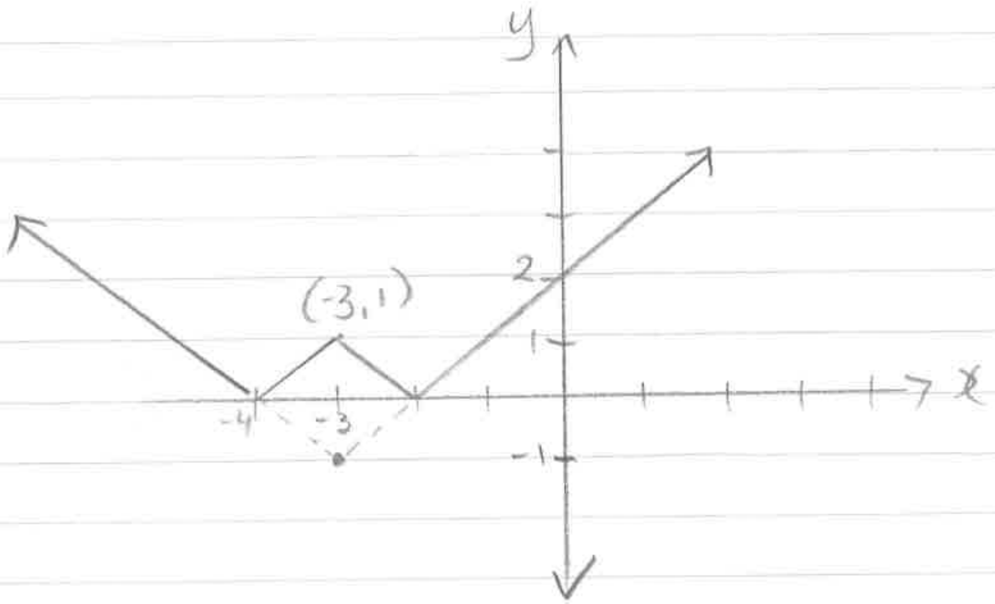
Q14 e) $f(x) = ||x+3|-1|$

let $y = f(x)$.

Standard curve $y = |x|$ is shifted left 3 units

then $y = |x+3|$ is shifted down 1 unit

and then curve below $x$-axis is reflected in $x$-axis.




$||x+3|-1| = k$ exactly four solutions

$\underline{0 < k < 1}$