SYDNEY TECHNICAL HIGH SCHOOL

YEAR 11 MATHEMATICS EXTENSION 1

COMMON TEST

MAY 2002

TIME ALLOWED: 70 MINUTES

Instructions:

- Show all necessary working
- Start each question on a new page
- Marks may be deducted for careless or badly arranged work.

Name:_____ **Teacher:**_____

Question1	Question 2	Question 3	Question 4	Question 5	Question 6	Total
/11	/10	/10	/11	/11	/9	/62

Question 1 **Marks**

(i) $(2+3\sqrt{3})(6-5\sqrt{3}) = a+\sqrt{b}$. Evaluate a and b. 2

(ii) Fully factorise $2a(b-2c)-3d(2c-b)$. 2

(iii) Factorise fully $20-9x-20x^2$. 2

(iv) Simplify the fraction $\dfrac{1-x^{-1}}{x^{-1}-x^{-2}}$ 2

(v) Graph $y=|x|+1$ and $y=|x-1|$ on the same number plane. Use this to solve the equation $|x|+1=|x-1|$. 3

Question 2

(i) (a) Show that $\dfrac{2x+1}{x+1}$ can be written as $2-\dfrac{1}{x+1}$. 2

(b) Hence, or otherwise find the equations of all asymptotes for the curve $y=\dfrac{2x+1}{x+1}$ 2

(c) Sketch the curve showing intercepts and asymptotes. 2

(ii) Calculate the cosine of the smallest angle of the triangle whose sides are 4cm, 5cm and 6cm. 2

(iii) (a) Determine algebraically whether the function $f(x)=\dfrac{x}{1+x^2}$ is odd, even or neither. 1

(b) If $f(x)=\begin{cases}-1 & x\le -1\\ x^2 & -1<x<1\\ 1 & x\ge 1\end{cases}$

Evaluate $f(-1)+f(0)+f(1)$. 1

Question 3

(i) (a) Find the domain of $f(x)=\dfrac{x^2+6x+9}{x+3}$ and by simplifying, find the range 2

(b) Hence sketch the function. 2

(ii) Solve $\dfrac{x}{x+3}\le x$ 4

(iii) Graph the region defined by $y\ge -\sqrt{16-x^2}$ on the number plane. 2

Question 4

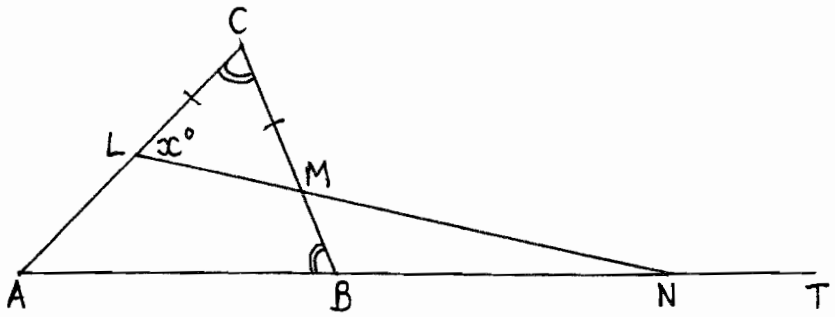
(i) Sketch the main features of ;

(a) $y = (x-1)^3$ 2

(b) $3x - 2y - 4 = 0$ 2

(ii) Given $2^x = 8^{y+1}$ *and* $9^y = 3^x$, *find* x *and* y. 3

(iii) Given $CL = CM, \angle ACB = \angle ABC$ *and* $\angle CLM = x°$,

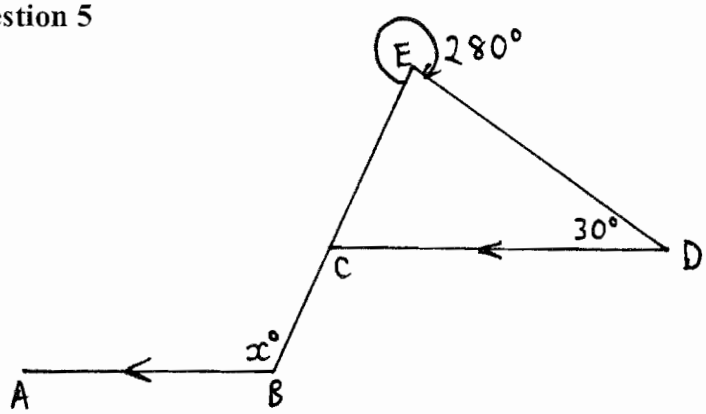


(a) show that $\angle ABC = 180 - 2x$ 2

(b) Hence show $\angle TNL = 3x°$ 2

Question 5

(i)



Find x giving reasons 2

(ii) Sketch $y = \dfrac{|x|}{x^2}$ 2

(iii) Simplify $\dfrac{\sin(180+\theta)}{\sin(90-\theta)}$ 2

(iv) Find the exact value of $\cos(-150°)$

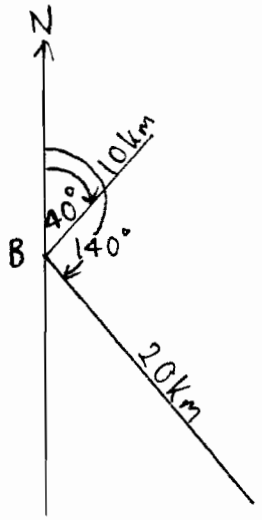
Q5 continued

(v) Two yachts sail in a straight line from a bouy B. The first sails 10km in the direction 040°T, and the second sails 20km in the direction 140°T in the same time.

(a) Copy the diagram below onto your answer sheet.

(b) How far apart are they? (correct to nearest km) 2

(c) What is the bearing of the 1st yacht from the 2nd?(correct to nearest degree) 2

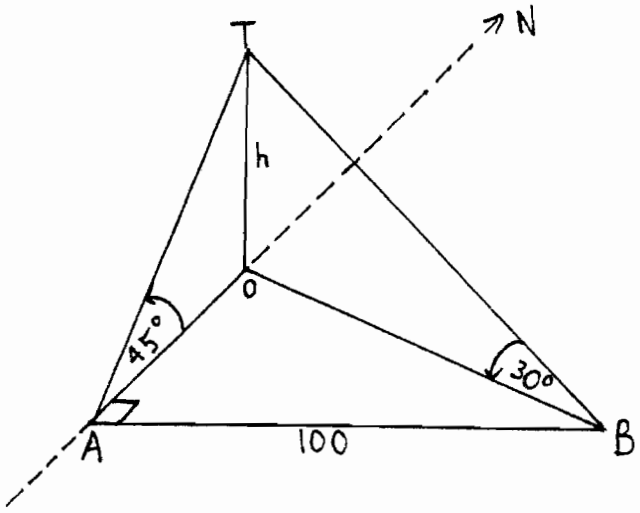


Not to scale

Question 6

(i) In the triangle PQR, PQ = 18cm, QR = 24cm and $\angle R = 12°$.Find the possible size(s) of $\angle P$ correct to the nearest degree. 3

(ii)



Not to scale

A surveyor stands at a point A, which is due south of a Tower OT of height h m. The angle of elevation of the top of the tower from A is 45°. The surveyor then walks 100m due east to point B, from where she measures the angle of elevation of the top of the tower to be 30°.

(a) Express the length OB in terms of h. 1

(b) Show that h = $50\sqrt{2}$. 3

(c) Calculate the bearing of B (to nearest degree) from the base of the tower. 2

Solution to Extension Task 1 (Preliminary) 2002

Question 1

(i) $(2+3\sqrt{3})(6-5\sqrt{3})$

$= 12 - 10\sqrt{3} + 18\sqrt{3} - 15\times 3$

$= -33 + 8\sqrt{3}$ ①

$= -33 + \sqrt{192}$

$\therefore a = -33,\ b = 192$ ①

(ii) $2a(b-2c) - 3d(2c-b)$

$2a(b-2c) + 3d(b-2c)$ ①

$(2a+3d)(b-2c)$ ①

(iii) $20 - 9x - 20x^2$

$-20x^2 - 9x + 20$

$+ \mid -9$

$\times \mid -400$

$-25, 16$

$\frac{(-20x-25)(-20x+16)}{-20}$

$= \frac{-5(4x+5)\times(4-5x)}{-20}$

$(4x+5)(4-5x)$

① ①

(iv) $\frac{1-x^{-1}}{x^{-1}-x^{-2}}$

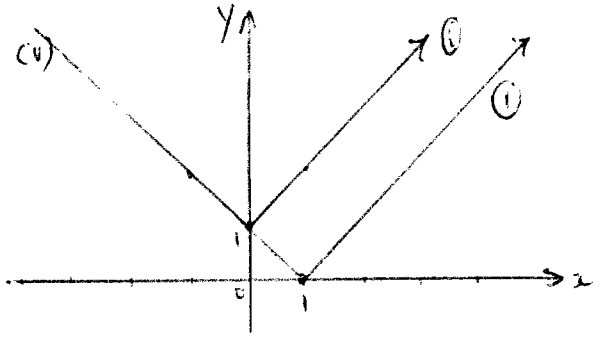
$= \frac{1-\frac{1}{x}}{\frac{1}{x}-\frac{1}{x^2}} \times \frac{x^2}{x^2}$

$= \frac{x^2-x}{x-1}$ ①

$= \frac{x(x-1)}{(x-1)}$

$= x$ ①

(v)



① ①

$x \neq 0$ ①

Question 2

(i) (a) $\frac{2x+1}{x+1}$

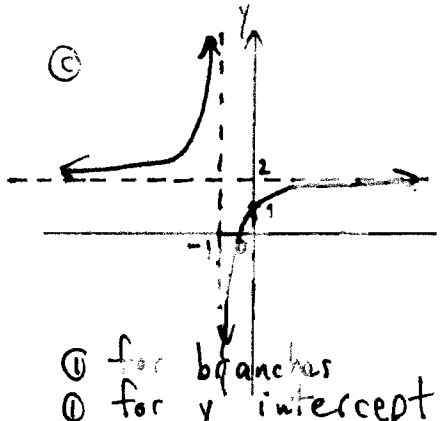
$= \frac{2(x+1)}{x+1} - \frac{1}{x+1}$ ①

$= 2 - \frac{1}{x+1}$ ①

(b) $x = -1$ ①

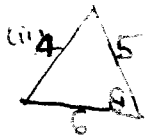
$y = 2$ ①

(c)



① for branches
① for y intercept

(ii)



$\cos\theta = \frac{6^2+5^2-4^2}{2\times 6\times 5}$ ①

$\cos\theta = \frac{3}{4}$ ①

(iii) $f(x) = \frac{x}{1+x^2}$

odd if $f(-x) = -f(x)$

$-x \quad -x$ ①

(b) $f(-1) + f(0) + f(1)$

$= -1 + 0^2 + 1$

$= 0$ (1)

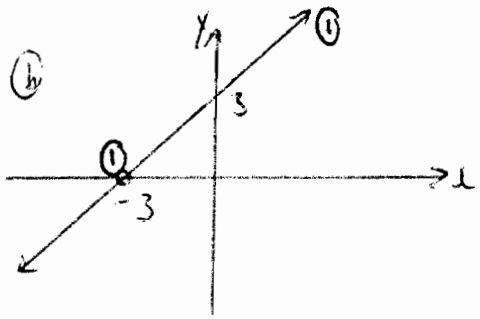
Question 3

(a) $f(x) = \dfrac{x^2+6x+9}{x+3}$

D: All real x, $x \neq -3$ (1)

R: All real y, $y \neq 0$ (1)

(b)



(1) (1)

(c)(i) $\dfrac{x}{x+3} \leq x$

Critical pts. $x \neq -3$ (1)

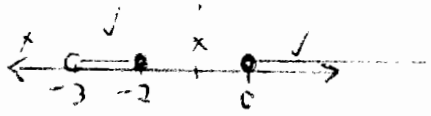
$\dfrac{x}{x+3} = x$

$x = x^2 + 3x$

$0 = x^2 + 2x$

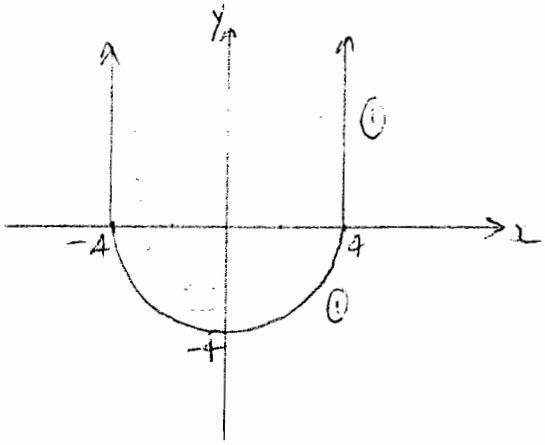
$0 = x(x+2)$

$x = 0, -2$ (2)



$x \geq 0, \quad -3 < x \leq -2$ (1)

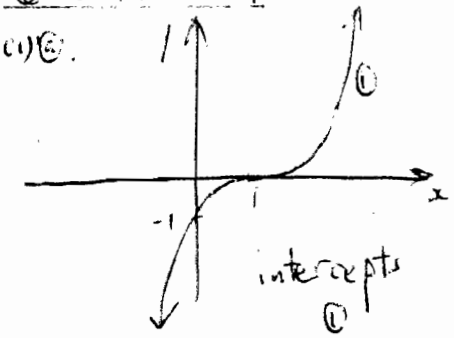
(c)(iii)



(1) (1) (1)

Question 4

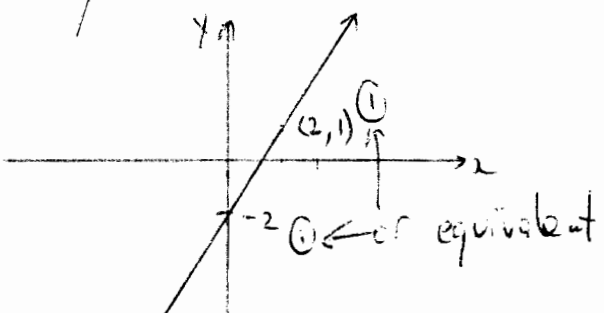
(a)(a)



(1) intercepts (1)

(b) $3x - 2y - 4 = 0$

$y = \frac{3}{2}x - 2$



(1) (1) or equivalent

(iii) $2^x = 8^{y+1}$ $9^y = 3^x$

$2^x = 2^{3(y+1)}$ $3^{2y} = 3^x$ ①

$\Rightarrow x = 3y + 3$,

$x = 2y$

$\Rightarrow 3y + 3 = 2y$ ①

$y = -3$ $\therefore x = -6$ ①

(iii) ⓐ $\angle LMC = x°$ (base $\angle$'s of isos. $\triangle$) ①

$\angle LCM = 180 - 2x$ ($\angle$ sum of $\triangle$)

$\angle ABC = 180 - 2x$ (base $\angle$'s of isos. $\triangle ABC$) ①

ⓑ $\angle BMN = x$ (vert. opp. $\angle$'s =)

$\angle MBN = 2x$ ($\angle$ sum on straight line) ①

$\angle TNL = 2x + x$

$= 3x$ (Ext. $\angle$ of $\triangle$) ①.

Question 5

(i) $\angle BCD = x$ (alt. $\angle$'s = in // lines)

$\angle ECD = 180 - x$) ($\angle$ sum on straight line)

$\angle CED = 180 - 30 - (180 - x)$

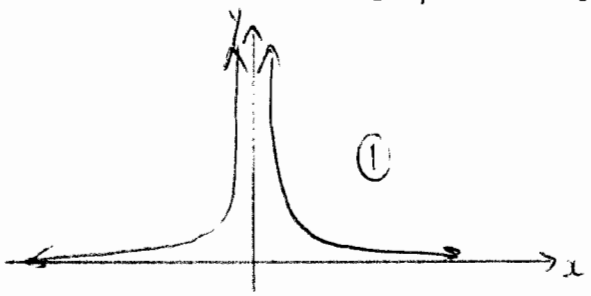
$= x - 30$ ①

$= 80$ (revolution $= 360$)

$x = 110°$ ①

(ii) $y = \frac{|x|}{x^2} = \frac{1}{x}$, $x > 0$ ①

$= \frac{-1}{x}$, $x < 0$



①

(iii) $\frac{\sin(180+\theta)}{\sin(90-\theta)}$

$= \frac{-\sin\theta}{\cos\theta}$ ①

$= -\tan\theta$ ①

(iv) $\cos(-150)$

$= \cos 210$

$= -\cos 30$

$= -\frac{\sqrt{3}}{2}$ ①

(v) $c^2 = a^2 + b^2 - 2.a.b.\cos C$

$c^2 = 10^2 + 20^2 - 2.10.20.\cos 100$ ①

$c^2 = 569.4$

$c = 24$ km ①

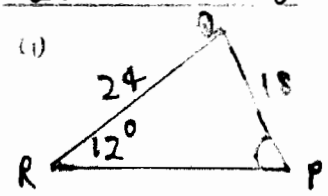
(c) 

$\frac{\sin\theta}{10} = \frac{\sin 80}{24}$

$\theta = 24°$ ①

Question 6

(i)



$\frac{\sin P}{24} = \frac{\sin 12}{18}$ ①

$P = 16°$ or $164°$

① ①

(ii) (a) $\tan 30 = \frac{h}{OB}$

$\frac{1}{\sqrt{3}} = \frac{h}{OB}$

$OB = h\sqrt{3}$ ①

(b) Similarly $\tan 45 = \frac{h}{OA}$

$OA = h$ ①

By Pythagoras'

$OB^2 = OA^2 + 100^2$

$h^2 3 = h^2 + 10000$ ①

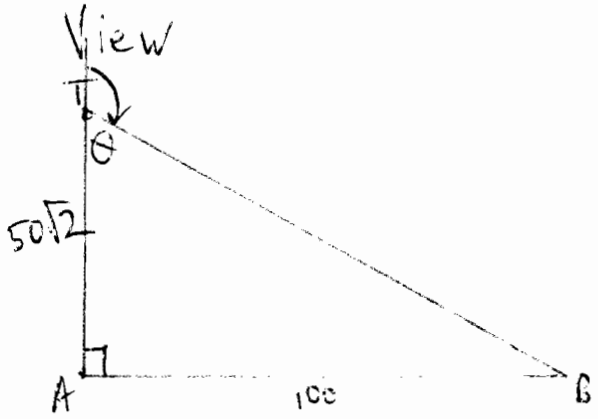
$2h^2 = 10000$

$h^2 = 5000$

$h = \sqrt{5000}$

$h = 50\sqrt{2}$ as req'd. ①

(c) Top View



$\tan\theta = \frac{100}{50\sqrt{2}}$

$\theta = 54°44'$ ①

∴ Bearing $= 180 - 54°44'$

$= 125°$ from true north. ①