SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS EXTENSION 1

COMMON TEST

MAY 2003

Time Allowed: 70 minutes

Instructions:

- Show all necessary working
- Start each question on a new page
- All questions are of equal value
- Approximate marks are shown alongside each question

Q1	Q2	Q3	Q4	Q5	Q6	Total

Question 1

(a) If $\sqrt[3]{p} = 32$ *and* $\sqrt{q} = 243$ *find* $\sqrt[5]{pq}$ (2)

(b) Factorise $x^4 + 8x^2 - 9$ (2)

© Solve $\cos 2x = -\frac{\sqrt{3}}{2}$ *for* $0 \le x \le 360$ (2)

(d) Simplify $\frac{\cos(360-\theta)^\circ}{\sin(-\theta)^\circ}$ (2)

(e) If $x - \frac{1}{x} = 2\sqrt{2}$ find the value of $x^2 + \frac{1}{x^2}$ (2)

Question 2

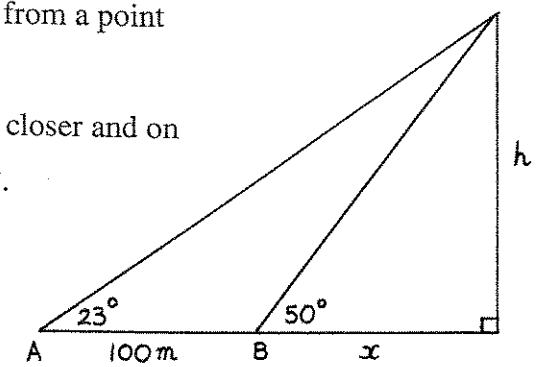
(a) (i) Factorise $2^{n+1} + 2^n$ (3)

(ii) Hence, write $\frac{2^{1001} + 2^{1000}}{3}$ as a power of 2

(b) Given that $\tan A = p$ *and* $180^\circ < A < 270^\circ$ (2)

find an expression for *cos A* in terms of p.

© The angle of elevation of a tower top from a point A is 23°
The angle of elevation from B, 100m closer and on the same horizontal plane as A is 50°. (5)



(i) Find an expression for tan 67° *and* tan 40° in terms of x and h.

(ii) Hence show that $h = \dfrac{100}{\tan 67° - \tan 40°}$

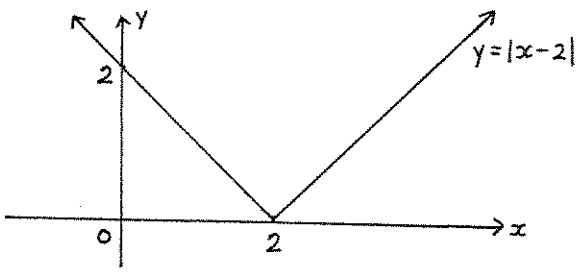
(iii) Hence find the height of the tower (correct to 2 decimal places.).

Question 3

(a) Solve $\dfrac{\sqrt{x+1}+\sqrt{x-1}}{\sqrt{x+1}-\sqrt{x-1}} = 3$ (2)

(b) Solve $2\sin^2 x + \cos x - 1 = 0$ for $-180° \leq x \leq 180°$ (4)

© The graph of $y = |x-2|$ is given below (4)



i) Sketch $y = |x-2|-1$

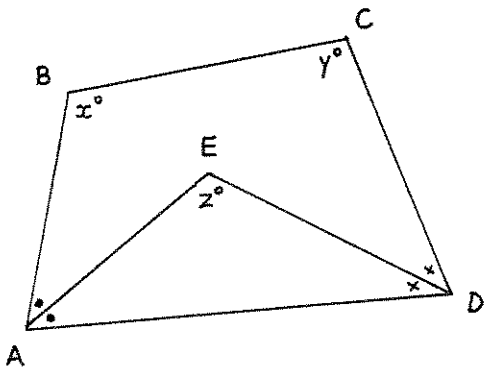
ii) Hence, sketch $y = \big|\,|x-2|-1\,\big|$ on a separate number plane

iii) If $\big|\,|x-2|-1\,\big| = a$, where a is an integer, has 3 solutions, find the value of a.

Question 4

(a) Simplify $\dfrac{15^n \times 3^{n+1}}{5^{n-1}}$ (2)

(b)



$\angle BAE = \angle EAD$ and $\angle CDE = \angle EDA$. (5)

i) Copy and complete: $\angle EAD + \angle EDA =$ _____(_____)

ii) Hence find an expression for $x + y$.

Give a reason.

(c) Prove $(1 + \tan A + \sec A)(1 + \tan A - \sec A) = 2\tan A$ (3)

Question 5

(a) State the natural domain of $y = \dfrac{\sqrt{x+4}}{x}$ (2)

(b) Factorise $x^2 - y^2 + 6y - 9$ (2)

© Solve $\dfrac{x^2 - 4}{x} \leq 0$ (3)

(d) Solve simultaneously $\begin{aligned} 2^x + 3^y &= 5 \\ 2^{x+3} - 3^{y+2} &= 23 \end{aligned}$ (3)

Question 6

(a) Solve $\frac{1}{|x-3|} > \frac{1}{2}$ (3)

(b) Consider the function $f(x) = \frac{x}{x^2 - 1}$ (7)

i) For what values of x is $f(x)$ undefined?

ii) Show that $y = f(x)$ is an odd function

iii) What is the graphical significance of part ii.

iv) Hence, sketch the function showing important features.
Use a ruler to draw the axes.
Use about $\frac{1}{3}$ of a page.

EXTENSION 1 - MAY 2003

Question 1

(a) $p = 32^3 \quad q = 243^2$

$\sqrt[5]{pq} = \sqrt[5]{32^3 \times 243^2}$

$= \underline{\underline{72}}$

(b) $x^4 + 8x^2 - 9$

$= (x^2+9)(x^2-1)$

$= \underline{\underline{(x^2+9)(x+1)(x-1)}}$

(c) $\cos 2x = -\frac{\sqrt{3}}{2}$

$2x = 150, 210, 510, 570$

$\underline{\underline{x = 75, 105, 255, 285}}$

(d) $\frac{\cos(360-\theta)}{\sin(-\theta)} = \frac{\cos\theta}{-\sin\theta}$

$= \underline{\underline{-\cot\theta}}$

(e) $x - \frac{1}{x} = 2\sqrt{2}$

$\left(x - \frac{1}{x}\right)^2 = x^2 - 2 + \frac{1}{x^2}$

$x^2 + \frac{1}{x^2} = (2\sqrt{2})^2 + 2$

$= 8 + 2$

$= \underline{\underline{10}}$

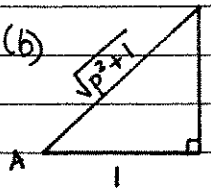
Question 2

(a) i. $2^n(2+1) = \underline{\underline{3 \times 2^n}}$

ii. $\frac{2^{1001} + 2^{1000}}{3} = \frac{3 \times 2^{1000}}{3}$

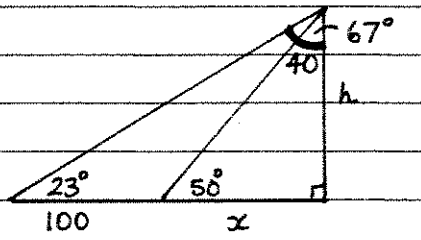
$= \underline{\underline{2^{1000}}}$

(b)



$\cos A = \underline{\underline{\frac{-1}{\sqrt{p^2+1}}}}$

(c)



i. $\tan 67° = \underline{\underline{\frac{100+x}{h}}}$

$\tan 40° = \underline{\underline{\frac{x}{h}}}$

ii. $h \tan 67° - 100 = x$

$h \tan 40° = x$

$h \tan 40° = h \tan 67° - 100$

$100 = h \tan 67° - h \tan 40°$

$100 = h(\tan 67° - \tan 40°)$

$\therefore h = \underline{\underline{\frac{100}{\tan 67° - \tan 40°}}}$

iii. $h = \underline{\underline{65 \cdot 93}}$

Question 3

(a) $\frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} = 3$

$\sqrt{x+1} + \sqrt{x-1} = 3\sqrt{x+1} - 3\sqrt{x-1}$

$4\sqrt{x-1} = 2\sqrt{x+1}$

$2\sqrt{x-1} = \sqrt{x+1}$

$4(x-1) = (x+1)$

$4x - 4 = x + 1$

$3x = 5$

$\therefore \underline{\underline{x = \frac{5}{3}}}$

(b) $2\sin^2 x + \cos x - 1 = 0$

$2(1 - \cos^2 x) + \cos x - 1 = 0$

$2 - 2\cos^2 x + \cos x - 1 = 0$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2} \qquad \cos x = 1$

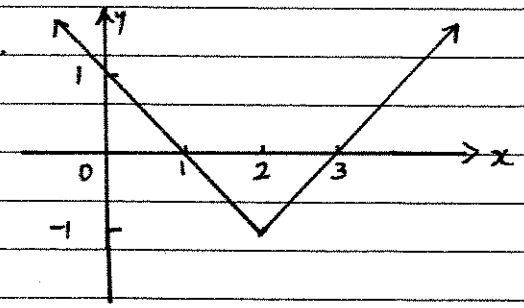
for $0 \leq x \leq 360$

$x = 120, 240$ and $0, 360$

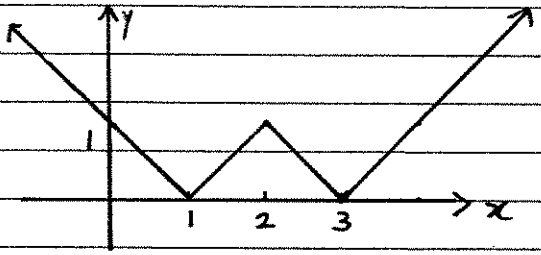
$\therefore$ for $-180 \leq x \leq 180$

$x = 120, -120, 0$

(c) i.



ii.



iii. $a = 1$

Question 4

(a) $\dfrac{15^n \times 3^{n+1}}{5^{n-1}} = \dfrac{3^n \times 5^n \times 3^{n+1}}{5^{n-1}}$

$= 3^{2n+1} \times 5$

(b) i. $180 - z$ (angle sum of $\triangle$)

ii. $2(\angle EAD + \angle EDA) + x + y = 360^\circ$ (angle sum of quad)

$2(180 - z) + x + y = 360$

$360 - 2z + x + y = 360$

$x + y = 2z$

(c) LHS $= (1 + \tan A + \sec A)(1 + \tan A - \sec A)$

$= (1 + \tan A)^2 - \sec^2 A$

$= 1 + 2\tan A + \tan^2 A - \sec^2 A$

$= 2\tan A + \sec^2 A - \sec^2 A$

$= 2\tan A$

$=$ RHS

Question 5

(a) $x \geq -4, \ x \neq 0$

(b) $x^2 - (y^2 - 6y + 9)$

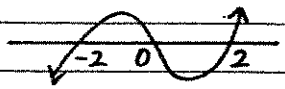
$= x^2 - (y-3)^2$

$= [x + (y-3)][x - (y-3)]$

$= (x + y - 3)(x - y + 3)$

(c) $x^2 \times \dfrac{x^2 - 4}{x} \leq 0 \times x^2$

$x(x+2)(x-2) \leq 0$



$\therefore x \leq -2, \ 0 < x \leq 2$

(d) $2^x + 3^y = 5$

$2^x = 5 - 3^y$

$2^x \cdot 2^3 - 3^y \cdot 3^2 = 23$

$8(5 - 3^y) - 9 \cdot 3^y = 23$

$40 - 8 \cdot 3^y - 9 \cdot 3^y = 23$

$17 = 17 \cdot 3^y$

$1 = 3^y$

$y = 0$

$2^x + 3^0 = 5$

$2^x = 4$

$x = 2$

$\therefore x = 2$ and $y = 0$

Question 6

(a) $\dfrac{1}{|x-3|} > \dfrac{1}{2}$

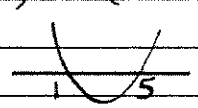
$(x-3)^2 \times \dfrac{1}{(x-3)^2} > \dfrac{1}{4} \times (x-3)^2$

$4 > (x-3)^2$

$(x-3)^2 - 4 < 0$

$(x-3-2)(x-3+2) < 0$

$(x-5)(x-1) < 0$



$\therefore 1 < x < 5, \ x \neq 3$

(b) $f(x) = \dfrac{x}{x^2-1}$

i. $x = \pm 1$

ii. $f(a) = \dfrac{a}{a^2-1}$

$f(-a) = \dfrac{-a}{(-a)^2-1}$

$= \dfrac{-a}{a^2-1}$

$\therefore$ function is odd since

$f(-a) = -f(a)$

iii. point symmetry about the origin

iv.