

# SYDNEY TECHNICAL HIGH SCHOOL



## MATHEMATICS EXTENSION 1

### YEAR 11 COMMON TEST

MAY 2005

**Time allowed:** 70 minutes

**Instructions:**

- Show all necessary working in every question
- Start each question on a new page
- Attempt all questions
- All questions are not of equal value
- Marks shown are approximate & may be changed
- Full marks may not be awarded for careless or badly arranged work
- Your sketches must be neat. Use a ruler to draw axes.
- Approved calculators may be used
- These questions are to be handed in with your answers.

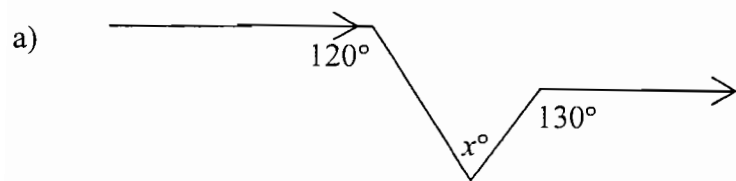
**Name:** \_\_\_\_\_

**Class:** \_\_\_\_\_

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Total
/10	/10	/10	/8	/8	/8	/54

**Question 1**

**Marks**



Find the value of  $x$  (no reasons necessary).

1

b) Factorise  $a^2 - b^2 - (a - b)^2$

2

c) The hyperbola  $y = \frac{3}{a - x}$  has a vertical asymptote at  $x = 1$ . What is the value of  $a$ ?

1

d) If  $\tan a = -\frac{1}{3}$  and  $\cos a > 0$ , find the exact value of  $\sin a$ .

2

e) Given that  $n$  is a positive number indicate

2

(i) the largest

(ii) the smallest

of the following numbers:

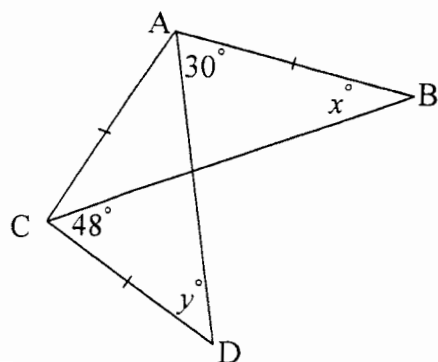
$$3^{-\frac{n}{3}}, 3^{\frac{n}{3}}, 3^n, 3^{-n}$$

f) Solve  $|5x - 3| = |3x + 1|$

2

**Question 2 (start a new page)**

a)



$\triangle ABC$  and  $\triangle ACD$  are isosceles.

3

By forming a pair of simultaneous equations or otherwise, find the value of  $x$ .

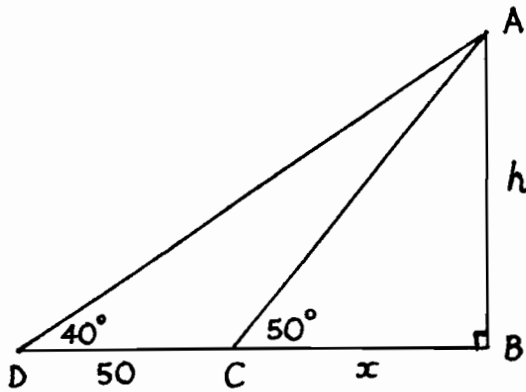
- b) If  $\frac{a^n + a^{n+2}}{a^n} = 10$  find  $a$  given that  $a > 0$ . 2
- c) Solve  $\frac{x-2}{x} \geq 1$  3
- d) There are two values of  $\theta$  in the domain  $0^\circ \leq \theta \leq 360^\circ$  where  $\sin \theta$  and  $\cos \theta$  are numerically equal. Find these two values. 2

**Question 3 (start a new page)**

- a) If  $p = \frac{\sqrt{3}}{4 - \sqrt{3}}$  and  $q = \frac{\sqrt{3}}{4 + \sqrt{3}}$  evaluate  $\frac{p+q}{1-pq}$  3
- b) How many solutions does the equation  $(\cos x - 2)(\sin^2 x - 1) = 0$  have in the domain  $0^\circ \leq x \leq 360^\circ$ ? 3  
There is no need to solve the equation.  
Justify your answer.
- c) i) Sketch the graph of  $y = x^2 + \frac{1}{2}$  4
- ii) On a separate diagram sketch  $y = \frac{2}{2x^2 + 1}$ .
- iii) Use your diagram or otherwise to write down the range of  $y = \frac{2}{2x^2 + 1}$

**Question 4 (start a new page)**

a)



We wish to find the height  $AB$  of a vertical cliff. From a point  $D$  the angle of elevation of  $A$  is  $40^\circ$ . From a point  $C$   $50\text{m}$  nearer the base of the cliff the angle of elevation is  $50^\circ$ .

4

- i) Show that  $h = (50 + x) \tan 40^\circ$
- ii) Show that  $h = x \tan 50^\circ$
- iii) Using simultaneous equations find  $h$

b)

- i) Sketch the graph of  $y = |x + 1|$ .
- ii) By using your graph or otherwise solve  $\frac{2}{x} > |x + 1|$

4

**Question 5 (start a new page).**

a)

- i) Sketch the graph of  $y = \cos x$  for  $0^\circ \leq x \leq 360^\circ$
- ii) Hence solve  $-\frac{\sqrt{3}}{2} \leq \cos x \leq \frac{\sqrt{3}}{2}$  for  $0^\circ \leq x \leq 360^\circ$

4

b)

- i) If  $xy = c^2$  prove that  $\frac{1}{c+x} + \frac{1}{c+y} = \frac{1}{c}$ .
- ii) Hence or otherwise simplify  $\frac{1}{6 + \sqrt{51} + \sqrt{15}} + \frac{1}{6 + \sqrt{51} - \sqrt{15}}$

4

**Question 6 (start a new page)**

a) Find the point/s of intersection for the graphs of  $y = x^2 - 1$  and  $y = \frac{1}{x^2 - 1}$  3

b) i) Sketch the graph of  $y = -\sqrt{2 - x^2}$  5

ii) On the same diagram shade the region where  $y \geq -\sqrt{2 - x^2}$ ,  $|x| \leq 1$   
and  $y \leq 0$  hold simultaneously.

iii) Find the exact value for the area of the shaded region.

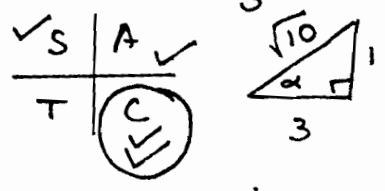
Question 1

a)  $x = 70$

b)  $a^2 - b^2 - (a-b)^2$   
 $(a-b)(a+b) - (a-b)^2$   
 $(a-b)((a+b) - (a-b))$   
 $\frac{2b(a-b)}{3}$

c)  $y = \frac{3}{a-x}$       $a-x=0$   
 $\therefore \underline{\underline{a=1}}$

d)  $\tan \alpha = -\frac{1}{3}$       $\cos \alpha > 0$



$\therefore \underline{\underline{\sin \alpha = -\frac{1}{\sqrt{10}}}}$

e)  $3^{\frac{-n}{3}}, 3^{\frac{n}{3}}, 3^n, 3^{-n}$   
 i) largest  $3^n$   
 ii) smallest  $3^{-n}$  } n a +ve integer

f)  $|5x-3| = |3x+1|$   
 $5x-3 = 3x+1$  or  $5x-3 = -3x-1$   
 $2x = 4$       $8x = 2$   
 $\underline{\underline{x=2}}$      or      $\underline{\underline{x=\frac{1}{4}}}$

b)  $\frac{a^n + a^{n+2}}{a^n} = 10$   
 $\cancel{a^n} (1 + a^2) = 10$

$\therefore a^2 = 9$

$\underline{\underline{a = 3 \text{ only } a > 0}}$

c)  $\frac{x-2}{x} \geq 1$

$x(x-2) \geq x^2$

$x(x-2) - x^2 \geq 0$

$x(x-2-x) \geq 0$

$-2x \geq 0$

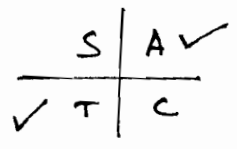
$x \leq 0$  but  $x \neq 0$

$\therefore \underline{\underline{x < 0}}$

d)  $\sin \theta = \cos \theta$

$\tan \theta = 1$

acute  $\theta = 45$



$\therefore \underline{\underline{\theta = 45^\circ, 225^\circ}}$

Question 3

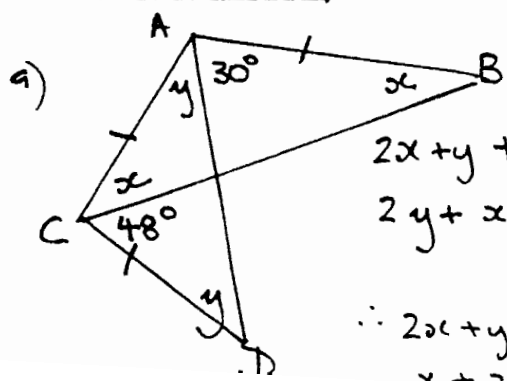
a)  $p = \frac{\sqrt{3}}{4-\sqrt{3}} \times \frac{4+\sqrt{3}}{4+\sqrt{3}} = \frac{4\sqrt{3}+3}{13}$

$q = \frac{\sqrt{3}}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{4\sqrt{3}-3}{13}$

$p+q = \frac{4\sqrt{3}+3+4\sqrt{3}-3}{13}$   
 $= \frac{8\sqrt{3}}{13}$

$1-pq = 1 - \frac{\sqrt{3}}{4-\sqrt{3}} \times \frac{\sqrt{3}}{4+\sqrt{3}}$

Question 2



$2x + y + 30 = 180$   
 $2y + x + 48 = 180$

$\therefore 2x + y = 150 \text{ --- (1)}$   
 $x + 2y = 132 \text{ --- (2)}$

$$\frac{p+q}{1-pq} = \frac{8\sqrt{3}}{13} \div \frac{10}{13}$$

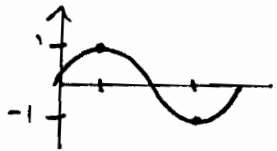
$$= \frac{8\sqrt{3}}{10}$$

$$= \frac{4\sqrt{3}}{5}$$

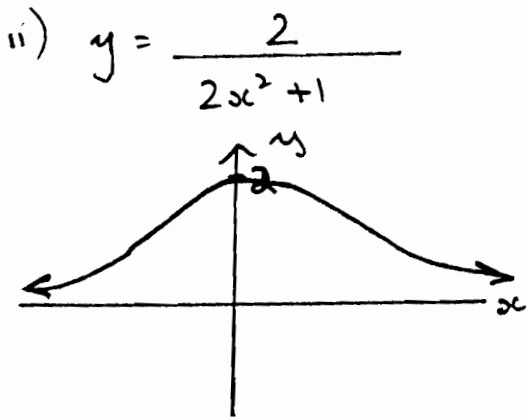
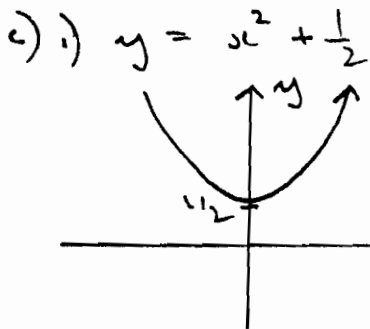
b)  $(\cos x - 2)(\sin^2 x - 1) = 0$

$\cos x = 2$  no solutions

$\sin^2 x = 1$   
 $\sin x = \pm 1$

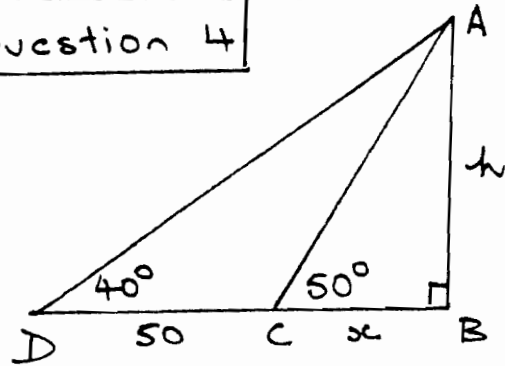


2 solutions



iii) Range  $0 < y \leq 2$

Question 4

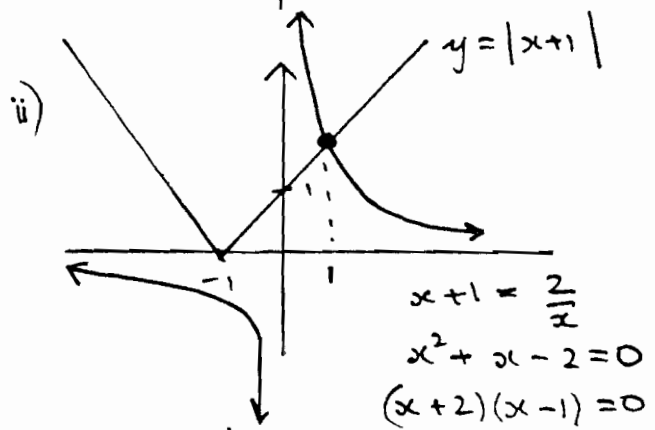
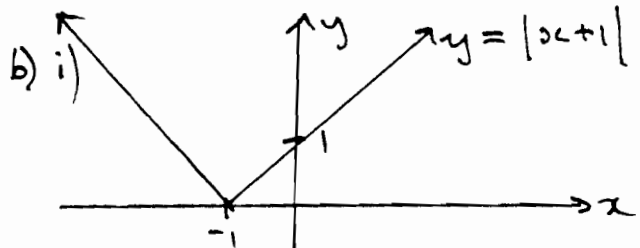


a)

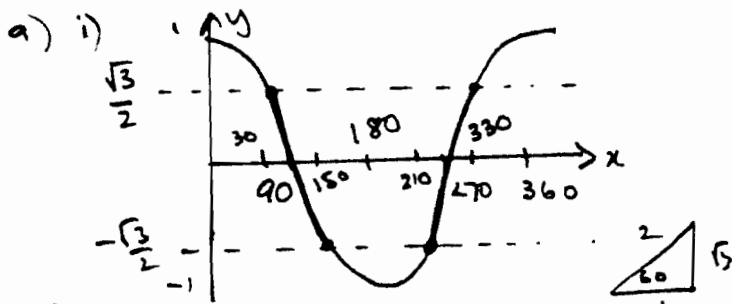
ii)  $\tan 50^\circ = \frac{h}{x}$   
 $\therefore h = x \tan 50^\circ$

i)  $\tan 40^\circ = \frac{h}{50+x}$   
 $(50+x) \tan 40^\circ = h$

iii)  $x \tan 50^\circ = (50+x) \tan 40^\circ$   
 $x \tan 50^\circ = 50 \tan 40^\circ + x \tan 40^\circ$   
 $x \tan 50^\circ - x \tan 40^\circ = 50 \tan 40^\circ$   
 $x (\tan 50^\circ - \tan 40^\circ) = 50 \tan 40^\circ$   
 $x = \frac{50 \tan 40^\circ}{\tan 50^\circ - \tan 40^\circ}$   
 $x = 118.97$  units (2 dec pl)



### Question 5



ii)

$$\cos x = \frac{\sqrt{3}}{2}, \quad \cos x = -\frac{\sqrt{3}}{2}$$

acute  $x = 30^\circ$

$$\underline{\underline{x = 30^\circ, 150^\circ}} \quad \underline{\underline{x = 210^\circ, 330^\circ}}$$

$\therefore$  Solution:  $30^\circ \leq x \leq 150^\circ$   
and  $210^\circ \leq x \leq 330^\circ$

b)

$$\frac{1}{c+x} + \frac{1}{c+y}$$

$$= \frac{c+y + c+x}{(c+x)(c+y)}$$

$$= \frac{2c + x + y}{c^2 + cx + cy + xy}$$

$$= \frac{2c + x + y}{2c^2 + cx + cy}$$

$$= \frac{(2c + x + y)}{c(2c + x + y)}$$

$$= \frac{1}{c}$$

$\therefore \frac{1}{c+x} + \frac{1}{c+y} = \frac{1}{c}$

ii) If  $x = \sqrt{51} + \sqrt{15}$

$y = \sqrt{51} - \sqrt{15}$

$$\frac{1}{6+x} + \frac{1}{6+y} = \frac{1}{6}$$

### Question 6

a)  $y = x^2 - 1$        $y = \frac{1}{x^2 - 1}$

$$x^2 - 1 = \frac{1}{x^2 - 1}$$

$$(x^2 - 1)^2 = 1$$

$$x^2 - 1 = 1 \quad \text{OR} \quad x^2 - 1 = -1$$

$$x^2 = 2$$

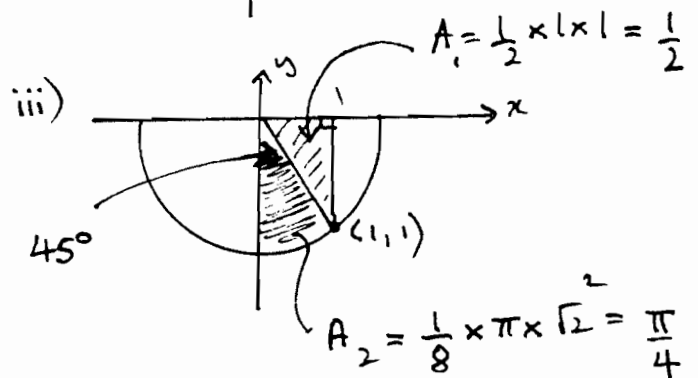
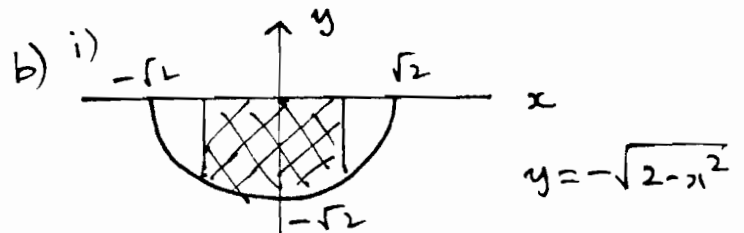
$$x^2 = 0$$

$$x = \pm\sqrt{2}$$

$$x = 0$$

$\therefore$  pts intersection

$(\sqrt{2}, 1)$     $(-\sqrt{2}, 1)$     $(0, -1)$



$\therefore$  Required area =  $2 \left( \frac{1}{2} + \frac{\pi}{4} \right)$