

SYDNEY TECHNICAL HIGH SCHOOL
(Est. 1911)

Year 11 Assessment Task 1

May 2007

Extension 1 Mathematics

Time allowed: 70 mins.

Instructions:

- Write neatly and do not cramp your work. Use a ruler to draw straight lines in diagrams.
- Marks may not be awarded for careless or badly arranged work.
- **Staple these questions to the front of your answers.**
- All necessary working must be shown.
- Indicated marks are for guidance and may be changed slightly during the marking process.

Q1	Q2	Q3	Q4	Q5	TOTAL
/11	/11	/12	/12	/12	/58

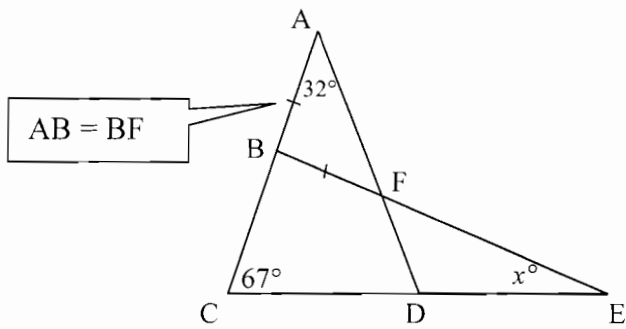
Question 1.

a) Factorise fully: $4a^2 - 9b^2 - 4a - 6b$ 2

b) Simplify $\frac{|x|}{x}$ ($x \neq 0$), stating the values of x for which your answer applies. 2

c) Test the function $f(x) = \frac{x}{2+x^2}$ and state whether it is *odd*, *even* or *neither*. 1

d)



From the following solution, copy the lines i), ii) and iii) onto your page and fill in the missing parts:

To find x :

$$\angle BAF = 32^\circ$$

i) \therefore _____ = 32° (_____) 1

ii) \therefore _____ = 64° (_____) 1

iii) Now $64^\circ +$ ___ + ___ = 180° (_____) 1

$$\therefore x = 49$$

e) Solve $\frac{4}{x-3} - \frac{6}{x+1} = 1$ 3

Question 2. (Begin a new page)

- a) Simplify $\frac{\frac{1}{a-1} + \frac{1}{a}}{\frac{1}{a-1} - \frac{1}{a}}$ 1
- b) i) Expand and simplify $(\sqrt{x} + \sqrt{y})^2$ 1
- ii) Hence or otherwise find values for x and y such that 2
- $$\sqrt{7+2\sqrt{10}} = \sqrt{x} + \sqrt{y}$$
- c) i) Sketch the function $y = \sqrt{4-x}$ showing clearly any x intercepts. 1
(Use a ruler to draw the axes)
- ii) Shade in the region $y \geq \sqrt{4-x}$ 2
- d) Solve for x : $\frac{3x+1}{x-3} \geq 2$ 4

Question 3. (Begin a new page)

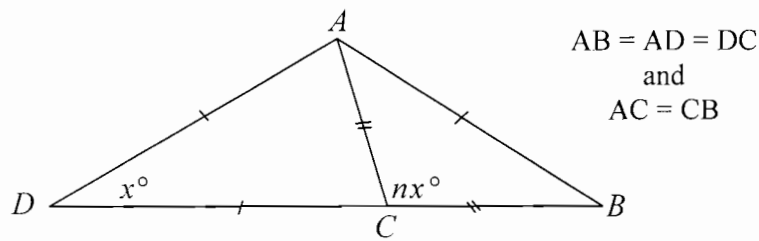
a) Solve $|x+2| = |3x-6|$ 2

b) Simplify fully:

i) $\frac{\tan A}{\sec A}$ 2

ii) $\cot A(\sec^2 A - 1)$ 2

c) Copy the following diagram onto your writing page.



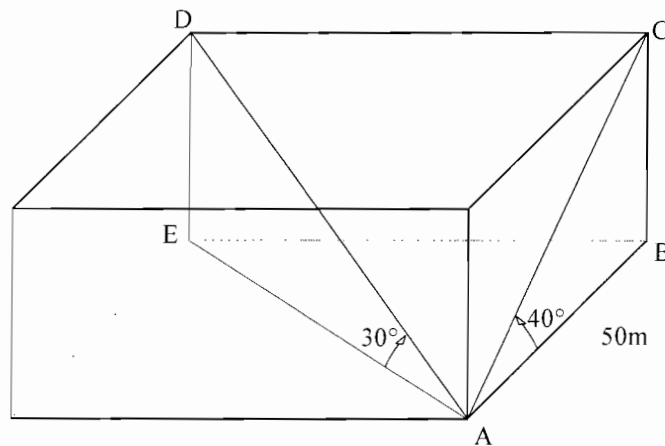
i) $\angle ACB$ is a multiple of x . Find the value of n . 1

ii) Determine any values of x which make this configuration of triangles possible. 2

d) Solve: $2\sin^2 2x = 1$ for $0^\circ \leq x \leq 360^\circ$ 3

Question 4. (Begin a new page)

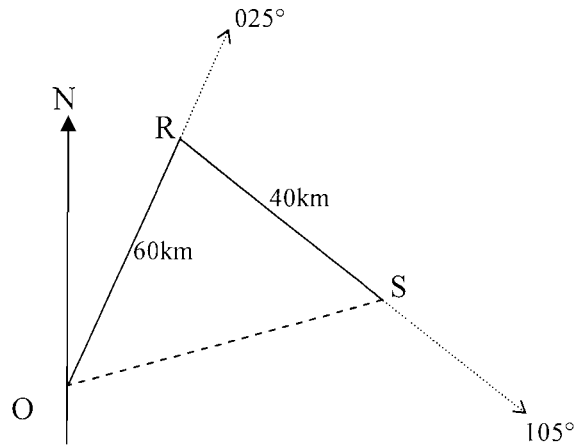
- a) If $\sin x = \frac{3}{5}$ and $90^\circ \leq x \leq 270^\circ$, find the value of $\tan x$. 2
- b) Find the sum of the interior angles of a regular polygon if each interior angle is 150° . 2
- d) i) Sketch $y = |2x - 4|$ showing clearly the x and y intercepts. 1
(Use a ruler to draw the axes).
- ii) Use your sketch to help solve $|2x - 4| < x + 4$. 3
- d) In a rectangular prism the face diagonal AC is inclined 40° to the base while the internal diagonal AD is inclined 30° to the base. The length of AB is 50m.



- i) Use trigonometry to find an *expression* for the height CB . 1
- ii) Use the result of part i) to find an *expression* for the length of EA . 1
- iii) Hence find the length of EB to the nearest metre. 2

Question 5. (Begin a new page)

- a) A ship sails for 60 kilometres from O to R on a bearing of 025° . It then changes course and sails for 40 kilometres to S on a bearing of 105° .



- | | | |
|-------|--|---|
| i) | Calculate the ship's distance from O to the nearest kilometre. | 3 |
| ii) | Find the size of $\angle ROS$ to the nearest degree. | 2 |
| iii) | Hence find the bearing of the ship from O (to the nearest degree). | 1 |
| | | |
| b) i) | Sketch the graph of $y = x^2 - 1$ (Use a ruler to draw the axes) | 1 |
| ii) | On the same diagram sketch the graph of $y = \frac{1}{x^2 - 1}$ showing clearly any intercepts and asymptotes. | 2 |
| iii) | The two graphs intersect at $(0, -1)$ and at two other points. Find the other two points of intersection. | 3 |

End of Test

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Question 1

a)
$$4a^2 - 9b^2 - 4a - 6b = (2a - 3b)(2a + 3b) - 2(2a + 3b)$$

$$= (2a + 3b)[(2a - 3b) - 2]$$

$$= (2a + 3b)(2a - 3b - 2)$$
2 marks
allow 1 for diff. of 2 squares

b)
$$\frac{|x|}{x} = 1 \text{ for } x > 0,$$

$$= -1 \text{ for } x < 0$$
1 mark for each

$$f(-x) = \frac{-x}{2 + (-x)^2}$$

c)
$$= -\frac{x}{2 + x^2}$$

$$= -f(x)$$

$$\therefore \text{function is odd}$$
1 if odd (even if test not shown)

- d) i) $\therefore \angle AFB = 32^\circ$ (angles opposite equal sides *or similar*) 1
 ii) $\therefore \angle CBE$ (*or equivalent*) = 64° (exterior angle of triangle) 1
 iii) Now $64^\circ + 67^\circ + x^\circ = 180^\circ$ (angle sum of triangle) 1

e)
$$\frac{4}{x-3} - \frac{6}{x+1} = 1$$
3 marks

$$4(x+1) - 6(x-3) = (x-3)(x+1)$$
 1

$$-2x + 22 = x^2 - 2x - 3$$
 1

$$x^2 - 25 = 0$$

 Therefore $x = \pm 5$ 1

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$$\frac{1}{a-1} + \frac{1}{a} = \frac{\left(\frac{1}{a-1} + \frac{1}{a}\right) \times a(a-1)}{\left(\frac{1}{a-1} + \frac{1}{a}\right) \times a(a-1)}$$

$$\frac{1}{a-1} - \frac{1}{a} = \frac{\left(\frac{1}{a-1} - \frac{1}{a}\right) \times a(a-1)}{\left(\frac{1}{a-1} - \frac{1}{a}\right) \times a(a-1)}$$

a)

$$= \frac{a+a-1}{a-(a-1)}$$

$$= \frac{2a-1}{1}$$

$$= 2a-1$$

1 mark (*working need not be shown*)

b) i) $(\sqrt{x} + \sqrt{y})^2 = x + y + 2\sqrt{xy}$

ii)

$$\sqrt{7+2\sqrt{10}} = \sqrt{x} + \sqrt{y}$$

$$\therefore 7+2\sqrt{10} = x+y+2\sqrt{xy}$$

$$\therefore x+y=7 \text{ and } xy=10$$

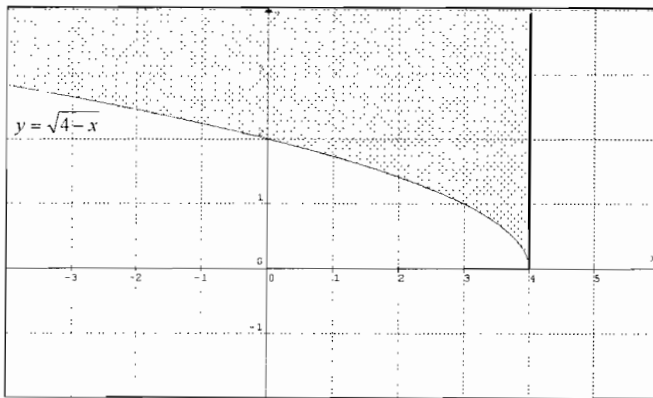
$$\therefore x=2, 5$$

$$y=5, 2$$

1 mark

2 marks,
only 1 set of values need be shown

c)



1 for correct graph

1 for shading above

1 for correct right boundary

d) $\frac{3x+1}{x-3} \geq 2$ Critical values at $x=3$ ($x \neq 3$) and when $3x+1 = 2x-6$ i.e. $x=-7$ 2 marks

When $x < -7$, $\frac{3x+1}{x-3} > 2$ ✓

When $-7 < x < 3$, $\frac{3x+1}{x-3} < 2$ ✗

When $x > 3$, $\frac{3x+1}{x-3} > 2$ ✓

$\therefore x \leq -7 \text{ or } x > 3$

1 mark

1 mark

Correct answer – 4 marks total
- suggestions shown.

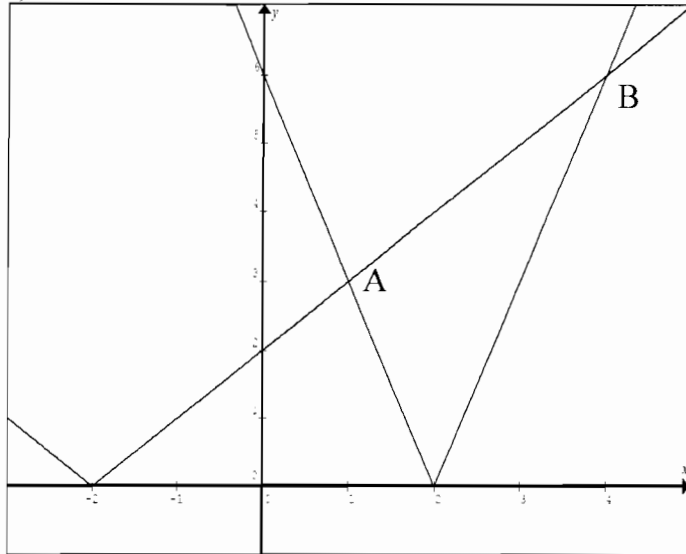
Allow 1 for excluding 3 from
the solution

Suggest in general, 1 mark off
for each error.

Marking rubric here should be

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a)



Solve $|x + 2| = |3x - 6|$

Solutions occur when

$$x + 2 = 6 - 3x \quad A$$

and

$$x + 2 = 3x - 6 \quad B$$

Which give $x = 1$

and $x = 4$

1 for each correct solution

b)

i)
$$\frac{\tan A}{\sec A} = \frac{\sin A}{\cos A} \times \cos A$$

$$= \sin A$$

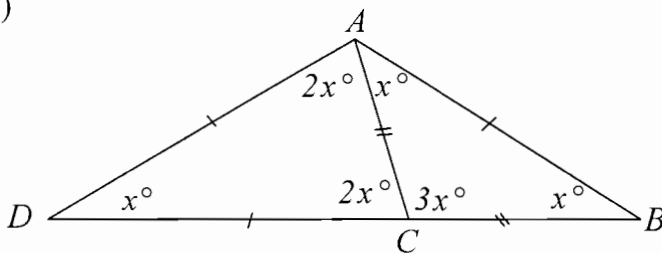
2 marks
Allow 1 for substitution

ii)
$$\cot A(\sec^2 A - 1) = \frac{1}{\tan A} \times \tan^2 A$$

$$= \tan A$$

2 marks
Allow 1 for substitution

c)



i) $n = 3$ 1 mark

ii) $5x = 180$

$\therefore x = 36$

2 marks
Allow 1 for correct equation

d) Solve: $2\sin^2 2x = 1$ for $0^\circ \leq x \leq 180^\circ$

$$\sin^2 2x = \frac{1}{2}$$

$$\sin 2x = \pm \frac{1}{\sqrt{2}}$$

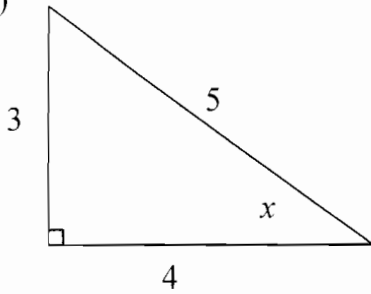
$$2x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

$$x = 22\frac{1}{2}^\circ, 67\frac{1}{2}^\circ, 112\frac{1}{2}^\circ, 157\frac{1}{2}^\circ$$

3 marks
(1 mark each step)

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a)



$$\sin x = \frac{3}{5} \text{ and } 90^\circ \leq x \leq 270^\circ,$$

x must be in the second quadrant

$$\therefore \tan x = -\frac{3}{4}$$

1 for $\frac{3}{4}$

1 for correct sign

b) Exterior angle = $30^\circ \therefore$ polygon has 12 sides

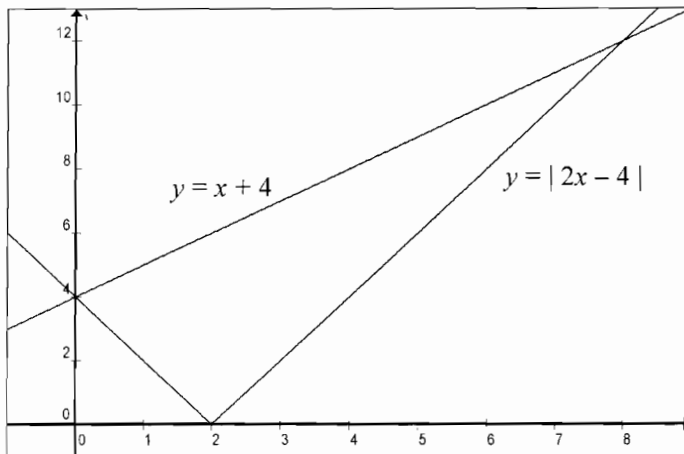
1 mark

$$\text{Interior sum} = 12 \times 150^\circ$$

$$= 1800^\circ$$

1 mark

c)



1 mark for $y = |2x - 4|$

Points of intersection occur when

$$x = 0 \text{ and when } x + 4 = 2x - 4$$

$$\text{i.e. } x = 0 \text{ and } x = 8$$

2 marks

$$\therefore |2x - 4| \leq x + 4 \text{ when } 0 \leq x \leq 8$$

1 mark

d)

i) $CB = 50 \tan 40^\circ.$

1 mark

ii)

$$\frac{EA}{DE} = \tan 60^\circ$$

$$\therefore EA = DE \tan 60^\circ$$

$$= CB \tan 60^\circ$$

$$= 50 \tan 40^\circ \tan 60^\circ$$

1 mark

iii)

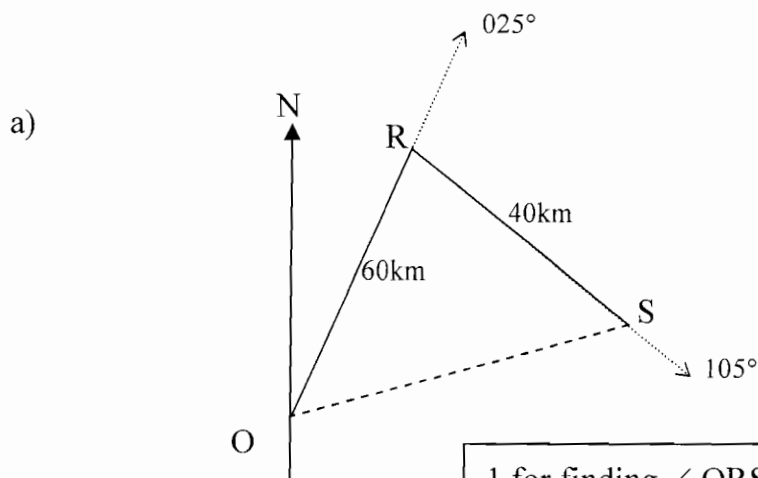
$$EB = \sqrt{EA^2 - AB^2}$$

2 marks

$$= \sqrt{2500 \tan^2 40^\circ \tan^2 60^\circ - 2500}$$

Allow 1 for correct

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1 for finding $\angle ORS$ and 1 for cosine formula

i) $OS^2 = 60^2 + 40^2 - 2 \cdot 60 \cdot 40 \cdot \cos 100^\circ$ 2 marks

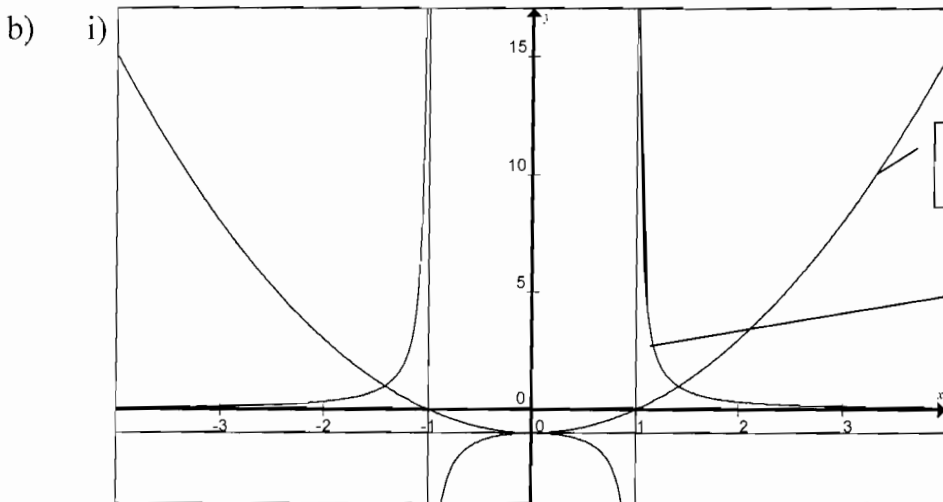
$\therefore OS = 78 \text{ km}$ 1 mark

ii) $\frac{\sin \angle ROS}{40} = \frac{\sin 100^\circ}{78}$ $\cos \angle ROS = \frac{78^2 + 60^2 - 40^2}{2 \cdot 78 \cdot 60}$

$\angle ROS = \sin^{-1}\left(\frac{40 \sin 100^\circ}{78}\right)$ OR $\therefore \angle ROS = \cos^{-1}\left(\frac{78^2 + 60^2 - 40^2}{2 \cdot 78 \cdot 60}\right)$

$\approx 30^\circ$ $\therefore \angle ROS \approx 30^\circ$ 3 marks

iii) Bearing of the ship from O is 055° . 1 mark



$y = x^2 - 1$ 1 mark

ii) $y = \frac{1}{x^2 - 1}$
1 for graph
1 for asymptotes

iii) $\frac{1}{x^2 - 1} = x^2 - 1$
 $\therefore (x^2 - 1)^2 = 1$
i.e. $x^2 - 1 = \pm 1$
 $\therefore x^2 = 0 \text{ or } 2$