

SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS EXTENSION 1

YEAR 11 COMMON TEST

MAY 2008

Time allowed: 70 minutes

Instructions:

- Show necessary working in every question .
- Start each question on a new page .
- Attempt all questions .
- Marks shown are approximate and may be changed .
- Full marks may not be awarded for careless or badly arranged work .
- Your sketches must be neat. Use a ruler to draw axes. Label the axes .
- Approved calculators may be used .
- These questions are to handed in with your answers .

Name: _____

Class: _____

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Total
/8	/8	/8	/9	/9	/9	/51

QUESTION 1

- a) Expand $(2x - 1)^3$ 1
- b) Solve $8^{x+1} = \frac{1}{4\sqrt{2}}$ 2
- c) Fully factorise
- i) $8x^3 + 1$ 1
- ii) $x^2 - xy - 2y^2$ 1
- iii) $y^2 - x^2 + 2x - 1$ 2
- d) Simplify $2^{n+3} - 2^{n+2}$ 1

QUESTION 2

- a) Simplify $\frac{\sqrt{3}-1}{\sqrt{3}+1} - \frac{\sqrt{3}+1}{\sqrt{3}-1}$ 2
- b) State the natural domain of $y = \frac{1}{\sqrt{4-x^2}}$ 2
- c) i) Sketch $y = x^2 - 3x - 4$ for $0 \leq x \leq 5$ 1
- ii) State the range for the above function for the given domain . 1
- d) Express $\frac{3^n + 3^n + 3^n}{9^n}$ in simplest index form . 2

QUESTION 3

a) True or false: " $|a + b| = |a| + |b|$ for all values a and b ".

Give an example to illustrate your response .

1

b) Solve $\frac{x}{x^2-4} < 0$

2

c) i) Simplify $\frac{x+3}{x^2-9}$

1

ii) Hence sketch $y = \frac{x+3}{x^2-9}$

Clearly label the axes, any asymptotes, intercepts and critical points.

2

d) Given that $\cos \theta = -\frac{2}{3}$ and $\tan \theta > 0$, find the exact value of $\sin \theta$.

2

QUESTION 4

a) Solve $\frac{2}{|2x-1|} > \frac{1}{2}$

3

b) i) On the same axes, neatly sketch $y = 4 - x^2$ and $y = \frac{1}{4-x^2}$

Label axes, asymptotes and intercepts .

3

ii) Find the x values of the points of intersection of the two curves above.

3

QUESTION 5

- a) Simplify $\frac{\cos(90^\circ - \theta)}{\sin(90^\circ + \theta)}$ 2
- b) Sketch $y = \frac{x+1}{|x+1|}$ 2
- c) i) Sketch $y = 2 - |x|$ 1
- ii) Hence or otherwise, solve $|1 - x| + |x| > 2$ 2
- d) Prove that $\cot \alpha + \tan \alpha = \sec \alpha \operatorname{cosec} \alpha$ 2

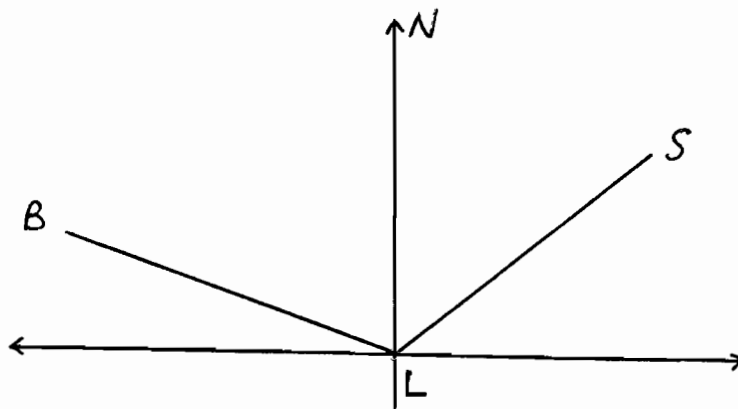
QUESTION 6

- a) Solve for $0^\circ \leq \theta \leq 360^\circ$. Give answers correct to the nearest minute where applicable.

i) $\frac{3}{\sin^2 \theta} = 4$ 2

ii) $3 \sec^2 \theta = 3 + 5 \tan \theta$ 3

- b)



From a lighthouse L a ship S bears 053° T and is at a distance of 8 nautical miles.

From L a boat B bears 293° T at a distance of 6 nautical miles.

- i) Draw the diagram above, marking the information supplied.
- ii) Find the distance of ship S from boat B. Give your answer as a surd. 2
- iii) Find the bearing of ship S from boat B. Give your answer to the nearest degree. 2

EXTENSION SOLUTIONS

Corrected Version

①

a) $\underline{8x^3 - 12x^2 + 6x - 1}$

b) $(2^3)^{x+1} = \frac{1}{2^2 \cdot 2^{\frac{1}{2}}}$

$2^{3x+3} = 2^{-5/2}$

$3x+3 = -2\frac{1}{2}$

$3x = -5\frac{1}{2}$

$\underline{\underline{x = -1\frac{5}{6}}}$

c) i) $\underline{(2x+1)(4x^2-2x+1)}$

ii) $\underline{(x+y)(x-2y)}$

iii) $y^2 - (x^2 - 2x + 1)$

$= y^2 - (x-1)^2$

$= \underline{\underline{(y+x-1)(y-x+1)}}$

d) $2 \cdot 2^{n+2} - 2^{n+2} = \underline{\underline{2^{n+2}}}$

②

a) $\frac{(\sqrt{3}-1)^2 - (\sqrt{3}+1)^2}{3-1}$

$= \frac{3 - 2\sqrt{3} + 1 - (3 + 2\sqrt{3} + 1)}{2}$

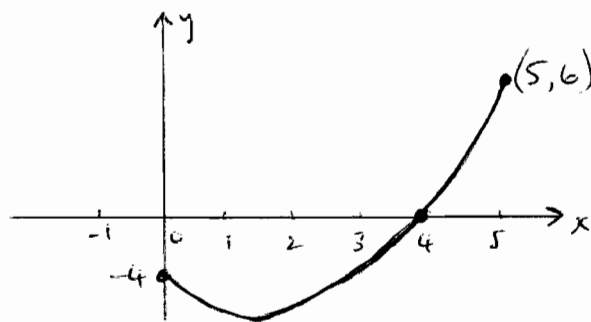
$= \frac{-4\sqrt{3}}{2}$

$= \underline{\underline{-2\sqrt{3}}}$

b) $4 - x^2 > 0$

$\therefore \underline{\underline{-2 < x < 2}}$

c) i) $y = (x-4)(x+1)$



ii) Vertex at $x = 1\frac{1}{2}, y = -6\frac{1}{4}$

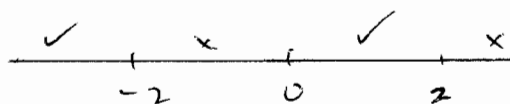
$\therefore R : \underline{\underline{-6\frac{1}{4} \leq y \leq 6}}$

d) $\frac{3 \times 3^{n+1}}{(3^2)^n} = \frac{3^{n+1}}{3^{2n}}$
 $= 3^{n+1-2n}$
 $= \underline{\underline{3^{1-n}}}$

③ a) False, eg. $|-3+4| \neq |-3|+|4|$

b) $\frac{x}{x^2-4} \times (x^2-4)^2 < 0 \times (x^2-4)^2$

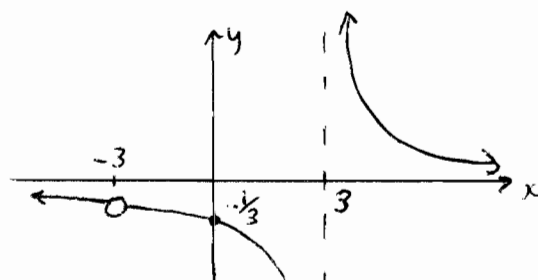
$x(x+2)(x-2) < 0$



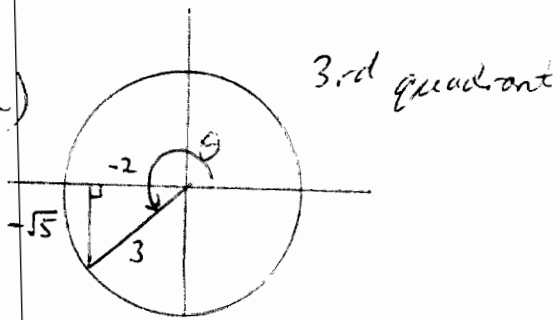
$\therefore \underline{\underline{x < -2 \text{ or } 0 < x < 2}}$

c) i) $\frac{x+3}{(x+3)(x-3)} = \frac{1}{x-3}$

ii)



d)



$$\therefore \sin \theta = \underline{\underline{-\frac{\sqrt{5}}{3}}}$$

4) a) $|2x-1|$ is positive and $x \neq \frac{1}{2}$

$$\therefore 2 > \frac{1}{2} |2x-1|$$

$$\therefore |2x-1| < 4$$

$$\therefore 2x-1 < 4 \text{ or } -(2x-1) < 4$$

$$x < 2\frac{1}{2}$$

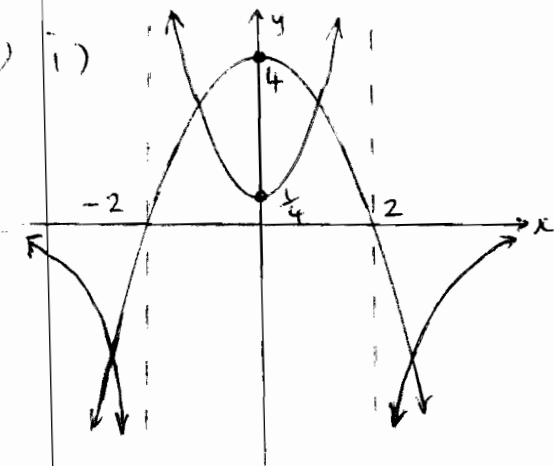
$$-2x+1 < 4$$

$$-2x < 3$$

$$x > -\frac{1}{2}$$

$$\therefore \underline{\underline{-\frac{1}{2} < x < 2\frac{1}{2}, x \neq \frac{1}{2}}}$$

b) i)



$$ii) 4 - x^2 = \frac{1}{4 - x^2}$$

$$(4 - x^2)^2 = 1$$

$$16 - 8x^2 + x^4 = 1$$

$$x^4 - 8x^2 + 15 = 0$$

$$(x^2 - 5)(x^2 - 3) = 0$$

5)

$$a) \frac{\sin \theta}{\sin(90 - \theta)} = \frac{\sin \theta}{\cos \theta} = \underline{\underline{\tan \theta}}$$

$$b) y = \frac{x+1}{|x+1|}$$

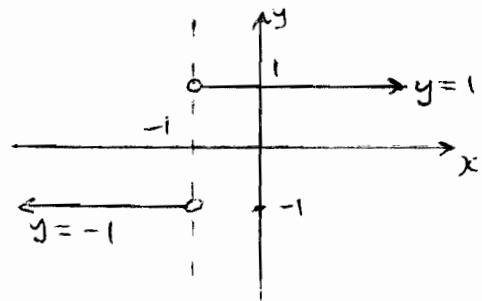
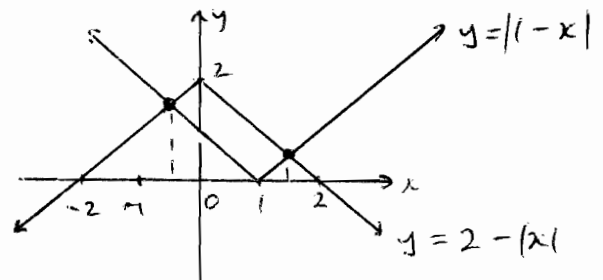
$$= \frac{x+1}{x+1} \text{ for } x+1 > 0 (x > -1)$$

$$\therefore y = 1 \text{ for } x > -1$$

$$\text{OR } y = \frac{x+1}{|x+1|}$$

$$= \frac{x+1}{-(x+1)} \text{ for } x+1 < 0 (x < -1)$$

$$\therefore y = -1 \text{ for } x < -1$$

c) i) $y = -|x| + 2$ 

$$ii) |1-x| > 2 - |x|$$

$$\therefore \underline{\underline{x < -\frac{1}{2} \text{ or } x > \frac{1}{2}}}$$

$$d) \text{LHS} = \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$$

$$= \frac{1}{\sin x \cos x}$$

$$= \frac{1}{\sin x} \times \frac{1}{\cos x}$$

6

a) i) $3 = 4 \sin^2 \theta$

$\sin^2 \theta = \frac{3}{4}$

$\therefore \sin \theta = \pm \frac{\sqrt{3}}{2}$

$\therefore \theta = 60^\circ$ (all 4 quadrants)
 $= 60^\circ, 120^\circ, 240^\circ, 300^\circ$

ii) $3(1 + \tan^2 \theta) = 3 + 5 \tan \theta$

$3 + 3 \tan^2 \theta - 5 \tan \theta - 3 = 0$

$3 \tan^2 \theta - 5 \tan \theta = 0$

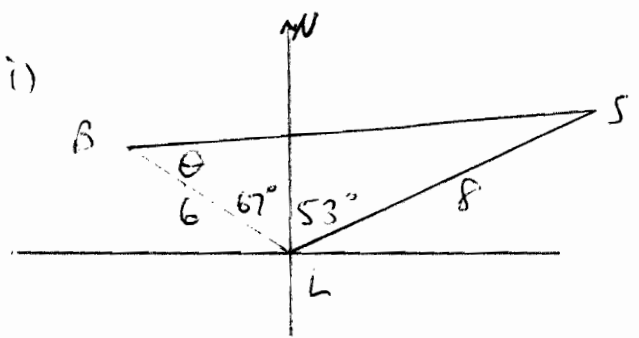
$\tan \theta (3 \tan \theta - 5) = 0$

$\therefore \tan \theta = 0$ or $\frac{5}{3}$

$\therefore \theta = 0^\circ, 180^\circ, 360^\circ$ or $59^\circ 2'$ (1st, 3rd quadrants)

$\therefore \theta = 0^\circ, 180^\circ, 360^\circ, 59^\circ 2', 239^\circ 2'$

b) i)

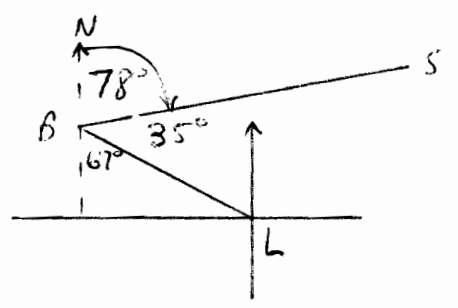


ii) $BS^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 120^\circ$
 $= 100 - 96(-\frac{1}{2})$
 $= 148$

$\therefore BS = \sqrt{148}$ n.m.

iii) $\frac{8}{\sin \theta} = \frac{\sqrt{148}}{\sin 120^\circ}$

$\therefore \theta = 35^\circ$



\therefore Bearing of S from B is $078^\circ T$ or $N 78^\circ E$