

SYDNEY TECHNICAL HIGH SCHOOL



Mathematics Extension 1

YEAR 11 PRELIMINARY HSC
ASSESSMENT TASK 1
MAY 2009

General Instructions

- Working time allowed – 70 minutes
- Write using black or blue pen
- Approved calculators may be used
- All necessary working should be shown
- Start each question on a new page
- Attempt all questions

NAME : _____

TEACHER : _____

QUESTION 1	QUESTION 2	QUESTION 3	QUESTION 4	QUESTION 5	QUESTION 6	TOTAL

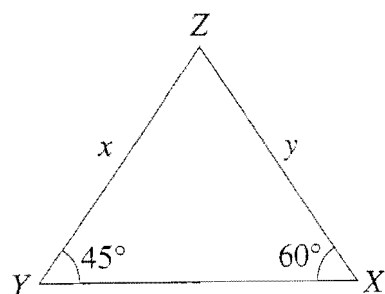
Question 1 (10 marks)

Marks

a) Simplify $\frac{1}{1-2x} - \frac{2x}{1-4x^2}$

2

b)



NOT TO
SCALE

In the diagram, XYZ is a triangle where $\angle ZYX = 45^\circ$ and $\angle ZXY = 60^\circ$.

2

Find the exact value for the ratio $\frac{x}{y}$.

c) Write $(\sqrt{6} + 2)^3$ in the form $a + b\sqrt{6}$ where a and b are integers.

2

d)

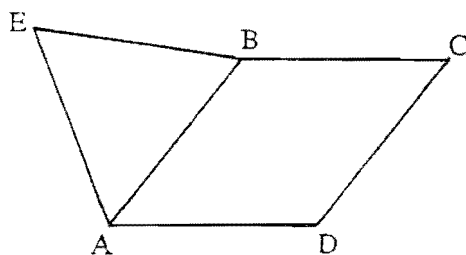


Figure not
to scale.

$ABCD$ is a rhombus with $\angle CDA = 134^\circ$.

ABE is an equilateral triangle.

i) Find the size of $\angle EAD$, giving reasons for your answer.

2

ii) Find the size of $\angle EDA$, giving reasons for your answer.

2

Question 2 (9 marks) Start a new page

Marks

a) Fully factorise $x^2 - 4ax + 4a^2 - 4b^2$ 2

b) If $\tan \theta = \frac{2}{5}$ and $\cos \theta < 0$ find the exact value of $\sin \theta$. 2

c) Find a value of x that satisfies the equation $\cos 2x = \sin 80^\circ$ 1

d) Solve for x : $2^{-4x} \times 4^{3x-2} = 16$ 2

e) Draw a neat sketch of the function $y = g(x)$ 2

given that
$$g(x) = \begin{cases} 1-x, & x < 1 \\ \sqrt{x}, & x \geq 1 \end{cases}$$

Question 3 (9 marks) Start a new page

a) Solve $2 \cos 2\theta + 1 = 0$ for $0^\circ \leq \theta \leq 360^\circ$ 3

b) Prove that $\frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x} = \tan x$ 2

c) Consider the function $y = \frac{x^2}{1-x^2}$. 4

i) State the equations of the vertical asymptotes.

ii) State the equation of the horizontal asymptote.

iii) Draw a neat sketch of $y = \frac{x^2}{1-x^2}$

Question 4 (9 marks) Start a new page

Marks

a) Show that $8^{120} - 8^{119} - 8^{118}$ is divisible by 55. 1

b) Solve the equation $3\sin^2\theta = 2\cos\theta + 2$ for $0^\circ \leq \theta \leq 360^\circ$. 4

Give your answers correct to the nearest degree.

c) A regular polygon has an interior angle of 162° . 1

How many sides does the polygon have ?

d) Solve simultaneously for x and y : 3

$$25x^2 - y^2 = 84 \quad \text{and} \quad 5x - y = 6.$$

Question 5 (9 marks) Start a new page

Marks

a) Solve $\frac{x}{x+1} \geq 2$ 3

b) Solve $8 \sin \theta \cos \theta \tan \theta = \operatorname{cosec} \theta$ for $0^\circ \leq \theta \leq 360^\circ$ 2

c) Solve $y^{\frac{2}{3}} = 64$ 1

d) Simplify $\frac{(1 - \tan x)^2 + (1 + \tan x)^2}{(1 + \cot x)^2 + (1 - \cot x)^2}$ 3

Please turn over for question 6

Question 6 (9 marks) Start a new page

Marks

a)

5

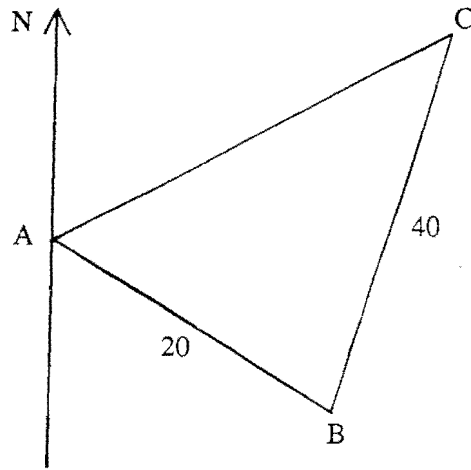


Figure not
to scale

A ship sails 20 kilometres from point A on a bearing of $146^\circ T$ to point B.

The ship then sails 40 kilometres from point B on a bearing of $015^\circ T$ to point C.

- i) Find the size of angle ABC.
- ii) Find the distance between point A and point C
correct to the nearest hundred metres.
- iii) Find the bearing of point A from point C
correct to the nearest degree.

b) i) Draw a neat sketch of the function $y = \frac{20}{|x|}$ 1

ii) Solve $\frac{20}{|x|} < 17 - 3x$ 3

End of Paper

QUESTION 1

$$a) \frac{1}{1-2x} - \frac{2x}{(1-2x)(1+2x)}$$

$$= \frac{1+2x-2x}{(1-2x)(1+2x)}$$

$$= \frac{1}{(1-2x)(1+2x)}$$

$$b) \frac{x}{\sin 60^\circ} = \frac{y}{\sin 45^\circ}$$

$$\therefore \frac{x}{y} = \frac{\sin 60^\circ}{\sin 45^\circ}$$

$$= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}}$$

$$= \frac{\sqrt{6}}{2}$$

$$c) (\sqrt{6} + 2)^3$$

$$= 6\sqrt{6} + 36 + 12\sqrt{6} + 8$$

$$= 44 + 18\sqrt{6}$$

d) i) $\angle BAD = 46^\circ$
 (co-interior angles, $AB \parallel DC$)
 $\angle EAB = 60^\circ$
 (angle in equilateral triangle)
 $\therefore \angle EAD = 60^\circ + 46^\circ$
 $= 106^\circ$

ii) $\angle EDA = \angle AED$
 (angles opposite equal sides are equal)

$$\therefore 2 \times \angle EDA + 106^\circ = 180^\circ$$

(angle sum of a triangle)

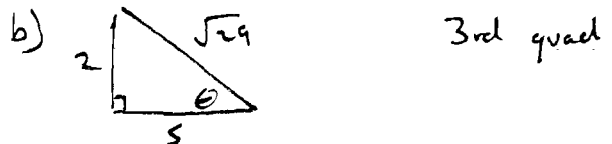
$$\therefore \angle EDA = 37^\circ$$

QUESTION 2

$$a) x^2 - 4ax + 4a^2 - 4b^2$$

$$= (x-2a)^2 - 4b^2$$

$$= (x-2a+2b)(x-2a-2b)$$



$$\therefore \sin \theta = -\frac{2}{\sqrt{29}}$$

$$c) 2x + 80 = 90$$

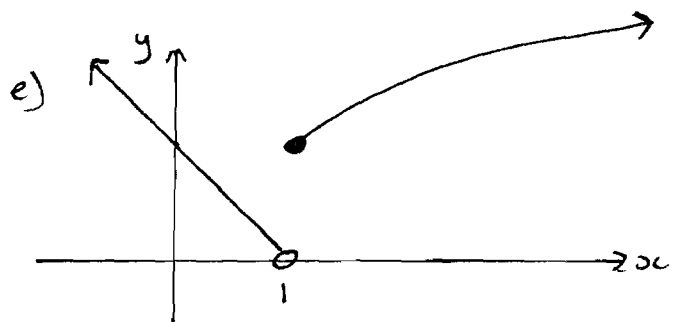
$$x = 5^\circ$$

$$d) 2^{-4x} \times 4^{3x-2} = 16$$

$$2^{-4x} \times 2^{6x-4} = 2^4$$

$$-4x + 6x - 4 = 4$$

$$x = 4$$



QUESTION 3

a) $2 \cos 2\theta + 1 = 0$

$$\cos 2\theta = -\frac{1}{2}$$

$$2\theta = 120^\circ, 240^\circ, 480^\circ, 600^\circ$$

$$\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$$

b)

$$\text{LHS} = \frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x}$$

$$= \frac{1 - \cos^2 x}{\sin x \cos x}$$

$$= \frac{\sin^2 x}{\sin x \cos x}$$

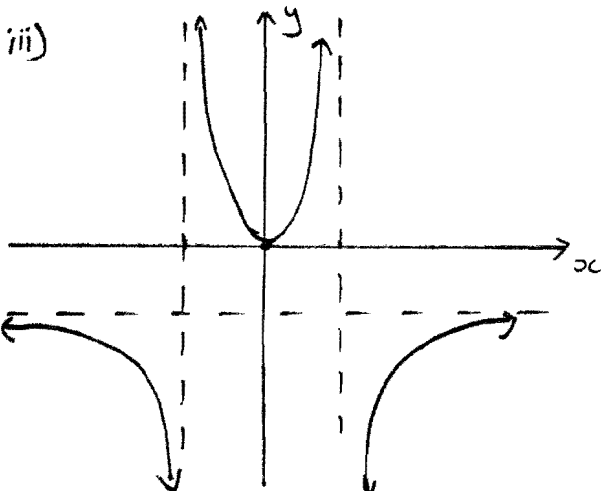
$$= \frac{\sin x}{\cos x}$$

$$= \tan x$$

$$= \text{RHS}$$

c) i) $x = 1$ and $x = -1$

ii) $y = -1$



QUESTION 4

a) $8^{120} - 8^{119} - 8^{118}$

$$= 8^{118} (8^2 - 8 - 1)$$

$$= 55 \times 8^{118}$$

which is divisible by 55.

b) $3 \sin^2 \theta = 2 \cos \theta + 2$

$$3(1 - \cos^2 \theta) = 2 \cos \theta + 2$$

$$3 \cos^2 \theta + 2 \cos \theta - 1 = 0$$

$$(3 \cos \theta - 1)(\cos \theta + 1) = 0$$

$$\cos \theta = \frac{1}{3}, -1$$

$$\theta = 71^\circ, 289^\circ, 180^\circ$$

c) exterior angle = $180^\circ - 162^\circ$
 $= 18^\circ$

$$\therefore \text{number of sides} = \frac{360}{18}$$
$$= 20$$

d) $25x^2 - y^2 = 84$ — (1)
 $5x - y = 6$

$$\therefore y = 5x - 6 \text{ sub. into (1)}$$

$$25x^2 - (5x - 6)^2 = 84$$

$$60x - 36 = 84$$

$$x = 2$$

$$\therefore y = 4$$

QUESTION 5

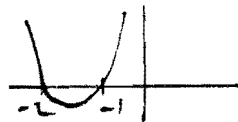
a) $\frac{x}{x+1} \geq 2$

$$\frac{x(x+1)^2}{x+1} \geq 2(x+1)^2$$

$$x(x+1) \geq 2x^2 + 4x + 2$$

$$x^2 + 3x + 2 \leq 0$$

$$(x+2)(x+1) \leq 0$$



$$-2 \leq x < -1$$

b) $8 \sin \theta \cos \theta \tan \theta = \operatorname{cosec} \theta$

$$8 \sin \theta \cos \theta \frac{\sin \theta}{\cos \theta} = \frac{1}{\sin \theta}$$

$$8 \sin^3 \theta = 1$$

$$\sin^3 \theta = \frac{1}{8}$$

$$\sin \theta = \frac{1}{2}$$

$$\theta = 30^\circ, 150^\circ$$

c) $y = \pm 512$

d)

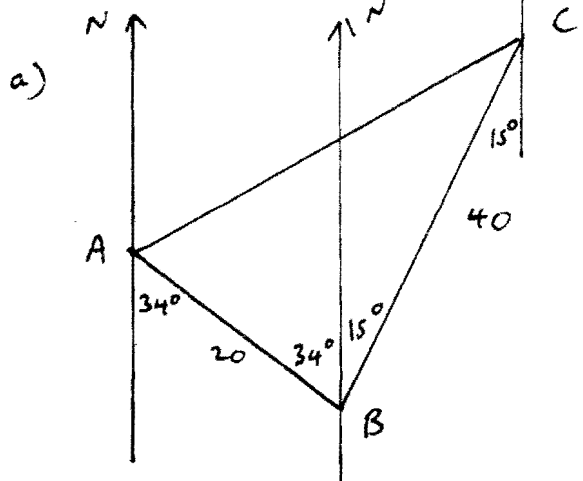
$$\frac{1 - 2 \tan x + \tan^2 x + 1 + 2 \tan x + \tan^2 x}{1 + 2 \cot x + \cot^2 x + 1 - 2 \cot x + \cot^2 x}$$

$$= \frac{2(1 + \tan^2 x)}{2(1 + \cot^2 x)}$$

$$= \frac{2 \sec^2 x}{2 \operatorname{cosec}^2 x}$$

$$= \tan^2 x$$

QUESTION 6



i) $\angle ABC = 49^\circ$

ii) $AC^2 = 20^2 + 40^2 - 2 \times 20 \times 40 \times \cos 49^\circ$

$$AC = 30.8 \text{ km}$$

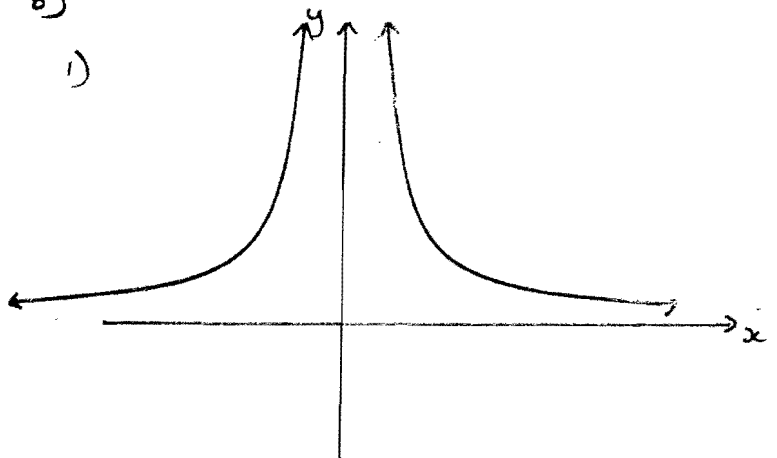
iii) $\frac{\sin C}{20} = \frac{\sin 49^\circ}{30.8}$

$$\angle C = 29^\circ$$

$$\therefore \text{bearing} = 180^\circ + 15^\circ + 29^\circ = 224^\circ$$

b)

i)



$$ii) \quad \frac{20}{|x|} < 17 - 3x$$

$$\text{Solve } \frac{20}{|x|} = 17 - 3x$$

$$\frac{20}{x} = 17 - 3x \quad \text{or} \quad \frac{20}{-x} = 17 - 3x$$

$$20 = 17x - 3x^2$$

$$20 = -17x + 3x^2$$

$$3x^2 - 17x + 20 = 0$$

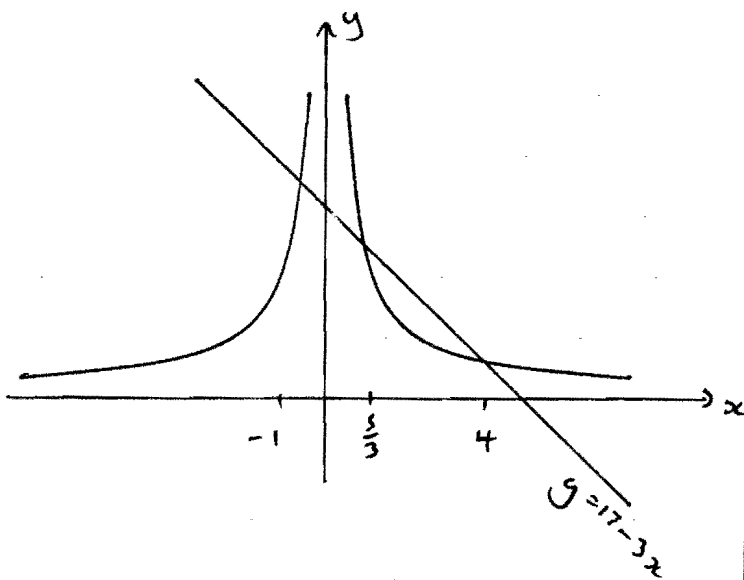
$$3x^2 - 17x - 20 = 0$$

$$(3x-5)(x-4) = 0$$

$$(3x-20)(x+1) = 0$$

$$= \frac{5}{3}, 4$$

$$x = \frac{20}{3}, -1$$



$$\therefore x < -1, \quad \frac{5}{3} < x < 4$$