Name: ______________________

Teacher: ______________________

SYDNEY TECHNICAL HIGH SCHOOL

MANNERS MAKYTH MAN

Mathematics Extension 1

Preliminary Assessment 1
May 2010

Time allowed – 70 minutes

Instructions

- Use a blue or black pen.
- Approved calculators may be used.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks awarded are shown on each question.
- Total marks – 55
- Attempt all questions.
- Start each question on a new page.
- Hand in your examination paper and solutions in one bundle.

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Total

Question 1 **Marks 8**

a) Solve: $-12 \leq 1 - 2x \leq -3$. *(2)*

b) Sketch the function $f(x) = \begin{cases} 1-x, for\ x<0 \\ -1, for\ x=0 \\ 1-x^2\ for\ x>0 \end{cases}$ *(2)*

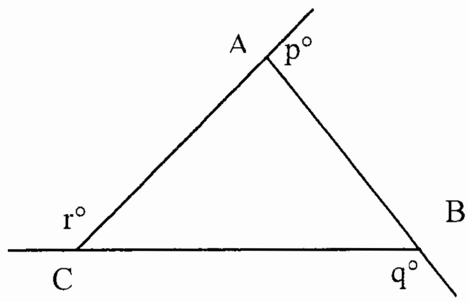
c) Solve $\frac{3}{x-3} - \frac{5}{x+1} = 1$. *(2)*

d) Solve by completing the square $x^2 - 4x + 1 = 0$ *(2)*

Question 2 (Start a new page) **Marks 10**

a) Find the values x and y such that: $6 + \sqrt{x-y} = x + y + 3\sqrt{2}$. *(3)*

b) For the diagram shown, prove giving reasons that $p + q + r = 360$. *(2)*



c) Solve for x and show on a number line: *(3)*

$$\frac{4x+1}{x-4} \geq 2$$

d) Write down the exact value of *cosec 240°*, leaving denominator irrational. *(2)*

Question 3 (Start a new page) **Marks 9**

a) For the function $y = \frac{1}{x^2-1}$:

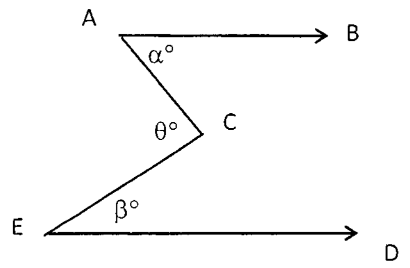
(i) State the domain. *(1)*

(ii) Find all asymptotes. *(2)*

(iii) Hence, sketch the curve. *(2)*

b) Solve $|2x+1| = |x-5|$. *(2)*

c) Express θ in terms of α and β, giving reasons. *(2)*



Question 4 (Start a new page) **Marks 10**

a) Solve $2\cos 2x = \sqrt{3}$, for $0° \leq x° \leq 360°$. *(3)*

b) Solve for $0° \leq \theta \leq 360°$ *(2)*

$\sin\theta = \cos\theta$

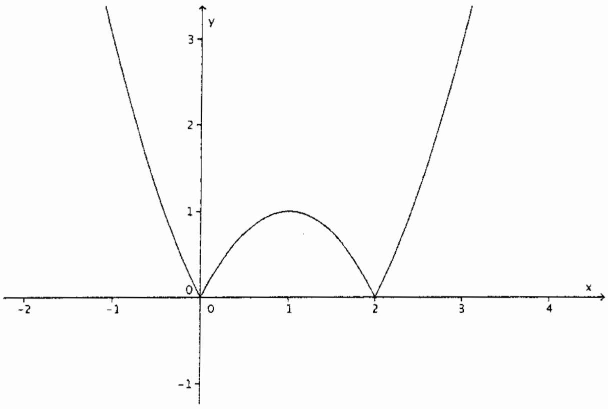
c) Show that $(1 + \tan A + \sec A)(1 + \tan A - \sec A) = 2\tan A$ *(3)*

d) If $x - \frac{1}{x} = 3\sqrt{5}$, find the value of $x^2 + \frac{1}{x^2}$. (2)

Question 5 (Start a new page) **Marks 8**

a) Show algebraically whether the function $f(x) = 3x(3 - x^2)$ is *odd*, *even* or neither. *(2)*

b) Write a possible equation for the curve shown: *(2)*



c) Simplify $\frac{3^{n+2} \times 12^n}{4^{n-1}}$ *(2)*

d) Eliminate θ from the pair of equations. (Leave in factorised form). *(2)*

$x = 1 + \sec\theta$

$y = 2 + \tan\theta$

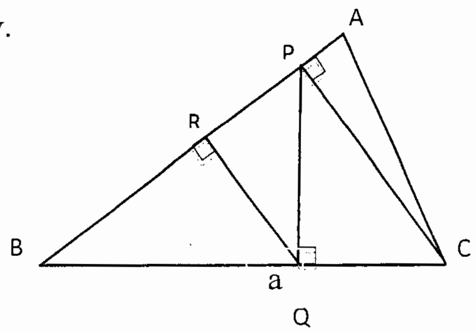
Question 6 (Start a new page) **Marks 10**

a) By treating the expression as a difference of two squares, express it as the product of four factors: $x^6 - y^6$. *(2)*

b) In triangle *ABC*, lines *CP*, *PQ* and *QR* are drawn perpendicular to *AB*, *BC* and *AB* respectively.

(i) Explain why $\angle RBQ = \angle RQP = \angle QPC$. *(3)*

(ii) Show that $QR = a \sin B \cos^2 B$. *(2)*



c) Simplify $\frac{1}{(a-b)(a-c)} + \frac{1}{(b-c)(b-a)} + \frac{1}{(c-a)(c-b)}$ *(3)*

Q1 a) Solve: $-12 \le 1-2x \le -3$

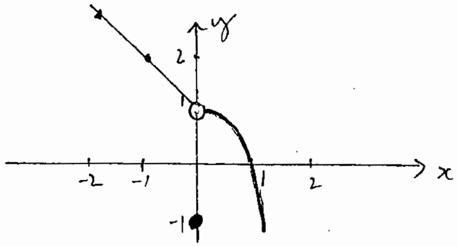
$-12 \le 1-2x \le -3$

$\therefore -13 \le -2x \le -4$

$\therefore 4 \le 2x \le 13$

$\therefore 2 \le x \le 6.5$

b)



c) $\dfrac{3}{x-3} - \dfrac{5}{x+1} = 1$

$\therefore 3(x+1) - 5(x-3) = (x-3)(x+1)$

$\therefore 3x+3-5x+15 = x^2-2x-3$

$\therefore x^2 = 21$

$\therefore x = \pm\sqrt{21}$

d) $x^2 - 4x + 1 = 0$

$\therefore (x-2)^2 - 4 + 1 = 0$

$\therefore (x-2)^2 = 3$

$\therefore x-2 = \pm\sqrt{3}$

$\therefore x = 2 \pm \sqrt{3}$

Q2

a) $6 + \sqrt{x-y} = x+y+3\sqrt{2}$
$= x+y+\sqrt{18}$

$\therefore x+y = 6$

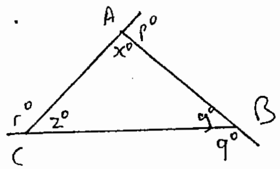
$x-y = 18$

$\therefore 2x = 24$

$\therefore x = 12$

$\therefore y = -6$

b.



Let $C\hat{A}B = x^\circ$
$A\hat{B}C = y^\circ$
$B\hat{C}A = z^\circ$

$p+x = 180$ (straight angle)

$q+y = 180$ (" ")

$r+z = 180$ (" ")

$\therefore p+x+q+y+r+z = 3\times180$
$= 540$

$\therefore p+q+r+x+y+z = 540$

But $x+y+z = 180$ (angle sum of Δ)

$\therefore p+q+r + 180 = 540$

$\therefore p+q+r = 360$ QED

c) $\dfrac{4x+1}{x-4} \ge 2$ NB $x \ne 4$

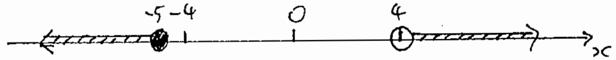
$\therefore (4x+1)(x-4) \ge 2(x-4)^2$

$\therefore 4x^2 - 15x - 4 \ge 2x^2 - 16x + 32$

$\therefore 2x^2 + x - 36 \ge 0$

$\therefore (2x+9)(x-4) = 0$] provides boundaries

$\therefore x = -4.5, 4$



For $x < -4.5$, $\dfrac{4x+1}{x-4} \ge$

For $-4.5 < x < 4$, $\dfrac{4x+1}{x-4}$

For $x > 4$, $\dfrac{4x+1}{x-4} \ge 2$

For $x = 4$, $\dfrac{4x+1}{x-4}$ is undef

d) $\text{cosec } 240^\circ = \dfrac{1}{\sin 240^\circ}$

$= \dfrac{-1}{\sin 60^\circ}$

$= \dfrac{-2}{\sqrt{3}}$

$\left(= \dfrac{-2\sqrt{3}}{3} \right)$

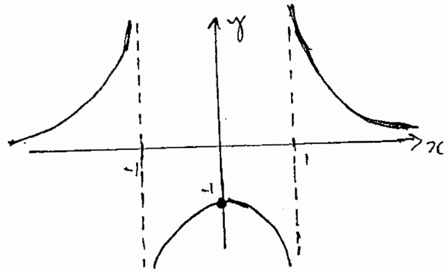
Q3 a) $y = \dfrac{1}{x^2-1}$

(i) domain $\Rightarrow$ all real x except $x = \pm1$

(ii) vertical asymptotes when $x = \pm1$

horizontal asymptotes when $y = 0$

(iii)



As $x \to 1^-$, $y \to -\infty$

As $x \to -1^+$, $y \to -\infty$

As $x \to 1^+$, $y \to \infty$

As $x \to -1^-$, $y \to \infty$

As $x \to \infty$, $y \to 0^+$

As $x \to -\infty$, $y \to 0^+$

Q3 cont

b) $|2x+1| = |x-5|$

$(2x+1)^2 = (x-5)^2$

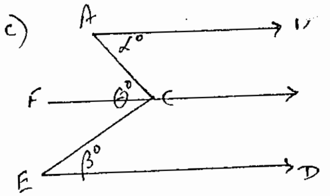
$\therefore 4x^2+4x+1 = x^2-10x+25$

$\therefore 3x^2+14x-24=0$

$\therefore (3x-4)(x+6)=0$

$\therefore x = \frac{4}{3}, -6$

c)



Construct FC parallel to AB and ED

$F\hat{C}A = \alpha°$ (alternate angles)

$F\hat{C}E = \beta°$ (" ")

$\therefore \theta = \alpha + \beta$

Q4

a) $2\cos 2x = \sqrt{3}$, for $0° \le x \le 360°$

$\therefore \cos 2x = \frac{\sqrt{3}}{2}$

$\therefore 2x = 30°, 330°, 390°, 690°$

$\therefore x = 15°, 165°, 195°, 345°$

b) $\sin\theta = \cos\theta$ $\therefore$ first quadrant.

$\therefore \frac{\sin\theta}{\cos\theta} = 1$

$\therefore \tan\theta = 1$

$\therefore \theta = 45$ only

c) RTS $(1+\tan A+\sec A)(1+\tan A-\sec A) = 2\tan A$

LHS $= (1+\tan A+\sec A)(1+\tan A-\sec A)$

$= (1+\tan A)^2 - \sec^2 A$

$= 1+2\tan A+\tan^2 A-\sec^2 A$

$= 2\tan A+\sec^2 A-\sec^2 A$

$= 2\tan A$

$=$ RHS

QED

d) $x - \frac{1}{x} = 3\sqrt{5}$

$\therefore \left(x-\frac{1}{x}\right)^2 = 45$

$\therefore x^2 - 2 + \frac{1}{x^2} = 45$

$\therefore x^2 + \frac{1}{x^2} = 47$

Q5

a)

$f(x) = 3x(3-x^2)$

$f(-x) = -3x(3-(-x)^2)$

$= -3x(3-x^2)$

$= -f(x)$

$\therefore$ the function is odd.

b) $y = |x(x-2)|$

$= |x^2-2x|$

c) $\frac{3^{n+2} \times 12^n}{4^{n-1}} = \frac{3^{n+2} \times 3^n \times 4^n}{4^{n-1}}$

$= 3^{2n+2} \times 4$

$= 4 \times 3^{2(n+1)}$

d) $x = 1 + \sec\theta$

$y = 2 + \tan\theta$

$\therefore \sec\theta = x-1$ —(i)

$\tan\theta = y-2$ —(ii)

Now $\sec^2\theta = 1 + \tan^2\theta$

$\therefore (x-1)^2 = 1 + (y-2)^2$

Q6

a) $x^6 - y^6 = (x^3-y^3)(x^3+y^3)$

$= (x-y)(x^2+xy+y^2)(x+y)(x^2-xy-$

b) (i) $R\hat{Q}B = 90 - R\hat{B}Q$ (right-angled Δ)

$\therefore R\hat{Q}P = 90 - (90 - R\hat{B}Q)$ (complementary angles)

$= R\hat{B}Q$

Now $R\hat{P}Q = 90 - R\hat{Q}P$

So $C\hat{P}Q = 90 - (90 - R\hat{Q}P)$ (complementary angles)

$= R\hat{Q}P$

$= R\hat{B}Q$

$\therefore R\hat{B}Q = R\hat{Q}P = Q\hat{P}C$ QED

(ii) $PC = a\sin B$ in Δ PBC

$\therefore \frac{PQ}{a\sin B} = \cos B$ in Δ PQC

$\therefore PQ = a\sin B\cos B$

$\therefore \frac{RQ}{a\sin B\cos B} = \cos B$ in Δ RQP

$\therefore RQ = a\sin B\cos^2 B$

QED

c) $\frac{1}{(a-b)(a-c)} + \frac{1}{(b-c)(b-a)} + \frac{1}{(c-a)(c-b)}$

$= \frac{(b-c) - (a-c) + (a-b)}{(a-b)(b-c)(a-c)}$

$= \frac{b-c-a+c+a-b}{(a-b)(b-c)(a-c)}$

$= 0$