

Name: ..... Maths Class: .....

# SYDNEY TECHNICAL HIGH SCHOOL



## YEAR 11 PRELIMINARY COURSE

### Extension 1 Mathematics

#### Task 1

May 2011

**TIME ALLOWED:** 70 minutes

#### *Instructions:*

- Write your name and class at the top of this page, and on all your answer sheets.
- Hand in your answers attached to the rear of this question sheet.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied at the time of marking
- *Start each new question on a new page*

(FOR MARKERS USE ONLY)

1	2	3	4	5	6	TOTAL
/9	/9	/9	/9	/9	/9	/54

**QUESTION 1 (9 Marks):****Marks**

- (a) Fully factorise and simplify:

2

$$\frac{6^n - 2^n}{9^n - 3^n}$$

- (b) Show that the expression
- $\frac{1}{3\sqrt{2}-2} - \frac{1}{3\sqrt{2}+2}$
- is rational.

2

- (c) What is the Domain of the function given by
- $y = \frac{1}{\sqrt{x^2-1}}$
- ?

2

- (d) If
- $\sin \alpha = x$
- and
- $90^\circ \leq \alpha \leq 270^\circ$
- , find an expression for
- $\tan \alpha$

3

**QUESTION 2 (9 Marks): (Start a New Page)**

- (a) On a set of Cartesian axes, neatly graph
- $|x| + |y| = 1$
- , showing all intercepts on the axes

2

- (b) (i) Sketch the graph of
- $y = \sqrt{9 - x^2}$

2

- (ii) Give the Domain and Range of the function

2

- (iii) On your diagram, neatly shade the solution to
- $y < \sqrt{9 - x^2}$
- , carefully indicating whether boundaries are included or excluded from the solution.

3

**QUESTION 3 (9 marks): (Start a New Page)**

- (a) Solve the two equations below simultaneously to find their point(s) of intersection:

3

$$xy = 6$$

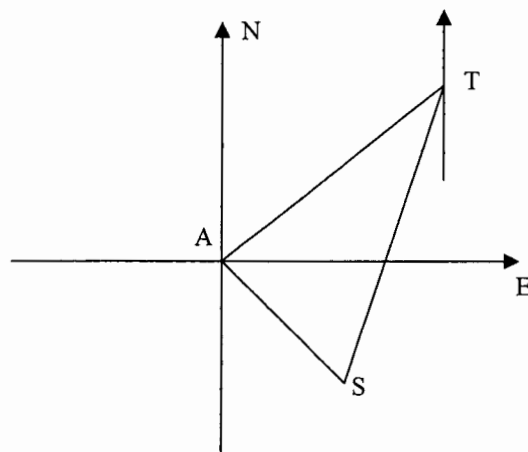
$$y = x - 5$$

- (b) The diagram below represents the information given in the following problem:

A ship S sails from a port A on a bearing of  $S18^\circ E$  at a speed of 10 km per hour.

A second ship, T, leaves at exactly the same time on a course of  $N72^\circ E$ .

After 2 hours the ship T sights S on a bearing of  $S24^\circ W$



- (i) Copy the diagram onto your answer page and put on it all the information given in the above question
- (ii) Find the size of angle ATS
- (ii) Find the distance between the two ships 2 hours after leaving port A (ie ST). (to the nearest km)

2

1

3

***You must show all necessary working to gain full marks***

**QUESTION 4 (9 Marks):** *(Start a New Page)*

**Marks**

- (a) (i) *On the same set of axes*, sketch the graphs of  $y = |2x - 1|$  and  $y = 1 - 2x$  3  
(ii) Use the diagram, or otherwise, to solve

$$|2x - 1| > 1 - 2x$$

for all real  $x$ .

- (b) Solve, for  $0^\circ \leq \theta \leq 360^\circ$ , 3

$$4\cos^2\theta - 3 = 0$$

- (c) Prove that  $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = \frac{2}{\cos x}$  3

**QUESTION 5 (9 Marks):** *(Start a New Page)*

- (a) You are given the function  $f(x) = \frac{2}{x^2-4}$
- (i) Is the function odd, even, or neither? (Give reasons) 1  
(ii) What are the vertical asymptotes for  $f(x)$ ? 1  
(iii) Where does this curve cut the y-axis? 1  
(iv) Sketch the curve  $y = f(x)$ , showing all important features and indicating the *horizontal* asymptote(s). 4
- (b) Find all solutions over the Real number system to: 2

$$\frac{1}{x-1} < \frac{1}{x}$$

*Show all necessary working*

**QUESTION 6 (9 Marks): (Start a New Page)**

(a) Simplify  $\frac{\cos(90^\circ - \theta)}{\sin(90^\circ + \theta)}$  2

(c) A function is defined by the following:

$$f(x) = \begin{cases} 3, & \text{if } x \leq -2 \\ -x, & \text{if } -2 < x < 0 \\ x^2, & \text{if } x \geq 0 \end{cases}$$

(i) Sketch  $y = f(x)$  for  $-4 \leq x \leq 2$  3

(ii) Find the value of  $f\left(\frac{1}{2}\right) + f\left(-\frac{1}{2}\right)$  1

(iii) Find the value of  $f(|a|)$  where  $a$  is real. 1

(c)  $g(x) = 2x + 4$  is a function for all real values of  $x$ . 2

A second function,  $f[x]$  is defined thus:

$$f[g(x)] = x$$

Find the function  $f[x]$  ( )

**END OF EXAMINATION PAPER**

SOLUTIONS - Maths Extension 1

YEAR 11 - EXT

May 2011

① (a)  $\frac{2^n(3^n-1)}{3^n(3^n-1)} = \left\{ \begin{array}{l} \frac{2^n}{3^n} \\ \left(\frac{2}{3}\right)^n \end{array} \right.$

1 mark for each factorisation  
= ②

(b)  $\frac{3\sqrt{2}+2}{14} - \frac{3\sqrt{2}-2}{14}$

1 mark for the rationalisation

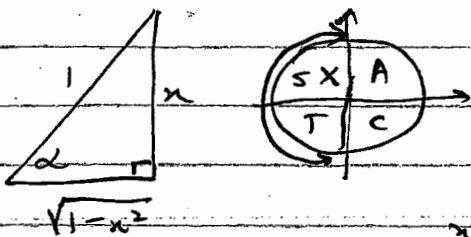
=  $\left\{ \begin{array}{l} \frac{4}{14} \text{ which is rational} \\ \frac{2}{7} \end{array} \right.$

1 mark for  $\frac{4}{14}$  or  $\frac{2}{7}$ .

(c) Q:  $x > 1$  or  $x < -1$

② MARKS - subtract 1 if they have "=" signs.

(d)



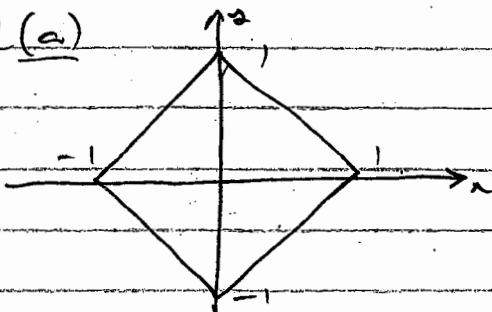
$\tan \alpha = -\frac{x}{\sqrt{1-x^2}}$

1 mark for  $\sqrt{1-x^2}$

1 mark for the expression for  $\tan \alpha$

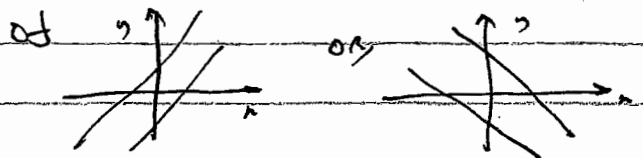
1 for the negative.

2(a)

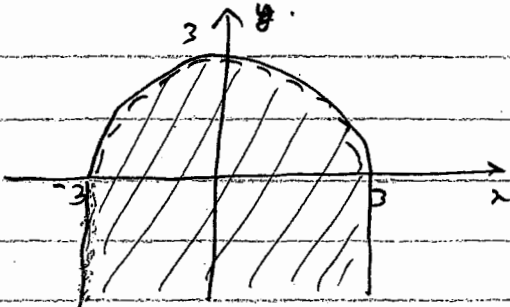


2 for the graph and its intercepts

1 MARK ONLY for either



(b)



② MARKS - 1 for the shape and 1 for intercepts. (1 only if full circle)

(ii) Q:  $-3 \leq x \leq 3$  R:  $0 \leq y \leq 3$

1 mark each.

(iii)

1 mark for shading inside the circle  
1 mark for the extension  
1 mark for dots and heavy line

$$\textcircled{3} \text{ (i) } \begin{cases} y = \frac{6}{x} \\ y = x - 5 \end{cases}$$

(1) into (2)  $\frac{6}{x} = x - 5$

$$6 = x^2 - 5x$$

$$x^2 - 5x - 6 = 0$$

$$(x - 6)(x + 1) = 0$$

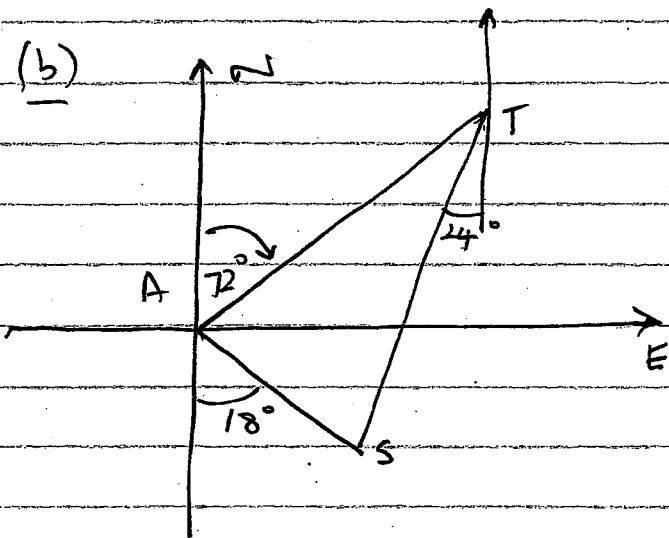
$\therefore x = 6$  or  $x = -1$

$\therefore$  Points are  $(6, 1)$  and  $(-1, -6)$

← 2 MARKS

← 1 MARK

(b)



All of these angles must be shown to get 2 MARKS

(no extra marks for  $\angle TAS = 90^\circ$  or  $AS = 20$ )

(c)  $\angle ATS = (72 - 24)^\circ = 48^\circ$

← 1 MARK

(ii)  $AS = 20$  and  $\angle TAS = 90^\circ$

① MARK for both

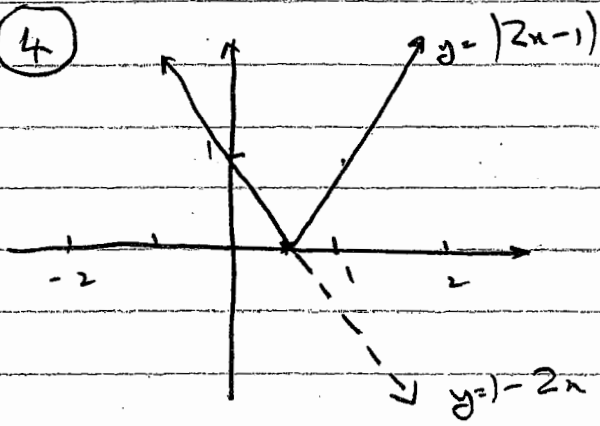
$\therefore \frac{ST}{20} = \frac{1}{\cos 48}$

← 1 MARK

$\therefore \left. \begin{aligned} ST &= 29.88 \\ &\approx 30 \text{ km} \end{aligned} \right\}$

1 for circle

(4)



← 1 MARK

$x > 1/2$

← 1 MARK

← 1 MARK

= (3)

(b)

$\cos \theta = \pm \sqrt{3}/2$

$\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$

← (1) MARK

← (2) MARKS (1) ONLY

for  $30^\circ$ , or for  $30^\circ, 150^\circ$

(c)

$$\begin{aligned} \text{LHS} &= \frac{\cos x (1 - \sin x)}{1 - \sin^2 x} + \frac{(1 + \sin x) \cos x}{\cos^2 x} \\ &= \frac{\cos x (1 - \sin x) + (1 + \sin x) \cos x}{\cos^2 x} \\ &= \frac{2}{\cos x} \end{aligned}$$

There are several methods. But for 3 marks, they cannot work down both sides (2 max.) and must show enough steps to explain what is happening



5 (a)  $f(x) = \frac{2}{x^2-4}$

(i)  $f(a) = \frac{2}{a^2-4}$   $f(-a) = \frac{2}{(-a)^2-4}$   
 $= \frac{2}{a^2-4}$   
 $= f(a)$

∴ EVEN.

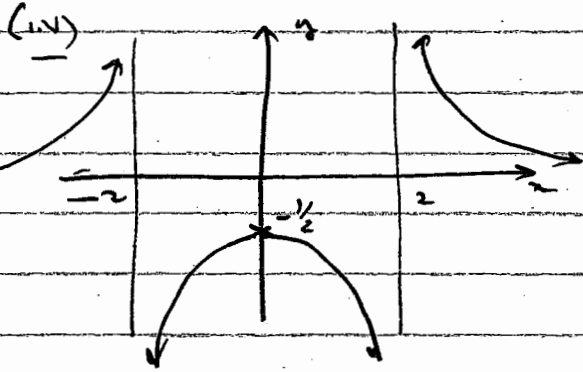
1 MARK but  
 some explanation as to  
why should be given  
 (may not be done as  
 this)

(ii)  $x \neq \pm 2$

← 1 MARK

(iii) cuts y-axis at  $(0, -\frac{1}{2})$

← 1 MARK



1 MARKS for each arm = (2)

① for the part between  
 $-2 < x < 2$

① for indicating  $y=0$  as  
 the asymptote.

(b)  $\frac{1}{x-1} < \frac{1}{x}$

C.V.  $x \neq 0$   
 $x \neq 1$

$x-1 = x$

NO SOLN



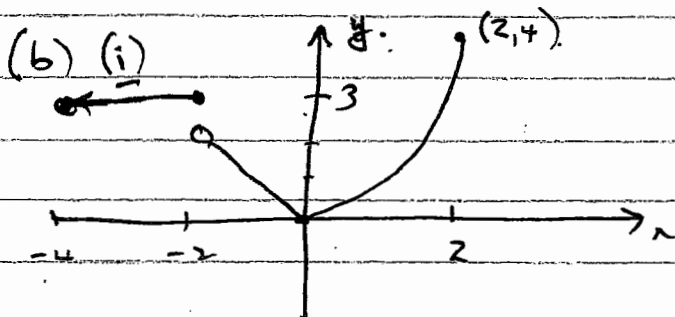
$0 < x < 1$

1 for any accurate  
 method shown.

← 1 for solution

$$\textcircled{6} \quad \textcircled{a} \quad \frac{\cos(90^\circ - \theta)}{\sin(90^\circ + \theta)} = \frac{\sin \theta}{\cos \theta} \\ = \tan \theta$$

$\textcircled{2}$  1 for each simplification



1 MARK for each "part"  
=  $\textcircled{3}$

Subtract 1 MARK for no open circle

Subtract 1 if the graph doesn't "end" at both ends

$$\textcircled{ii} \quad f\left(\frac{1}{2}\right) + f\left(-\frac{1}{2}\right) = \frac{1}{4} + \frac{1}{2} \\ = \frac{3}{4}$$

1 MARK

$$\textcircled{iii} \quad \text{Since } |a| \geq 0 \quad \forall a, \\ f(|a|) = a^2$$

1 MARK

$$\textcircled{c} \quad f[x] = \frac{x-4}{2}$$

$\textcircled{2}$  MARKS

1 for division by 2

1 for subtraction of 4