Name:_______________________________ Teacher:_________________________

SYDNEY TECHNICAL HIGH SCHOOL

Year 11 Mathematics Extension 1

Preliminary Assessment Task 1

May 2012

Time allowed: 70 minutes

Instructions: Please take notice and act upon ALL of these.

- Show full working.
- Start each question on a new page.
- Full marks may not be awarded for careless or badly arranged work.
- Non-programmable calculators may be used.
- This paper must be handed in on top of your answer booklets.
- Answers must be written in blue or black pen.
- Answers must be arranged in order and stapled securely.

Question 1 **9 marks**

(a) Factorise fully $x(x-y)^2 - xz^2$. **2**

(b) Simplify $\dfrac{\frac{x}{y}+\frac{y}{x}}{\frac{x}{y}-\frac{y}{x}}$. **2**

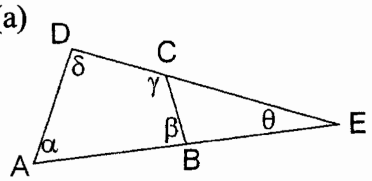
(c) Factorise $64 - 27k^3$. **1**

(d) Solve $\frac{x}{3} - 2 < \frac{x}{2} - 3$. **1**

(e) Find the points of intersection of $x^2+y^2 = 16$ and $x - y = 2$. **3**

Question 2 (Start a new page) **10 marks**

(a)



In quadrilateral ABCD there are no parallel sides. Use triangle and other geometric properties to show that the interior angle sum of such a quadrilateral is 360°. Show reasoning. **3**

(b) For $y = \sqrt{x-2}$:

(i) Is it a function? (Show reasoning) **1**

(ii) State its domain and range. **2**

(iii) Sketch the curve showing any significant features. **2**

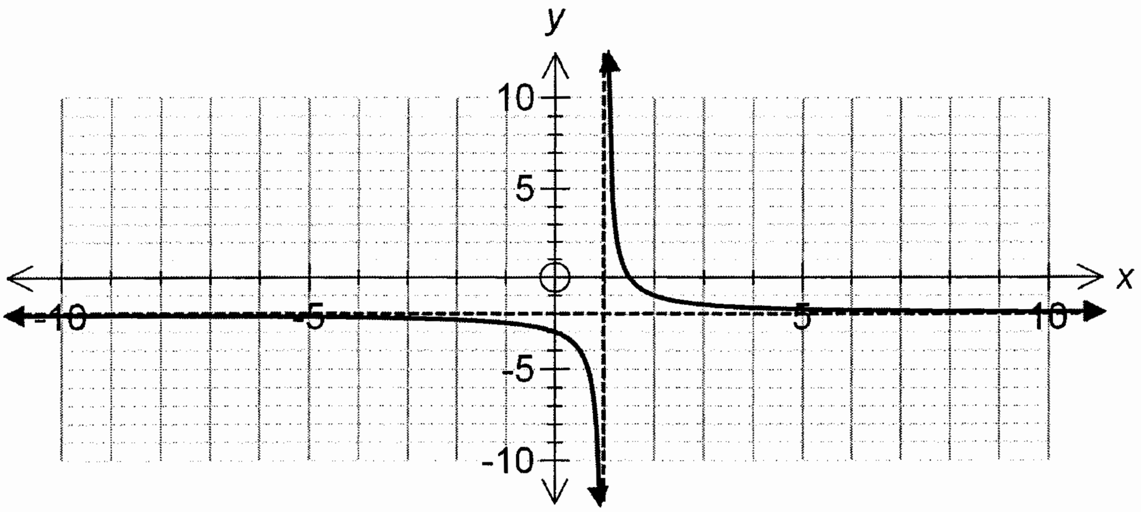
(c) (i) Factorise $x^2 - x - 6$. **1**

(ii) Hence sketch $y = \left| x^2 - x - 6 \right|$. **2**

Question 3 (Start a new page) **9 marks**

(a) Find rational numbers a and b such that $a + b\sqrt{5} = \dfrac{\sqrt{5}}{3+\sqrt{5}}$. **2**

(b) The curve shown is a translation of $y = \dfrac{1}{x}$. Write its equation. **2**



(c) Show whether the function $f(x) = \dfrac{8x}{x^2+9}$ is odd, even or neither. **2**

(d) Solve the inequality and graph the solution on a number line. **3**

$$\frac{3}{x-1} < \frac{5}{2}$$

Question 4 (Start a new page) **10 marks**

(a) Solve $2\sin\theta = \sqrt{2}$ for $0^\circ \le \theta \le 360^\circ$. **2**

(b) Solve $\cos 2\theta = \dfrac{1}{2}$ for $0^\circ \le \theta \le 360^\circ$. **3**

(c) Find the exact value of sec 315°. **2**

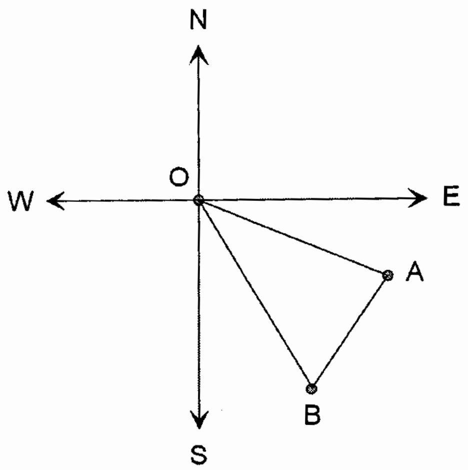
(d) Prove the identity $(1 - \cos\theta)(1 + \sec\theta) = \sin\theta\tan\theta$. **3**

Question 5 (Start a new page) **9 marks**

(a) If $\tan\theta = -\frac{3}{4}$ and θ is obtuse, find $\sin\theta$ and $\cos\theta$. **3**

(b) Eliminate θ from the set of equations $x = a\sec\theta$ and $y = b\tan\theta$. **2**

(c) A ship sails from O to a point A *60 km* on a bearing of 125^o. It then changes to a bearing of 200^o and sails to a point B, which has a bearing from O of 150^o.

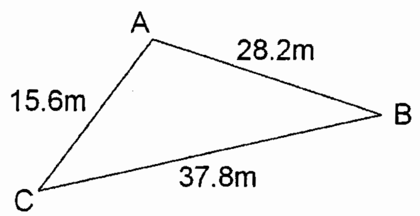


(i) Copy the diagram into your workbook and find the values of the internal angles of the triangle OAB. **2**

(ii) What is the distance of B from O? **2**

Question 6 (Start a new page) **10 marks**

(a) Find the largest angle in the triangle (to the nearest minute). **2**



(b) (i) Sketch $y = 2 - x^2$. **1**

(ii) Hence or otherwise, solve $|x| + x^2 \geq 2$ **3**

(c) (i) Find any vertical asymptotes of the curve $y = \dfrac{x^2 - 3}{x^2 - 4}$. **1**

(ii) Sketch the curve, including any vertical and horizontal asymptotes. **3**

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SOLUTIONS PRELIMINARY EXTENSION 1 ASSESSMENT 1

Q1

a) $x(x-y)^2 - xz^2$

$= x\left[(x-y)^2 - z^2\right]$

$= x(x-y-z)(x-y+z)$

b) $\dfrac{\frac{x}{y}+\frac{y}{x}}{\frac{x}{y}-\frac{y}{x}} = \dfrac{\frac{x^2+y^2}{xy}}{\frac{x^2-y^2}{xy}}$

$= \dfrac{x^2+y^2}{x^2-y^2}$

c) $64-27k^3 = 4^3-(3k)^3$

$= (4-3k)(16+12k+9k^2)$

d) $\frac{x}{3} - 2 < \frac{x}{2} - 3$

$\therefore 2x - 12 < 3x - 18$

$6 < x$

$\therefore x > 6$

e) $\left.\begin{array}{l} x^2+y^2=16 \\ x-y=2 \end{array}\right\}$

$y^2 = (2-x)^2$

$\therefore x^2 + (2-x)^2 = 16$

$\therefore x^2 + 4 - 4x + x^2 = 16$

$\therefore 2x^2 - 4x - 12 = 0$

$\therefore x^2 - 2x - 6 = 0$

$\therefore x = \dfrac{2 \pm \sqrt{4-4\times1\times-6}}{2}$

$= \dfrac{2 \pm \sqrt{28}}{2}$

$\left.\begin{array}{l} \therefore x = 1 \pm \sqrt{7} \\ \therefore y = -1 \pm \sqrt{7} \end{array}\right\}$

Q2

a) In ΔADE, $\theta + \alpha + \delta = 180$ (anglesum of Δ)

$\therefore \theta = 180 - (\alpha + \delta)$

In ΔCBE, $E\hat{C}B = 180 - \gamma$ (anglesum of Δ)

and $E\hat{B}C = 180 - \beta$ (anglesum of Δ)

$\therefore \theta = 180 - ((180-\gamma) + (180-\beta))$

$= \gamma - 180 + \beta$

$\therefore 180 - (\alpha + \delta) = \gamma - 180 + \beta$

$\therefore \alpha + \delta + \gamma + \beta = 360°$

$\therefore$ the interior anglesum of a quadrilateral with no parallel sides is $360°$.

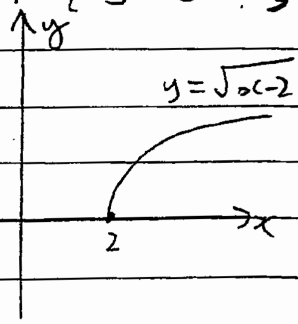
b) (i) $y = \sqrt{x-2}$

Yes, it is a function as the convention is to take the positive root and so for each x value there is at most one y value.

(ii) domain $\{x : x \geq 2\}$

range $\{y : y \geq 0\}$

(iii)



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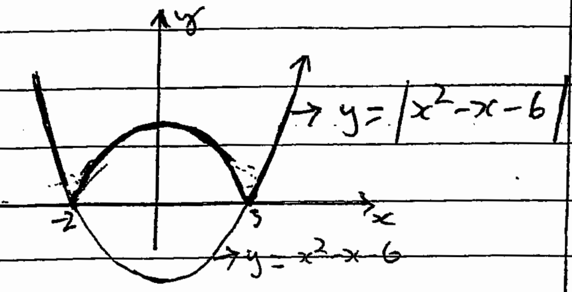
c) (i) $x^2 - x - 6 = (x-3)(x+2)$

(ii)



d) $6(x-1) < 5(x-1)^2$

$6x - 6 < 5x^2 - 10x + 5$

$5x^2 - 16x + 11 > 0$

let $(5x - 11)(x-1) = 0$

$\therefore x = 1, 2\frac{1}{5}$

x	0	2	3
	✓	×	✓

$\therefore x < 1, x > 2\frac{1}{5}$

$\overline{(-x)^2 + 9}$

$= \dfrac{-8x}{x^2+9}$

$= -f(x)$

$\therefore f(x)$ is odd

d) $LHS = (1 - \cos\theta)(1 + \sec\theta)$

$= 1 + \sec\theta - \cos\theta - 1$

$= \sec\theta - \cos\theta$

$= \dfrac{1}{\cos\theta} - \cos\theta$

$= \dfrac{1 - \cos^2\theta}{\cos\theta}$

$= \dfrac{\sin^2\theta}{\cos\theta}$

$= \sin\theta \tan\theta$

$= RHS$ QED

3 a) $a + b\sqrt{5} = \dfrac{\sqrt{5}}{3+\sqrt{5}} \times \dfrac{3-\sqrt{5}}{3-\sqrt{5}}$

$= \dfrac{3\sqrt{5} - 5}{9 - 5}$

$= -\dfrac{5}{4} + \dfrac{3\sqrt{5}}{4}$

$\therefore a = \dfrac{-5}{4}$ and $b = \dfrac{3}{4}$

c) $f(x) = \dfrac{8x}{x^2+9}$

Now, $f(-x) = \dfrac{-8x}{(-x)^2+9}$

$= \dfrac{-8x}{x^2+9}$

$= -f(x)$

$\therefore f(x)$ is odd

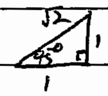
b) $y = \dfrac{1}{x-1}$

or $y \mp 2 = \dfrac{1}{x-1}$

Q4 a) $2\sin\theta = \sqrt{2}$ for $0° \le \theta \le 360°$

$\therefore \sin\theta = \dfrac{\sqrt{2}}{2}$

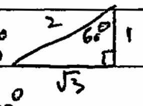
$= \dfrac{1}{\sqrt{2}}$



$\therefore \theta = 45°, 135°$

b) $\cos 2\theta = \frac{1}{2}$ for $0° \le \theta \le 360°$

$\therefore 2\theta = 60°, 300°, 420°, 660°$



$\therefore \theta = 30°, 150°, 210°, 330°$

c) $\sec 315° = \dfrac{1}{\cos 315°}$

$= \dfrac{1}{\cos 45°}$

$= \dfrac{1}{\frac{1}{\sqrt{2}}}$

$= \sqrt{2}$

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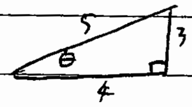
Q5 a) $\tan\theta = -\frac{3}{4}$ and θ is obtuse.



θ in 2nd quadrant

$\therefore \sin\theta > 0$ and $\cos\theta < 0$

$\therefore \sin\theta = \frac{3}{5}$ and $\cos\theta = \frac{-4}{5}$

b) $\left.\begin{array}{l} x = a\sec\theta \\ y = b\tan\theta \end{array}\right\}$

$x^2 = a^2\sec^2\theta \rightarrow \sec^2\theta = \frac{x^2}{a^2}$

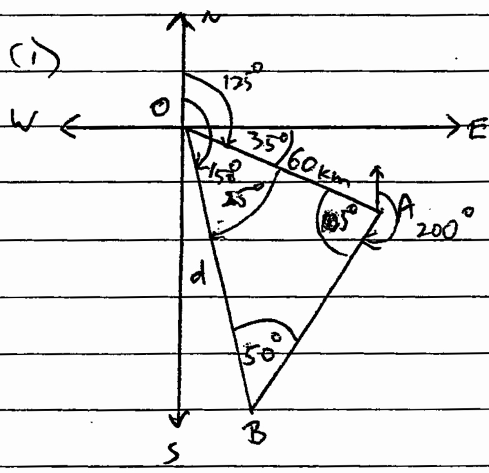
$y^2 = b^2\tan^2\theta \rightarrow \tan^2\theta = \frac{y^2}{b^2}$

Now, $\sec^2\theta = 1 + \tan^2\theta$

$\therefore \frac{x^2}{a^2} = 1 + \frac{y^2}{b^2}$

$\therefore \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ QED

c) (i)



(ii) $\frac{d}{\sin 105^\circ} = \frac{60}{\sin 50^\circ}$

$\therefore d = \frac{60\sin 105^\circ}{\sin 50^\circ}$

≈ 75.66

≈ 76 km to nearest km.

Teacher's Name: ____________ **Student's Name/N°:** ____________

Q6

a) $\cos A = \dfrac{15.6^2 + 28.2^2 - 37.8^2}{2 \times 15.6 \times 28.2}$

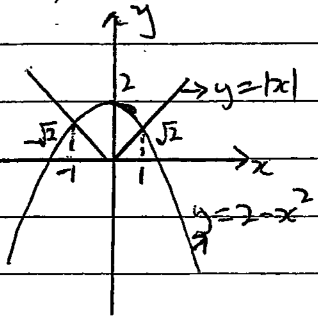
A is opposite the longest side.
∴ It is the longest angle.

$= -0.4435351887$

$\therefore A = 116.3296586$

$\approx 116° 20'$

b) (i) $y = 2 - x^2$



Only the parabola needed as answer to part (i)

(ii) $|x| + x^2 \geq 2$

$\therefore |x| \geq 2 - x^2$

from the graph $x \leq -1$ or $x \geq 1$

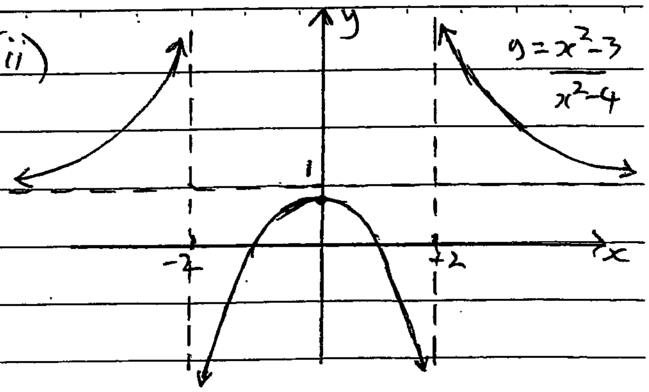
c) (i) $y = \dfrac{x^2-3}{x^2-4}$

vertical asymptotes when $x^2 - 4 = 0$

$\therefore (x-2)(x+2) = 0$

$\therefore x = \pm 2$

(ii)



As $x \to -2^-$, $y \to \infty$

As $x \to -2^+$, $y \to -\infty$

As $x \to 2^-$, $y \to -\infty$

As $x \to 2^+$, $y \to \infty$

Now $y = \dfrac{x^2-3}{x^2-4}$

$= \dfrac{1 - \frac{3}{x^2}}{1 - \frac{4}{x^2}}$

As $x \to \infty$, $y \to 1$

As $x \to -\infty$, $y \to 1$