Name: .... **Maths Class:** ....

SYDNEY TECHNICAL HIGH SCHOOL

(Est. 1911)

Year 11

Mathematics Extension 1

May 2013

Time allowed: 70 min

Instructions:

- Write your name and class at the top of this page.
- These questions must be handed in on the *top* of your answers
- Attempt all questions.
- Begin each question on a new page.

Use only blue or black pen for your answers

Total Marks – 60

Question 1 (10 marks) Use a SEPARATE page

(a) Simplify the following expression $\dfrac{x^3 - y^3}{x^2 - y^2}$. **2**

(b) (i) By factorising, simplify $2^{n+1} + 2^n$. **1**

(ii) Hence, or otherwise, write $\dfrac{2^{1001} + 2^{1000}}{3}$ as a power of 2. **2**

(c) Simplify: $\dfrac{10^x + 15^x}{2^5 \times 3^x + 2^{x+5}}$. **3**

(d) Find the exact value of $\sin 120^o - \tan 210^o$. Express your answer with a rational denominator. **2**

Question 2 (10 marks) Use a SEPARATE page.

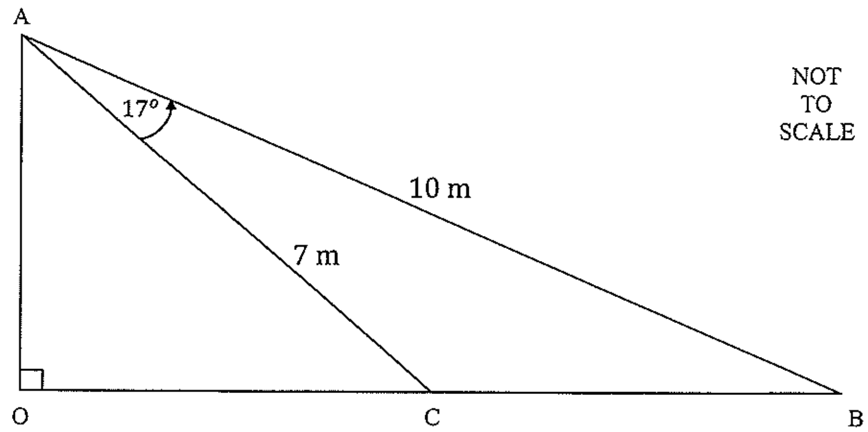
(a) The sum of the interior angles of a regular polygon is 3960^o.

(i) How many sides does the polygon have? **1**

(ii) Find the size of each interior angle. **1**

(iii) Hence or otherwise find the size of the exterior angle. **1**

(b)



(i) Find the area of ΔABC to 2 significant figures. **2**

(ii) Find the length of BC to 2 significant figures. **2**

(iii) Find the length of BO to 2 significant figures. **3**

Question 3 (10 marks) Use a SEPARATE page.

(a) Find the *exact* solutions of $x + 8 = \frac{6}{x}$ **3**

(b) (i) Draw the graph of $y = |x - 1|$ and $y = x + 3$ on the same axes. **2**

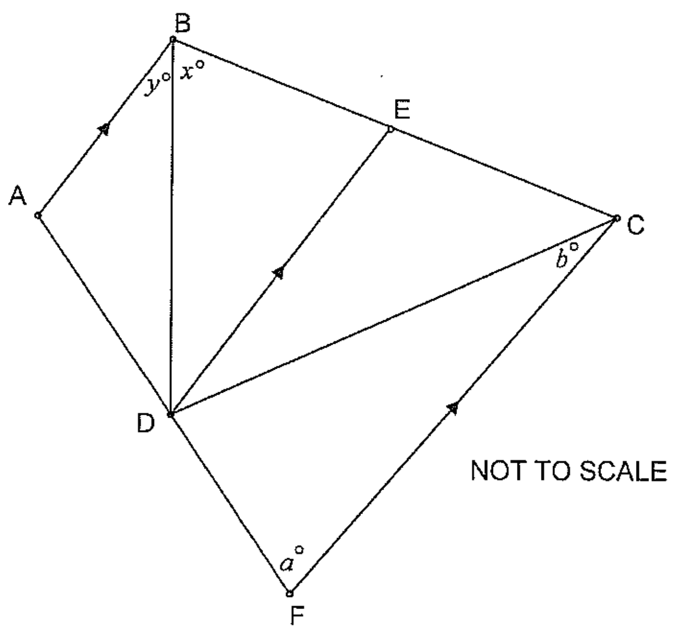
(ii) Hence or otherwise solve $|x - 1| > x + 3$. **2**

(c) Solve for x: **3**

$$\frac{4x - 1}{x + 2} \geq 3$$

Question 4 (10 marks) Use a SEPARATE page.

(a) In quadrilateral ABCD, AB = AD, CB = CD and FC is parallel to AB and DE.



(i) Show that $a = 2y$, giving reason(s). **3**

(ii) Show that $b = x - y$, giving reason(s). **2**

(b) (i) Use the method of grouping in pairs to factorise fully **2**

$$3x^3 + 3x^2 - x - 1.$$

(ii) Hence or otherwise solve **3**

$$3\tan^3\theta + 3\tan^2\theta - \tan\theta - 1 = 0 \text{ for } 0 \leq \theta \leq 180°.$$

Question 5 (10 marks) Use a SEPARATE page.

(a) For the function $f(x)=\dfrac{9}{9-x^2}$

(i) Giving reasons, is the function odd, even or neither? **1**

(ii) Find the equation(s) of the asymptotes. **2**

(iii) Using a ruler, sketch the graph of $y=f(x)$, showing all key features. **3**

(iv) Hence, or otherwise, state the domain and range of the function. **1**

(b) If $3\cos\theta+2=0$ and $\tan\theta > 0$, what is the exact value of $\sin\theta$? **3**

Question 6 (10 marks) Use a SEPARATE page.

(a) Jade is on a ship and observes two lighthouses on the shore. The lighthouse at Addison Head has a bearing of 224° from the ship. The lighthouse at Blake Beach has a bearing of 195° from the ship and 165° from Addison Head. The lighthouses are 3.4 km apart.

(i) Draw a diagram showing all necessary information. **2**

(ii) What is the distance of Jade's ship from the Addison Head lighthouse (1 decimal place)? **2**

(b) Prove **3**

$$\frac{(1+\tan^2\theta)\cot\theta}{\operatorname{cosec}^2\theta}=\tan\theta$$

(c) If $2^a+3^b=17$ and $2^{a+2}-3^{b+1}=5$, find the values of a and b. **3**

End of test

Student Name: ____________________ Teacher Name: ____________________

Q1

(a) $\dfrac{(x-y)(x^2+xy+y^2)}{(x-y)(x+y)}$

$= \dfrac{x^2+xy+y^2}{(x+y)}$

(b)(i) $2\times 2^n + 2^n$

$= 2^n(2+1)$

$= 3\times 2^n$

(ii) $\dfrac{2\times 2^{1000} + 2^{1000}}{3}$

$= \dfrac{3\times 2^{1000}}{3} = 2^{1000}$

(c) $\dfrac{2^x\times 5^x + 3^x\times 5^x}{2^5\times 3^x + 2^5\times 2^x}$

$= \dfrac{5^x(2^x+3^x)}{2^5(3^x+2^x)}$

$= \dfrac{5^x}{2^5}$

(d) $\sin(180-60) - \tan(180+30)$

$= \sin 60 - \tan 30$

$= \dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{3}.\sqrt{3}}\times\sqrt{3}$

$= \dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{3}}{3}$

$= \dfrac{3\sqrt{3}-2\sqrt{3}}{6} = \dfrac{\sqrt{3}}{6}$

Q2

(a) (i) $(3960 \div 180) + 2 = 24$

(ii) $3960 \div 24 = 165^\circ$

(iii) $180 - 165 = 15^\circ$

(b)(i) $A = \frac{1}{2}\times 10\times 7\times \sin 17$

$= 10.233$

$= 10\ m^2$ (2.s.f.)

(ii)

$BC^2 = 10^2 + 7^2 - 2\times 10\times 7\times \cos 17$

$= 15.117......$

$\therefore BC \doteq 3.9m.$

(iii) $\dfrac{\sin B}{7} = \dfrac{\sin 17}{3.9}$

$\sin B = 0.5247697.....$

$B \doteq 31.6527^\circ...$

$\therefore \dfrac{OB}{10} = \cos 31.6527$

$OB = 10\cos 31.6527$

$= 8.5\ m$

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Q3

(a) $x + 8 = \frac{6}{x} \times x$

$x^2 + 8x = 6$

$x^2 + 8x - 6 = 0$

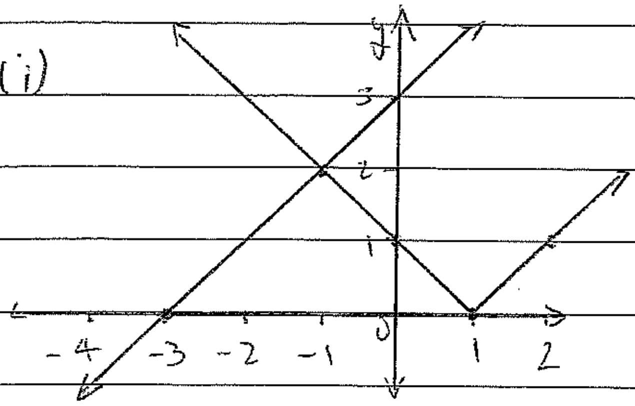
$x = \frac{-8 \pm \sqrt{8^2 - 4 \times 1 \times (-6)}}{2 \times 1}$

$= \frac{-8 \pm \sqrt{88}}{2}$

$= \frac{-8 \pm 2\sqrt{22}}{2}$

$x = -4 \pm \sqrt{22}$

b)(i)



(ii) when $x < -1$

c) $\frac{4x-1}{x+2} \times (x+2)^2 \geq 3(x+2)^2$

$(4x-1)(x+2) \geq 3(x+2)^2$.

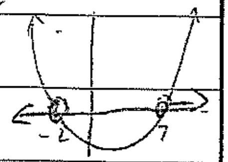
3c) continued.

$(4x-1)(x+2) - 3(x+2)^2 \geq 0$

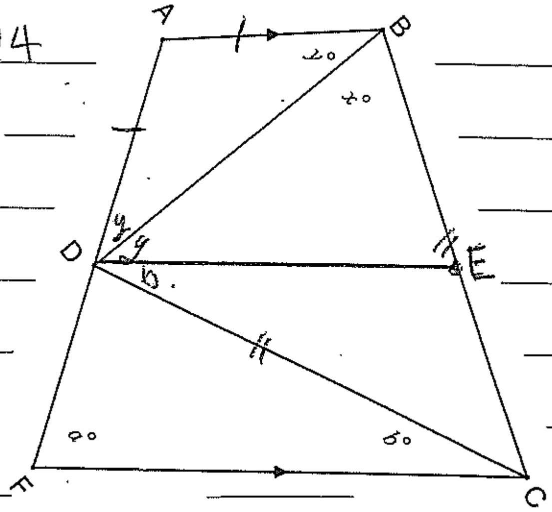
$(x+2)[4x-1-3(x+2)] \geq 0$.

$(x+2)(x-7) \geq 0$.

$\therefore x < -2$ & $x \geq 7$



Q4



(a)(i) AD = AB (given)

∠ABD = ∠ADB (equal angles of an isosceles ΔABD)
= y.

∠BDE = y (alternate angles, AB||DE)

∴ a = 2y (corresponding angles, DE||FC)

(ii) CB = CD (given)

∠CDE = b (alternate angles, DE||FC)

∠DBC = ∠CDB (equal angles of an isosceles ΔABD)

b + y = x

∴ b = x − y

CANDIDATE NO: ______________________

Q4 continued...

(b)(i) $3x^2(x+1) - (x+1)$

$(x+1)(3x^2-1)$

$(x+1)(\sqrt{3}x-1)(\sqrt{3}x+1)$

ii) $(\tan\theta+1)(\sqrt{3}\tan\theta-1)(\sqrt{3}\tan\theta+1)=0$

∴ $\tan\theta=-1$ $\quad\tan\theta=\frac{1}{\sqrt{3}}$ $\quad\tan\theta=\frac{-1}{\sqrt{3}}$

$\theta=135^\circ$ $\quad\theta=30^\circ$ $\quad\theta=150^\circ$

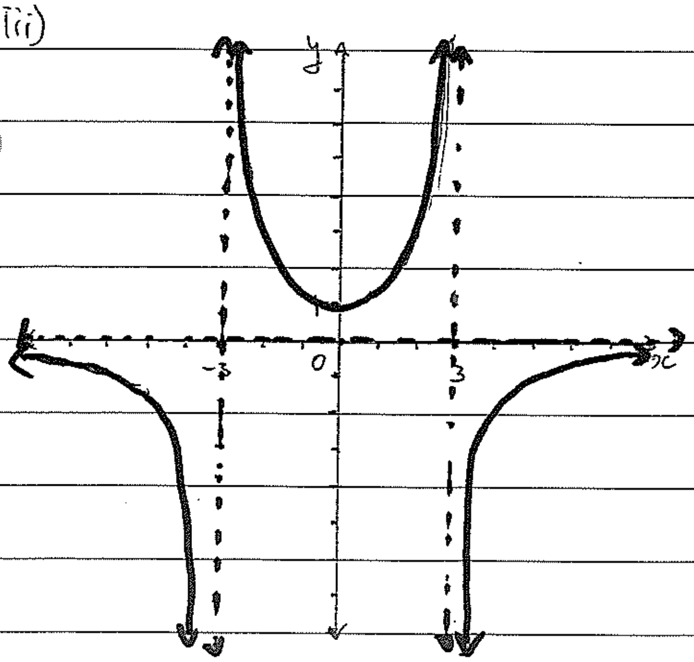
Q5

(i) $f(-x) = \frac{6}{9-(-x^2)} = \frac{6}{9-x^2} = f(x)$ ∴ even.

(ii) $9-x^2 \neq 0$

∴ $x \neq \pm 3$ $\quad y \neq 0$

(iii)

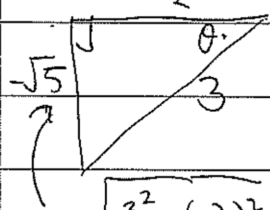


iv) R: $y \geq 1$ and $y < 0$ D: all x, except $x \neq \pm 3$

(b) $\cos\theta = \frac{-2}{3}$ $\quad\cos\theta < 0$

$\tan\theta > 0$

∴ 3rd quadrant

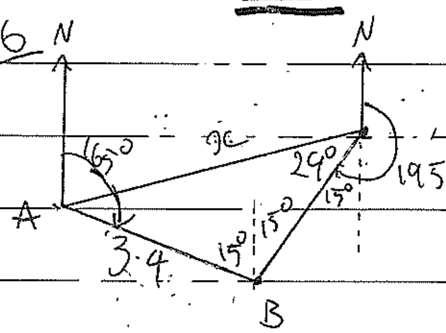


$\sqrt{3^2-(-2)^2} = \sqrt{5}$

∴ $\sin\theta = \frac{-\sqrt{5}}{3}$

Q6

(i)



ii) $\frac{x}{\sin 30} = \frac{3.4}{\sin 29}$

∴ $x \doteqdot 3.5$ km

(b) LHS $= \sec^2\theta \times \frac{\cos\theta}{\sin\theta} \div \frac{1}{\sin^2\theta}$

$= \frac{1}{\cos^2\theta} \times \frac{\cos\theta}{\sin\theta} \times \frac{\sin^2\theta}{1} = \frac{\sin\theta}{\cos\theta}$

$= \tan\theta = $ RHS

(c) Let $x=2^a$ & $y=3^b$

$x+y=17$ ①

$2^{a+2} - 3^{b+1} = 5 \Rightarrow 2^2\times 2^a - 3\times 3^b = 5$

∴ $4x-3y=5$ ②

$3\times$① $\quad 3x+3y = 51$ ③

③+② $\quad 7x = 56$ ∴ $x=8$

$8+y=17$ $\quad 2^a=8$

$y=9$ $\quad$ ∴ $a=3$

∴ $3^b=9$

∴ $b=3$