SYDNEY TECHNICAL HIGH SCHOOL

PRELIMINARY HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 1

MAY 2014

Mathematics Extension 1

General Instuctions

- Working time - 70 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- In questions 6 to 11, show relevant mathematical reasoning and/or calculations
- Start each question in section 2 on a new page

Total marks - 53

Section 1 - 5 marks

Attempt Questions 1 – 5.
Allow about 7 minutes for this section.

Section 2 - 48 marks

Attempt Questions 6 – 11.
Allow about 63 minutes for this section.

Name : _____

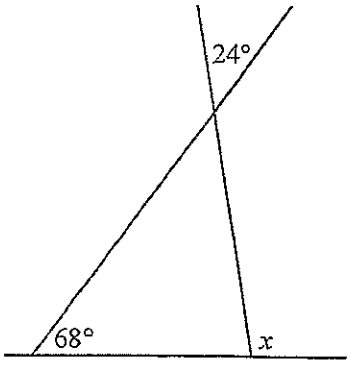
Teacher : _____

Section 1

5 marks
Attempt Questions 1 – 5
Allow about 7 minutes for this section

Use the multiple-choice answer sheet in your answer booklet for Questions 1 – 5.
Do not remove the multiple-choice answer sheet from your answer booklet.

1



The size of angle x is

A. 88°

B. 92°

C. 112°

D. 116°

2. How many asymptotes does the graph of the function $y = \frac{3x^2}{x(2-x)}$ have ?

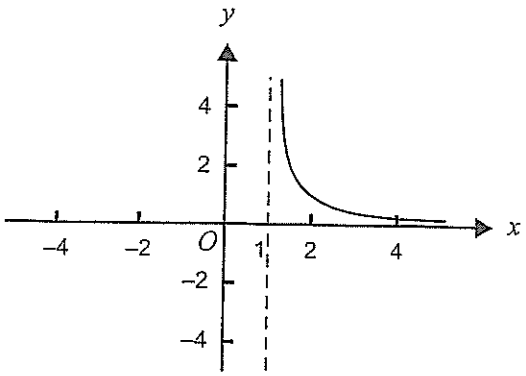
A. 0

B. 1

C. 2

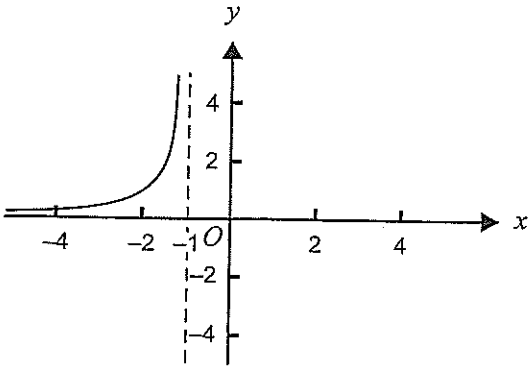
D. 3

3. Part of the graph of the function with rule $y = f(x)$ is shown below

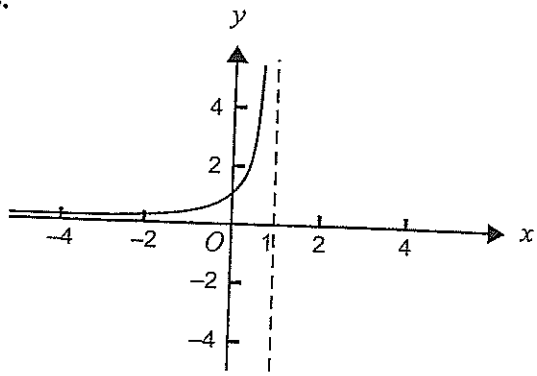


Which one of the following is most likely to be the corresponding part of the function with rule $y = f(-x)$?

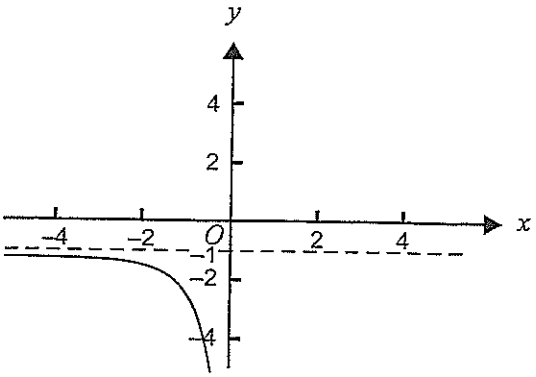
A.



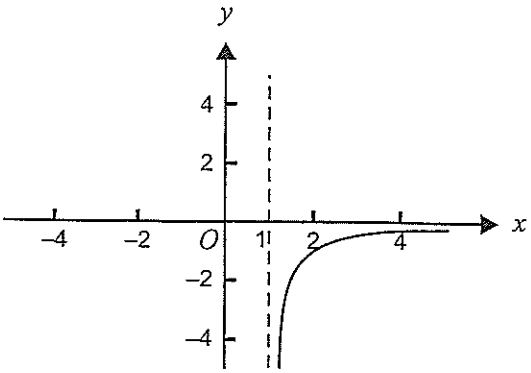
B.



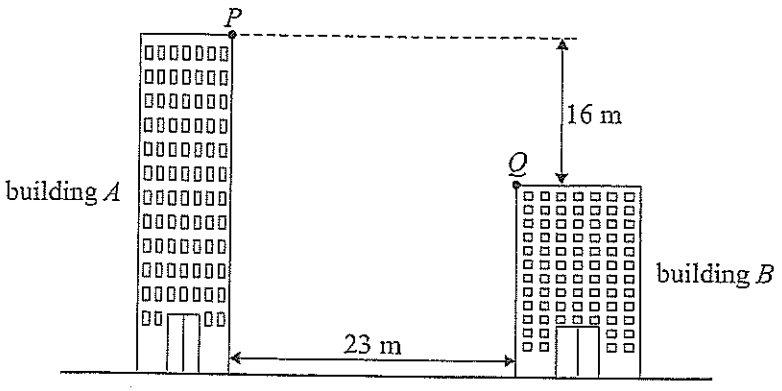
C.



D.



4.



In the diagram above, the angle of depression of point Q from point P is closest to

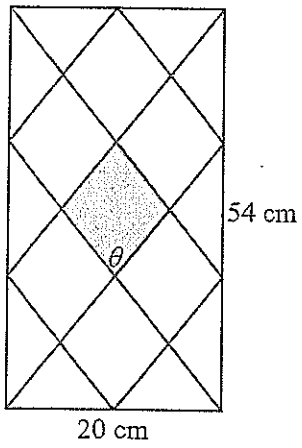
A. 35°

B. 41°

C. 46°

D. 55°

5. The rectangle shown below is 54 cm high and 20 cm wide.
The rhombuses drawn inside the rectangle are all the same size and shape.



The size of the angle θ, in the shaded rhombus, is closest to

A. 34°

B. 56°

C. 58°

D. 67°

Section 2

48 marks
Attempt Questions 6 – 11
Allow about 63 minutes for this section

Answer each question in your answer booklet. Start each question on a new page.

In Questions 6 – 11, your response should include relevant mathematical reasoning and/or calculations.

Question 6 (8 marks)

a) Fully factorise $2x^4 + 16x$ 1

b) If $\sec\theta = 3$ and $\tan\theta < 0$, find the exact value of $\sin\theta$. 2

c) Simplify $\left(\frac{x^{p+q}}{x^q}\right)^p \div \left(\frac{x^q}{x^{q-p}}\right)^{p-q}$ 2

d) If $g(5x) = 50x^2 + 10x + 1$, find an expression for $g(x)$. 1

e) Draw a neat sketch of $y = \frac{x-2}{x+2}$ 2

Question 7 (8 marks) Start a new page

a) Solve $\sin(\theta - 75°) = \frac{-\sqrt{3}}{2}$ for $0° \leq \theta \leq 360°$ 2

b) If A is an acute angle, simplify $\frac{\tan A}{\sqrt{1+tan^2 A}}$ 2

c) In pentagon $ABCDE$, angle $A = 120°$, angle $E = 140°$,

AB is parallel to DC , and BC is parallel to AE.

i) Draw a neat sketch clearly showing this information. 1

ii) Find the size of angle B, giving reasons. 1

iii) Find the size of angle D, giving reasons. 2

Question 8 (8 marks) Start a new page

a) Solve $2cos^2 x = \sin x + 1$, for $0° \leq x \leq 360°$. 3

b) Simplify $\frac{5^{-n} \times 25^{2n-2}}{5^{3n-2} \times 10^{-1}}$ 2

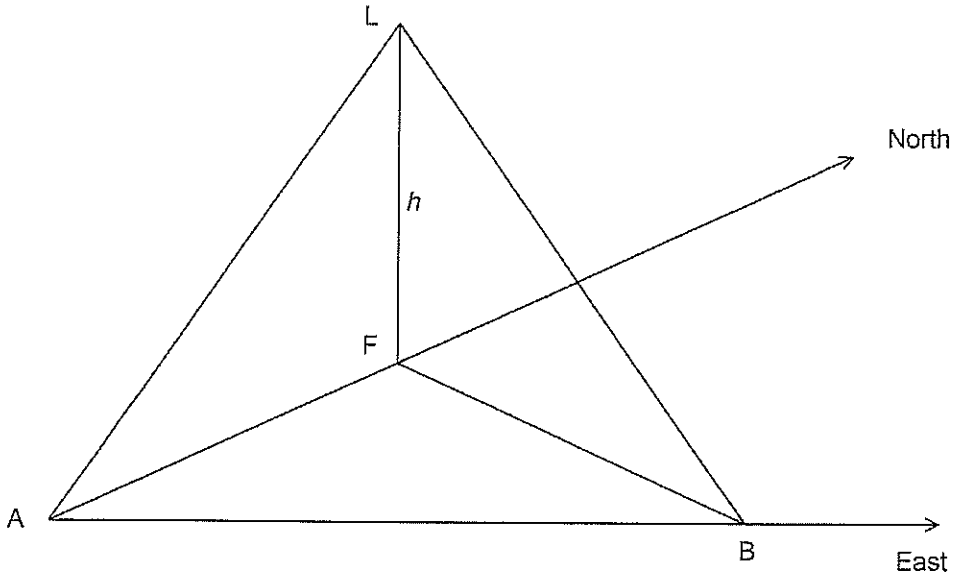
c) Solve $\frac{5}{4-x} \geq 1$ 3

Question 9 (8 marks) Start a new page

a) Solve $|2x - 1| = 3x + 6$ 2

b) Show that $\sec\alpha - \cos\alpha = \sin\alpha\tan\alpha$ 2

c)



A vertical flagpole, FL, of height h metres stands in the middle of a park. From point A, due South of the flagpole, the angle of elevation to the top of the flagpole is 35°. From point B, which is 45 metres due East of point A, the angle of elevation to the top of the flagpole is 28°.

i) Find an expression for the length of AF in terms of h. 1

ii) Find the height of the flagpole, in metres correct to 1 decimal place. 3

Question 10 (8 marks) Start a new page

a) Solve $(2x-1)^2 = 5$ 1

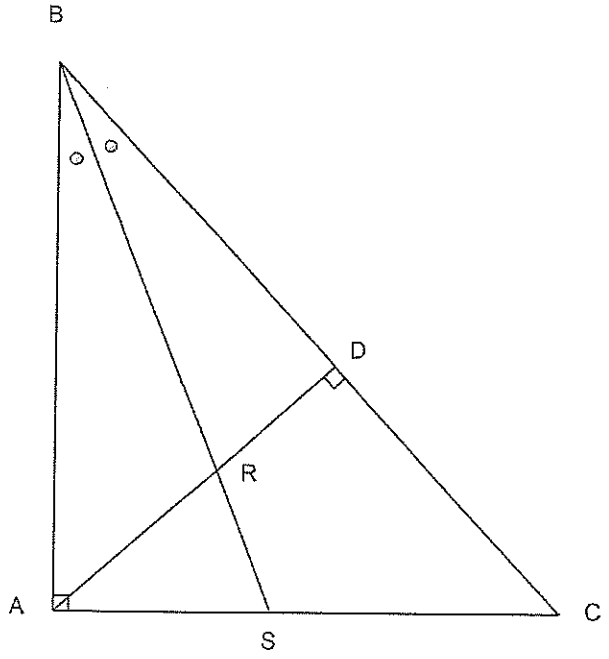
b) i) Draw a neat sketch of $y = x^2 - 6x + 8$, 2

clearly showing all intercepts and the vertex.

ii) On a separate diagram draw a neat sketch of $y = \frac{1}{x^2-6x+8}$ 2

clearly showing all important features.

c)



In triangle ABC, angle $A = 90°$, SB bisects angle B and AD is perpendicular to BC and meets SB at R. 3

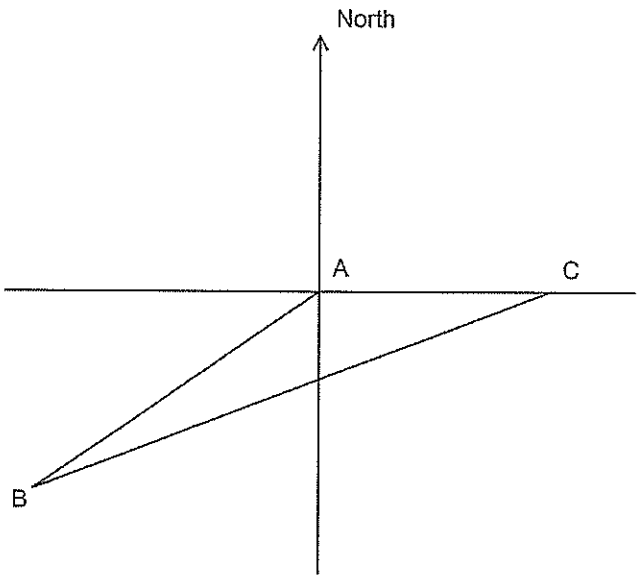
By letting angle $SBC = x$, or otherwise, prove that triangle ASR is isosceles.

Question 11 (8 marks) Start a new page

a) Solve simultaneously for x and y, 2

$y = x^2 - 2x - 1$ and $2x - y - 1 = 0$

b)



A surveyor standing at point A notes that, point B is on a bearing of $228°\ T$ and point C is due East of point A. The surveyor then walks 85 metres to point B where he notes that the bearing of point C from point B is $070°\ T$. 3

Find the distance from point B to point C.
(Give answer in metres correct to 1 decimal place)

c) Solve $|x + 1| > \sqrt{25 - x^2}$ 3

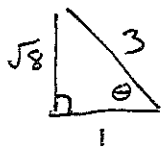
End of paper

Ext 1 Solutions May 2014

1. B
2. D
3. A
4. A
5. C

6. a) $2x(x+2)(x^2-2x+4)$

b) 4th quad



$\sin\theta = -\frac{\sqrt{8}}{3}$

c) $\left(\frac{x^{p+q}}{x^q}\right)^p \div \left(\frac{x^q}{x^{q-p}}\right)^{p-q}$

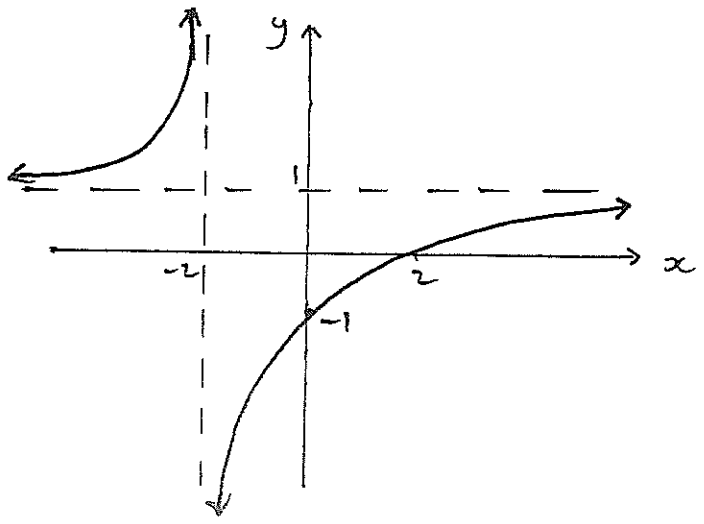
$= (x^p)^p \div (x^p)^{p-q}$

$= x^{pq}$

d) $g(5x) = 2(5x)^2 + 2(5x) + 1$

$\therefore g(x) = 2x^2 + 2x + 1$

e)



7.

a. $\theta - 75^\circ = 240^\circ, 300^\circ$

$\theta = 315^\circ, 375^\circ$

$\therefore \theta = 315^\circ, 15^\circ$

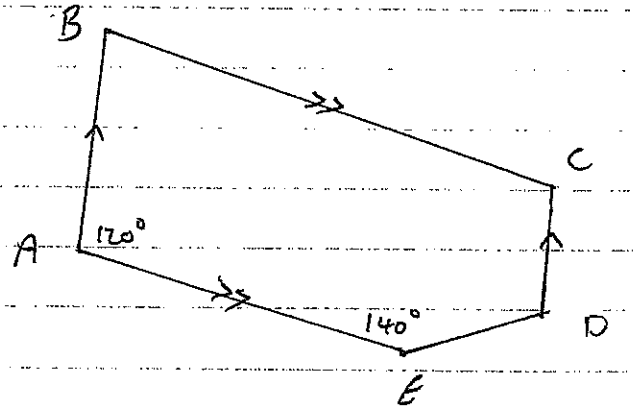
b. $\dfrac{\tan A}{\sqrt{1+\tan^2 A}}$

$= \dfrac{\tan A}{\sqrt{\sec^2 A}}$

$= \tan A \cos A$

$= \sin A$

c. i)



ii) $\angle B = 60^\circ$ (cointerior angles, $BC \parallel AE$)

iii) $\angle C = 120^\circ$ (cointerior angles, $AB \parallel DC$)

angle sum of pentagon $= 540^\circ$

$\therefore \angle D = 540^\circ - 120^\circ - 140^\circ - 60^\circ - 120^\circ$

$= 100^\circ$

8.

a. $2\cos^2 x = \sin x + 1$

$2(1-\sin^2 x) = \sin x + 1$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}, -1$

$x = 30^\circ, 150^\circ, 270^\circ$

b. $\dfrac{5^{-n} \times 25^{2n-2}}{5^{3n-2} \times 10^{-1}}$

$= \dfrac{5^{-n} \times (5^2)^{2n-2}}{5^{3n-2} \times 5^{-1} \times 2^{-1}}$

$= \dfrac{5^{-n} \times 5^{4n-4}}{5^{3n-2} \times 5^{-1} \times 2^{-1}}$

$= \dfrac{5^{3n-4}}{5^{3n-3} \times 2^{-1}}$

$= 5^{-1} \times 2$

$= \dfrac{2}{5}$

c. $\dfrac{5}{4-x} \geq 1$

$\dfrac{5(4-x)^2}{4-x} \geq (4-x)^2$

$5(4-x) \geq (4-x)^2$

$(4-x)^2 - 5(4-x) \leq 0$

$(4-x)(-x-1) \leq 0$

$-1 \leq x < 4$

9.

a. $|2x-1| = 3x+6$

$2x-1 = 3x+6$ $\qquad$ $2x-1 = -3x-6$

$x = -7$ $\qquad$ $5x = -5$

$x = -1$

test $x = -7$ ✗ $\qquad$ $x = -1$ ✓

$\therefore \quad x = -1$

b. $\text{LHS} = \sec\alpha - \cos\alpha$

$= \dfrac{1}{\cos\alpha} - \cos\alpha$

$= \dfrac{1-\cos^2\alpha}{\cos\alpha}$

$= \dfrac{\sin^2\alpha}{\cos\alpha}$

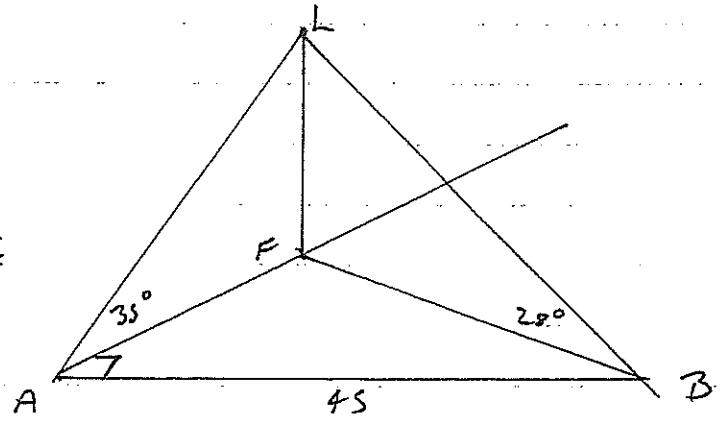
$= \sin\alpha \tan\alpha$

$= \text{RHS}$

c. i. $\tan 35° = \dfrac{h}{AF}$

$AF = \dfrac{h}{\tan 35°}$

$= h \tan 55°$ or



ii. $BF = h \tan 62°$

$AB^2 + AF^2 = BF^2$

$45^2 = h^2 \tan^2 62° - h^2 \tan^2 55°$

$h = \dfrac{45}{\sqrt{\tan^2 62° - \tan^2 55°}}$

$= 36.8 \text{ m}.$

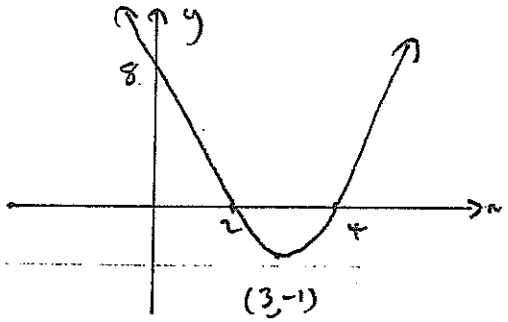
10.

a. $(2x-1)^2 = 5$

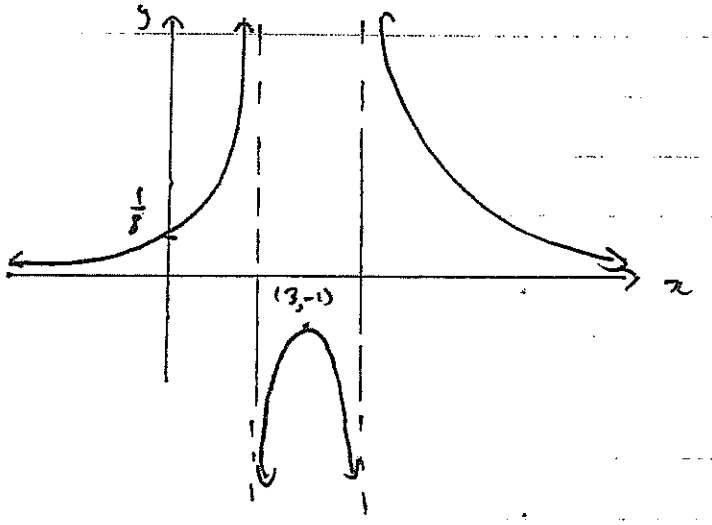
$2x - 1 = \pm\sqrt{5}$

$x = \frac{1 \pm \sqrt{5}}{2}$

b. i. $y = (x-4)(x-2)$



ii.



c. let $\angle SBC = x$

$\angle BRD = 90 - x$ (angle sum of $\triangle BDR$)

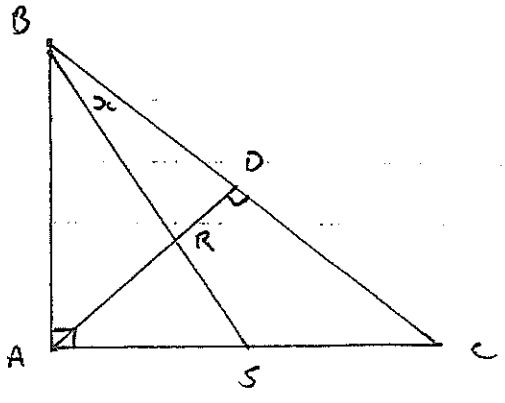
$\angle ARS = 90 - x$ (vertically opposite)

$\angle ABS = x$ (equal to $\angle SBC$)

$\therefore \angle ASB = 90 - x$ (angle sum of $\triangle ABS$)

$\therefore \angle ARS = \angle ASB$

$\therefore$ ASR is isosceles



11.

a. $y = x^2 - 2x - 1$

$2x - y - 1 = 0 \longrightarrow y = 2x - 1$ sub

$2x - 1 = x^2 - 2x - 1$

$x^2 - 4x = 0$

$x(x - 4) = 0$

$x = 0, 4$

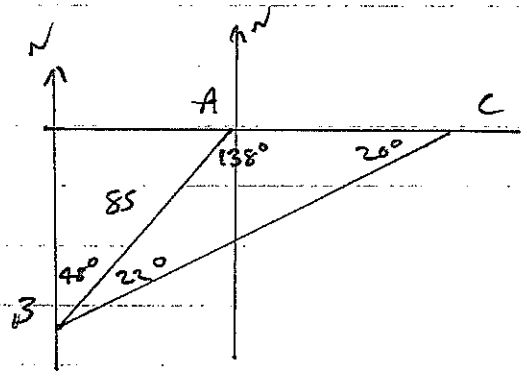
$\therefore y = -1, 7$

$\therefore$ solutions $x = 0, y = -1$ and $x = 4, y = 7$

b. $\frac{BC}{\sin 138^\circ} = \frac{85}{\sin 20^\circ}$

$BC = \frac{85 \times \sin 138^\circ}{\sin 20^\circ}$

$= 166.3$ m.



c. $y = |x+1|$

$y = \sqrt{25 - x^2}$

$x + 1 = \sqrt{25 - x^2}$

$(x+1)^2 = 25 - x^2$

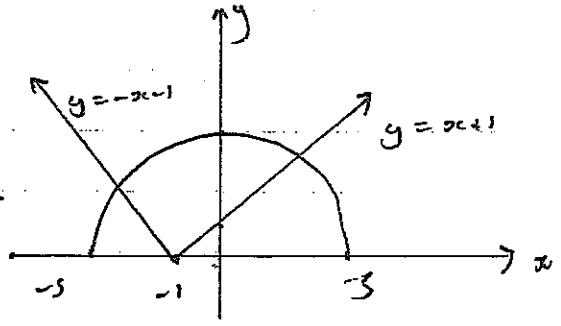
$2x^2 + 2x - 24 = 0$

$x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

$x = -4, 3$

$-x + 1 = \sqrt{25 - x^2}$ same



$\therefore -5 \leq x < -4, \quad 3 < x \leq 5$