

Name: Maths Class:

SYDNEY TECHNICAL HIGH SCHOOL



Year 11 Mathematics Extension 1

Preliminary Course

Assessment 1

May, 2015

Time allowed: 70 minutes

General Instructions:

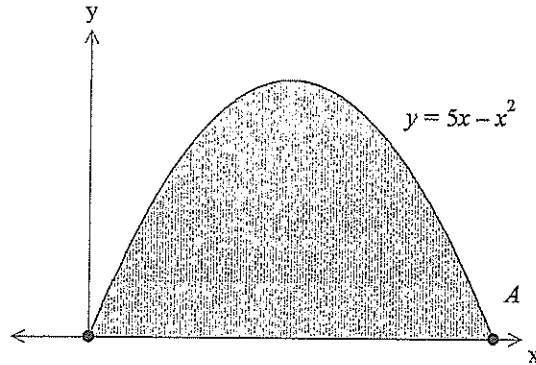
- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided

Section 1 Multiple Choice
Questions 1-5
5 Marks

Section II Questions 6-11
51 Marks

Section 1 – Multiple Choice – Answer on the sheet provided.

1 The diagram shows the graph of the function $y = 5x - x^2$.



Which pair of inequalities specify the shaded region?

- (A) $y \leq 5x - x^2$ and $y \leq 0$.
- (B) $y \leq 5x - x^2$ and $y \geq 0$.
- (C) $y \geq 5x - x^2$ and $y \leq 0$.
- (D) $y \geq 5x - x^2$ and $y \geq 0$.

2 What is the solution to the equation $|2x - 5| = x + 2$?

- (A) $x = 1$
- (B) $x = 7$
- (C) $x = 1$ or $x = 7$
- (D) $x = 1$ or $x = -7$

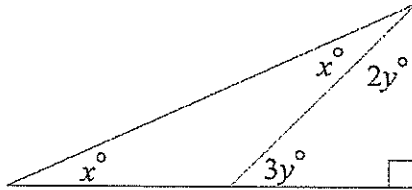
3 If $3 \cos \theta + 2 = 0$ and $\tan \theta > 0$, what is the exact value of $\sin(\theta + 180^\circ)$?

- | | |
|---------------------------|---------------------------|
| (A) $-\frac{\sqrt{5}}{3}$ | (B) $-\frac{\sqrt{5}}{2}$ |
| (C) $\frac{\sqrt{5}}{2}$ | (D) $\frac{\sqrt{5}}{3}$ |

4 A woman is standing on level ground 70 metres from the base of a vertical cliff. If the angle of elevation to the top of the cliff is 40° , what is the height of the cliff, correct to the nearest metre?

- (A) 58 metres
- (B) 59 metres
- (C) 60 metres
- (D) 61 metres

5



What is the value of x ?

- (A) 18°
- (B) 27°
- (C) 36°
- (D) 45°

End of section 1

SECTION II

(Start each new question on a new page)

QUESTION 6: (8 Marks)

| | Marks |
|---|-------|
| (a) Fully factorise, $x^4 - xy^3$ | 2 |
| (b) Write down the exact value of $\sin^2 225^\circ + \operatorname{cosec} 150^\circ$ | 2 |
| (c) Solve for x: $27^x \times \left(\frac{1}{3}\right)^{x-1} = 81$ | 2 |
| (d) State the Domain and Range of $y = \frac{2x+1}{x-2}$ | 2 |

QUESTION 7: (8 Marks) Start a new page

Marks

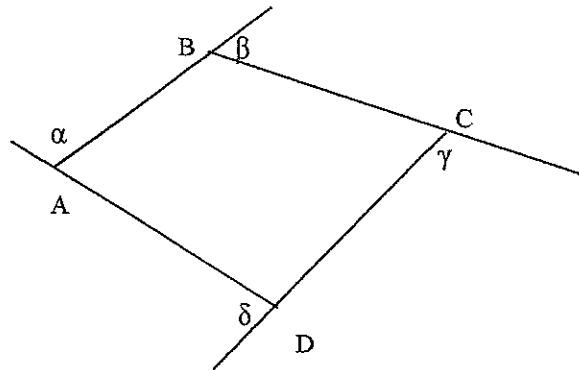
(a) If $\tan\theta = p$ and $\sec\theta < 0$, find an expression for $\sin\theta$

2

(b) ABCD is a quadrilateral with external angles α, β, γ and δ .

2

Explain why $\sin(\alpha + \beta + \gamma + \delta) = 0$

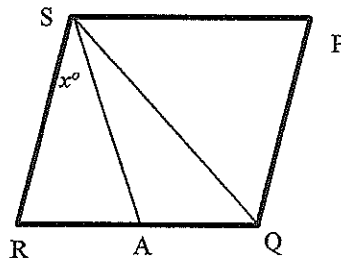


(c) PQRS is a rhombus. SA bisects $\angle RSQ$

$\angle RSA = x^\circ$

Prove: (i) $\angle RSP = 4x^\circ$

(ii) $\angle SAR = 3x^\circ$



2

2

QUESTION 8: (8 Marks) Start a new page

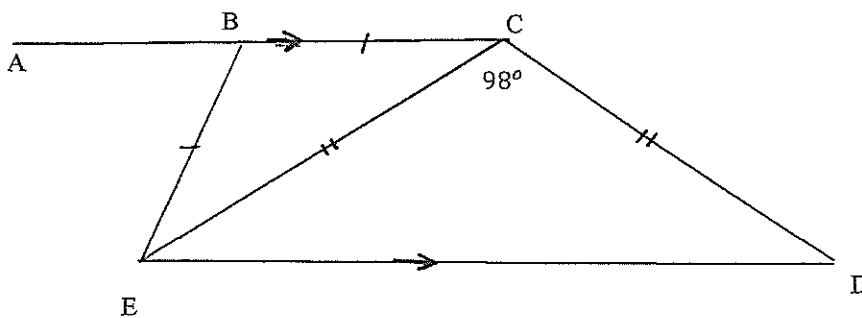
Marks

- (a) Solve for θ , if $\sin 2\theta = \cos \theta$ and $0^\circ < \theta < 90^\circ$ 1
- (b) If $f(x) = \frac{1}{x}$ write $\frac{f(x+h)-f(x)}{h}$ as a simplified fraction. 3
- (c) If $f(x) = 2x - 3$, find a simplified expression for $f(f(-x))$ 2
- (d) Sketch the function $y = \frac{1}{\sqrt{4-x}}$ showing all necessary information. 2

QUESTION 9: (8 Marks) Start a new page

Marks

- (a) Solve $\sec \theta = -2$ for $-180^\circ \leq \theta \leq 180^\circ$ 2
- (b) Consider the quadrilateral BCDE where BC is parallel to ED and CB is produced to A, $\angle ECD = 98^\circ$, $BC = BE$ and $EC = CD$



3

Copy the diagram showing all given information and find the size of angle ABE, giving reasons.

- (c) Solve the inequality $\frac{x-2}{x+3} > -2$ 3

QUESTION 10: (10 Marks) *Start a new page*

- | | <i>Marks</i> |
|---|--------------|
| (a) (i) Sketch the region $y \leq 6 - 2x $ on a number plane | 3 |
| (ii) Solve $6 - 2x = x $ | 2 |
| (iii) Find the area of the region held simultaneously by $y \leq 6 - 2x $ and $y \geq x $ | 2 |
| (b) Solve for θ , $2\sin^2\theta = \sin\theta\cos\theta$, $0^\circ \leq \theta \leq 360^\circ$, correct to the nearest minute. | 3 |

QUESTION 11: (9 Marks) *Start a new page*

- | | <i>Marks</i> |
|---|--------------|
| (a) Show that $\sec\theta + \tan\theta = \frac{\cos\theta}{1-\sin\theta}$ | 3 |
| (b) (i) Sketch the function $f(x) = \frac{1}{x^2+1}$ | 2 |
| (ii) On a separate number plane, sketch the function $y = -f(x) - 1$ | 2 |
| (c) Solve $ x + 2 + x - 2 = 6 - 4x$ | 2 |

End of Assessment task

11
2015

Section 1

Solutions.

1. B
2. C
3. D
4. B
5. B

Section 2

Question 6

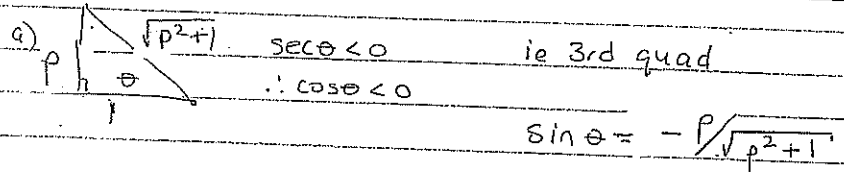
a) $x^4 - xy^3 = x(x-y)(x^2 + xy + y^2)$

b) $(\sin 225^\circ)^2 + \frac{1}{\sin 150^\circ} = (-\frac{1}{\sqrt{2}})^2 + (\frac{1}{\frac{1}{2}})$
 $= \frac{1}{2} + 2 = 2\frac{1}{2}$

c) $27^x \times (\frac{1}{3})^{x-1} = 81$
 $3^{3x} \times 3^{-x+1} = 3^4$
 $3^{2x+1} = 3^4$
 $2x = 3$
 $x = \frac{3}{2}$

d) $y = \frac{2x+1}{x-2}$ D: $x \in \mathbb{R}, x \neq 2$
R: $y \in \mathbb{R}, y \neq 2$

Question 7



b) exterior angles of a polygon equal 360°
 $\alpha + \beta + \delta + \delta = 360^\circ$
ie $\sin(\alpha + \beta + \delta + \delta) = \sin 360^\circ = 0$

c) $\angle RSA = x$ given
 $\angle ASQ = \angle RSA$ (given SA bisects $\angle RSQ$)
 $= x$

$\angle ASP = \angle RSQ$ (diagonal of a Rhombus bisects interior angles)
 $= 2x$

$\therefore \angle RSP = \angle RSA + \angle ASQ + \angle ASP$ (sum of adjacent angles)
 $= 4x$

$\angle PSA = \angle PSQ + \angle QSA$ (adjacent angles)
 $= 2x + x$

$\angle SAR = \angle PSA$ (alternate angles, $SP \parallel RQ$ opposite sides of Rhombus equal)
 $= 3x$

Question 8

a) $\sin 2\theta = \cos \theta$ as $\sin A = \cos(90 - A)$
then $2\theta + \theta = 90^\circ$
 $\theta = 30^\circ$

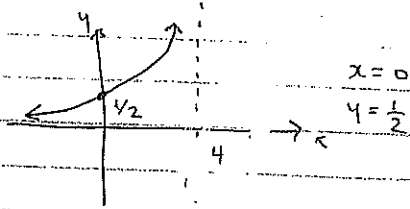
b) $\frac{\frac{1}{x+h} - \frac{1}{x}}{h} \div \frac{1}{x(x+h)x}$
 $= \frac{x - (x+h)}{h(x+h)x} \div \frac{1}{x(x+h)x}$
 $= \frac{-h}{h(x+h)x} \div \frac{1}{x(x+h)x}$
 $= -1$

c) $f(x) = 2x - 3$ then $f(-x) = -2x - 3$
and $f[f(-x)] = 2[-2x - 3] - 3$
 $= -4x - 6 - 3$
 $= -4x - 9$

$$y = \frac{1}{\sqrt{4-x}}$$

$$D: x < 4$$

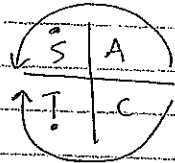
$$R: y > 0$$



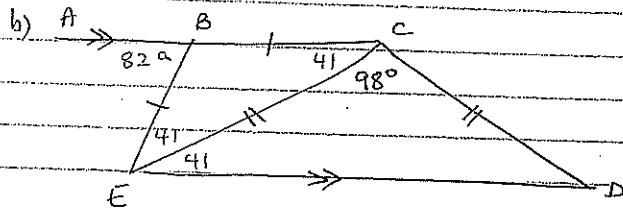
Question 9

$$\begin{aligned} a) \sec \theta &= -2 \\ \cos \theta &= -\frac{1}{2} \end{aligned}$$

$$-180^\circ \leq \theta \leq 180^\circ$$



$$\therefore \theta = \pm 120^\circ$$



$\angle CED = \angle CDE$ (base angles isosceles $\triangle CED$)

$$\therefore \angle CED = 41^\circ \text{ (angle sum } \triangle CED)$$

$\angle BCE = \angle CED$ (alternate angles $AC \parallel ED$)

$$= 41^\circ$$

$\angle BCE = \angle BEC$ (base angles of isosceles $\triangle BCE, BC = BE$)

$$= 41^\circ$$

$$\therefore \angle ABE = \angle BCE + \angle BEC \text{ (exterior angle } \triangle BCE)$$

$$= 82^\circ$$

$$c) \frac{x-2}{x+3} > -2 \quad x \neq -3$$

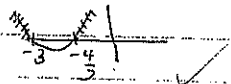
$$(x-2)(x+3) > -2(x+3)^2$$

$$(x-2)(x+3) + 2(x+3)^2 > 0$$

$$(x+3)[(x-2) + 2(x+3)] > 0$$

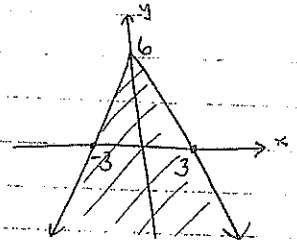
$$(x+3)(3x+4) > 0$$

$$\therefore x < -3, x > -4/3$$



Question 10

$$a. \quad i. \quad y = 6 - |2x|$$



$$ii. \quad y = |x| \text{ and } 6 - |2x|$$

$$6 - |2x| = |x|$$

$$\text{Let } 6 - 2x = x$$

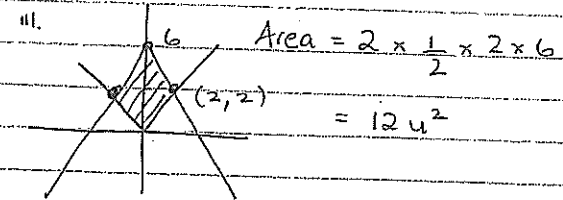
$$6 = 3x$$

$$x = 2$$

$$6 + 2x = -x$$

$$6 = -3x$$

$$x = -2$$



$$b. \quad 2\sin^2 \theta = \sin \theta \cos \theta \quad 0^\circ \leq \theta \leq 360^\circ$$

$$2\sin^2 \theta - \sin \theta \cos \theta = 0$$

$$\sin \theta (2\sin \theta - \cos \theta) = 0$$

$$\sin \theta = 0 \quad \text{or} \quad 2\sin \theta = \cos \theta$$

$$\tan \theta = \frac{1}{2}$$

$$\theta = 0^\circ, 180^\circ, 360^\circ \quad \theta = \tan^{-1}(1/2), 180 + \tan^{-1}(1/2)$$

$$= 26^\circ 34', 206^\circ 34'$$

11. a. Show that $\sec\theta + \tan\theta = \frac{\cos\theta}{1-\sin\theta}$

$$\text{LHS} = \sec\theta + \tan\theta$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1 + \sin\theta}{\cos\theta} \times \frac{1 - \sin\theta}{1 - \sin\theta}$$

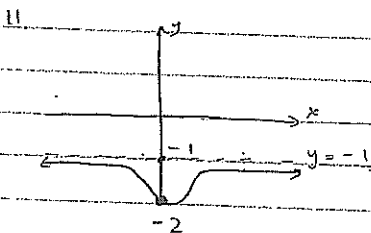
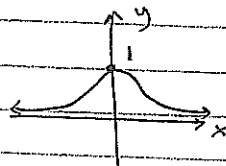
$$= \frac{1 - \sin^2\theta}{\cos\theta(1 - \sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta(1 - \sin\theta)}$$

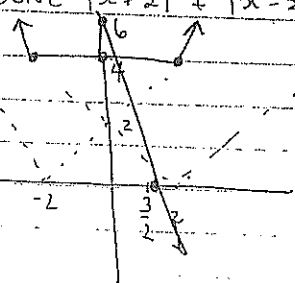
$$= \frac{\cos\theta}{1 - \sin\theta}$$

$$= \text{RHS.}$$

b. i. $f(x) = \frac{1}{x^2 + 1}$



c. Solve $|x+2| + |x-2| = 6 - 4x$



\therefore from the graph

$$\text{sol}^n \quad 6 - 4x = 4$$

$$4x = 2$$

$$x = \frac{1}{2}$$