GIRRAWEEN HIGH SCHOOL

YEAR 11 - TASK 1

2004

MATHEMATICS
2 UNIT

Time allowed – 90 minutes

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.

- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.

- Board-approved calculators may be used.

- Start each question on a *new* sheet of paper.

Question 1 (16 marks)

(a) Find $\sqrt{\dfrac{3}{7^2}} + 4(5)^3$ correct to 2 decimal places. **2**

(b) Evaluate $\sqrt{\dfrac{83.1}{0.2 + 5.4 \div 1.3}}$ correct to 3 significant figures. **2**

(c) Write in scientific notation.
(i) 364 000 **2**

(ii) 0.000912 **2**

(d) Simplify $2\sqrt{12}$. **2**

(e) If $\sqrt{800} = x\sqrt{2}$, find the value of x. **2**

(f) Write $64^{-\frac{2}{3}}$ as a rational number. **2**

(g) Find $\left(5.7 \times 10^{13}\right) \times \left(4.9 \times 10^{22}\right)$. Write your answer in scientific notation. **2**

Question 2 (16 marks)

(a)   Expand and simplify

(i) $4a(a - 2b) - 2a(b - 3a)$ **2**

(ii) $(3y - 1)(3y + 1)$ **2**

(iii) $(x - 5)(x^2 + 5x + 25)$ **2**

(iv) $(5t + 3)^2$ **2**

(v) $(2x - 3)(x + 8)$ **2**

(b)   If $d = \dfrac{c}{\sqrt{a^2 + b^2}}$. Find the exact value of d with a rational denominator if

$$a = 2,\ b = -1,\ c = 3,$$ **2**

(c)   If $A = \frac{1}{2} r^2 \theta$.   Find the value of θ, when $A = 12$ and $r = 4$.   **2**

(d)   If $V = \pi r^2 h$.   Find the *exact* value of r when $V = 9$ and $h = 16$.   **2**

Question 3 (14 marks)

(a) Factorise.

(i) $m^2 + 14m - 32$ **2**

(ii) $x^3 + x^2 + 3x + 3$ **2**

(iii) $b^3 - 8$ **2**

(iv) $3y^2 + 5y - 2$ **3**

(v) $z^2 - (1 + w)^2$ **3**

(b) Complete the square on, $x^2 - 18x + \square$?   **2**

Question 4 (23 marks)

(a) Simplify.

(i) $\sqrt{27} - \sqrt{12}$ **2**

(ii) $\sqrt{32} - 3\sqrt{2} + 2\sqrt{50}$ **2**

(iii) $\dfrac{\sqrt{2}}{\sqrt{8}}$ **2**

(b) Expand and simplify.

(i) $2\sqrt{3}(2 - \sqrt{12})$ **2**

(ii) $(2\sqrt{2} - \sqrt{3})^2$ **3**

(c) Express with a rational denominator.

(i) $\dfrac{5}{\sqrt{2}}$ **2**

(ii) $\dfrac{\sqrt{3}}{\sqrt{2} - 7}$ **3**

(iii) $\dfrac{\sqrt{3}}{\sqrt{5} + 2\sqrt{6}}$ **3**

(d) Express as a single fraction with rational denominator.

$\dfrac{\sqrt{2} - \sqrt{7}}{\sqrt{2} + \sqrt{3}} \times \dfrac{2}{\sqrt{2} - \sqrt{3}}$ **4**

Question 5 (20 marks)

(a)   Simplify

    (i) $\dfrac{4a}{7}\times\dfrac{21}{5a^2}$ **2**

    (ii) $\dfrac{5}{x^2-1}\div\dfrac{2}{x-1}$ **2**

    (iii) $\dfrac{3}{b+2}\times\dfrac{b^2+2b}{6a-3}$ **3**

    (iv) $\dfrac{3ab^2}{5xy}\div\dfrac{12ab-6a}{x^2y+2xy^2}$ **3**

(b)   Simplify

    (i) $\dfrac{a+4}{3}-\dfrac{a}{4}$ **2**

    (ii) $\dfrac{1}{a^2+2a+1}+\dfrac{1}{a+1}$ **2**

    (iii) $x-\dfrac{x^2}{x+2}$ **3**

    (iv) $\dfrac{1}{x-1}-\dfrac{2}{x}$ **3**

Question 6 (15 marks)

(a)   Simplify. $\dfrac{2xy+2x-6-6y}{4x^2-16x+12}$ **4**

(b)   Simplify. $\dfrac{2p^2-3p-2}{8p^3+1}$ **4**

(c)   Write $0.3\dot{0}\dot{4}$ as a rational number.   **3**

(d)   Write $1.6\dot{2}\dot{7}$ as a rational number.   **4**

Solutions

Yr 11 – Mathematics
TASK 1 (2004)

Q1 a) $= 500.2474\ldots$
$= 500.25$ (2 dec. pl.)

b) $= 4.368818\ldots$
$= 4.37$ (3 sign. figs.)

c) (i) 3.64×10^5

(ii) 9.12×10^{-4}

d) $= 2 \times 2\sqrt{3}$
$= 4\sqrt{3}$

e) $\sqrt{800} = \sqrt{400}\sqrt{2}$
$= 20\sqrt{2}$
$\therefore x = 20$

f) $64^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{64^2}} = \frac{1}{16}$

g) 2.793×10^{36}

Q2 a)(i) $4a(a-2b) - 2a(b-3a)$
$= 4a^2 - 8ab - 2ab + 6a^2$
$= 10a^2 - 10ab$

(ii) $(3y-1)(3y+1)$
$= 9y^2 - 1$

(iii) $(x-5)(x^2+5x+25)$
$= x^3 - 5^3$
$= x^3 - 125$

(iv) $(5t+3)^2 = 25t^2 + 30t + 9$

(v) $(2x-3)(x+8) = 2x^2 + 16x - 3x - 24$
$= 2x^2 + 13x - 24$

b) $d = \frac{3}{\sqrt{2^2 + (-1)^2}} = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$
$= \frac{3\sqrt{5}}{5}$

c) $12 = \frac{1}{2}(4)^2\theta$
$24 = 16\theta$
$\theta = \frac{24}{16}$
$\theta = 1.5$ or $1\frac{1}{2}$

d) $9 = \pi r^2 \left(\frac{\theta}{16}\right)$
$r^2 = \frac{9}{\pi(16)}$
$r = \pm\sqrt{\frac{9}{16\pi}} = \pm\frac{3}{4\sqrt{\pi}}$

Q3 a) (i) $m^2 + 14m - 32$
$= (m+16)(m-2)$

(ii) $x^3 + x^2 + 3x + 3$
$= x^2(x+1) + 3(x+1)$
$= (x+1)(x^2+3)$

Q3 a) (iii) $b^3 - 8$
cont. $= (b-2)(b^2 + 2b + 4)$

(iv) $3y^2 + 5y - 2$
or $(3y-1)(y+2)$ $= 3y^2 + 6y - y - 2$
$= 3y(y+2) - 1(y+2)$
$= (y+2)(3y-1)$

(v) $z^2 - (1+w)^2$
$= (z - (1+w))(z + (1+w))$
$= (z - 1 + w)(z + 1 + w)$

b) $x^2 + 18x + \square$
$\square = \left(\frac{-18}{2}\right)^2 = 81$

Q4 a) (i) $\sqrt{27} - \sqrt{12}$
$= 3\sqrt{3} - 2\sqrt{3}$
$= \sqrt{3}$

(ii) $\sqrt{32} - 3\sqrt{2} + 2\sqrt{50}$
$= 4\sqrt{2} - 3\sqrt{2} + 2\sqrt{25}\sqrt{2}$
$= \sqrt{2} + 10\sqrt{2}$
$= 11\sqrt{2}$

(iii) $\frac{\sqrt{2}}{\sqrt{8}} = \frac{\sqrt{2}}{2\sqrt{2}} = \frac{1}{2}$

b) (i) $2\sqrt{3}(2 - \sqrt{12})$
$= 4\sqrt{3} - 2\sqrt{36}$
$= 4\sqrt{3} - 12$

(ii) $(2\sqrt{2} - \sqrt{3})^2$
$= 8 - 4\sqrt{6} + 3$
$= 11 - 4\sqrt{6}$

c) (i) $\frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

(ii) $\frac{\sqrt{3}}{\sqrt{2} - 7} \times \frac{\sqrt{2} + 7}{\sqrt{2} + 7} = \frac{\sqrt{6} + 7\sqrt{3}}{2 - 49}$
$= \frac{\sqrt{6} + 7\sqrt{3}}{-47}$

(iii) $\frac{\sqrt{3}}{\sqrt{5} + 2\sqrt{6}} \times \frac{\sqrt{5} - 2\sqrt{6}}{\sqrt{5} - 2\sqrt{6}}$
$= \frac{\sqrt{15} - 2\sqrt{18}}{5 - 24}$
$= \frac{\sqrt{15} - 6\sqrt{2}}{-19}$

d) $\frac{\sqrt{2} - \sqrt{7}}{\sqrt{2} + \sqrt{3}} \times \frac{2}{\sqrt{2} - \sqrt{3}}$
$= \frac{2\sqrt{2} - 2\sqrt{7}}{2 - 3}$
$= \frac{2\sqrt{2} - 2\sqrt{7}}{-1}$ or $2\sqrt{7} - 2\sqrt{2}$

Q5 a) (i) $\dfrac{4a}{x} \times \dfrac{2x^3}{5a^2} = \dfrac{12}{5a}$

(ii) $\dfrac{5}{x^2-1} \div \dfrac{2}{x-1}$

$= \dfrac{5}{(x+1)(x-1)} \times \dfrac{(x-1)}{2}$

$= \dfrac{5}{2(x+1)}$ (oe) $\dfrac{5}{2x+2}$

(iii) $\dfrac{3}{b+2} \times \dfrac{b^2+2b}{6a-3}$

$= \dfrac{3}{b+2} \times \dfrac{b(b+2)}{3(2a-1)}$

$= \dfrac{b}{2a-1}$

(iv) $\dfrac{3ab^2}{5xy} \div \dfrac{12ab-6a}{x^2y+2xy^2}$

$= \dfrac{3ab^2}{5xy} \times \dfrac{xy(x+2y)}{6a(2b-1)}$

$= \dfrac{b^2(x+2y)}{10(2b-1)}$

(oe) $= \dfrac{b^2x+2yb^2}{20b-10}$

b) (i) $\dfrac{a+4}{3} - \dfrac{a}{4}$

$= \dfrac{4(a+4) - 3a}{12}$

$= \dfrac{4a+16-3a}{12}$

$= \dfrac{a+16}{12}$

(ii) $\dfrac{1}{a^2+2a+1} + \dfrac{1}{a+1}$

$= \dfrac{\dfrac{1}{(a+1)^2} + \dfrac{1}{a+1}}{}$

$= \dfrac{1+(a+1)}{(a+1)^2}$

$= \dfrac{a+2}{(a+1)^2}$

(iii) $x - \dfrac{x^2}{x+2} = \dfrac{x(x+2)}{x+2} - \dfrac{x^2}{x+2}$

$= \dfrac{x^2+2x-x^2}{x+2}$

$= \dfrac{2x}{x+2}$

(iv) $\dfrac{1}{x-1} - \dfrac{2}{x} = \dfrac{x-2(x-1)}{x(x-1)}$

$= \dfrac{x-2x+2}{x(x-1)}$

$= \dfrac{2-x}{x(x-1)}$

Q6 a) $\dfrac{2xy+2x-6-6y}{4x^2-16x+12}$

$\dfrac{2x(y+1)-6(y+1)}{4(x^2-4x+3)}$

$= \dfrac{(y+1)(2x-6)}{4(x-3)(x-1)}$

$= \dfrac{2(y+1)(x-3)}{2\cdot2(x-1)(x-3)}$

$= \dfrac{y+1}{2x-2}$

Q6 cont.

b) $\dfrac{2p^2-3p-2}{8p^3+1} \div \dfrac{2p}{p} \times \dfrac{1}{-2}$

$= \dfrac{(2p+1)(p-2)}{(2p+1)(4p^2-2p+1)}$

$= \dfrac{p-2}{4p^2-2p+1}$

c) $x = 0.\overset{.}{3}0\overset{.}{4}$

$x = 0.304304\ldots$

$1000x = 304.304304\ldots$

$1000x - x = 304.304\ldots - 0.304\ldots$

$999x = 304$

$x = \dfrac{304}{999}$

d) $x = 1.6\overset{.}{2}\overset{.}{7}$

$x = 1.62727\ldots$

$10x = 16.2727\ldots$

$1000x = 1627.2727\ldots$

$1000x - 10x = 1627.2727\ldots - 16.2727\ldots$

$990x = 1627-16$

$990x = 1611$

$x = \dfrac{1611}{990}$

$x = 1\dfrac{69}{110}$