

**GIRRAWEE HIGH SCHOOL
MATHEMATICS**

Year 11 Mathematics Task 1

Thursday 22nd March 2007

- Instructions: a) Write all your answers on your own paper.
b) Show all necessary working.
c) Marks may be deducted for careless or badly arranged work.

Time Allowed: 90 minutes

Question 1 (21 marks)

Marks

- a) Calculate $\sqrt{\frac{36.41 - 19.57}{23.62 - 11.39}}$ correct to four significant figures 2
- b) Classify each of these real numbers as rational or irrational;
- (i) π 1
- (ii) $\sqrt{\frac{4}{11}}$ 1
- (iii) $0.12\dot{5}$ 1
- (iv) $\sqrt[3]{64}$ 1
- c) Write 0.0000000607 in scientific notation 2
- d) Simplify;
- (i) $16^{\frac{3}{4}}$ 2
- (ii) $12a^7 \times 6a^9 \div 9a^5$ 2
- e) Find the exact value of $\frac{x^4 z}{y^4}$ where $x = \left(\frac{2}{3}\right)^2$, $y = \left(\frac{4}{3}\right)^4$, $z = \left(\frac{8}{3}\right)^3$ 3
- f) Express the following as a fraction on its simplest form;
- (i) $0.\dot{2}1$ 3
- (ii) $0.3\dot{4}\dot{5}$ 3

Question 2 (21 marks)**Marks**

a) Simplify;

(i) $6\sqrt{2} \times 5\sqrt{7}$

1

(ii) $\sqrt{18} + \sqrt{128} - \sqrt{242}$

2

(iii) $\frac{5\sqrt{7} \times \sqrt{3}}{\sqrt{28}}$

2

b) Expand and simplify;

(i) $\sqrt{6}(\sqrt{3} - 3)$

2

(ii) $(2\sqrt{6} + 3)(2\sqrt{6} - 3)$

3

(iii) $(3\sqrt{2} - \sqrt{5})^2$

3

c) Express with a rational denominator;

(i) $\frac{3}{5\sqrt{2}}$

2

(ii) $\frac{\sqrt{7}}{3 - \sqrt{7}}$

3

d) Show that $\frac{1}{9 - 4\sqrt{5}} - \frac{4}{2 + \sqrt{5}}$ is a rational number.

3

Question 3 (21 marks)

a) Expand and simplify;

(i) $4(x + 3) - 5(2x - 3)$

2

(ii) $(2 + 3a)^2$

2

(iii) $(m^2 - 4m + 16)(m + 4)$

2

(iv) $(x + 1)^2(x - 1)^2$

3

b) Factorise;

(i) $36 - 25k^2$

2

(ii) $x^2 + 5x - 36$

2

(iii) $a^3 + 8$

2

(iv) $4m^2 + 4m - 15$

3

(v) $x^3 + 9x^2 - 4x - 36$

3

Question 4 (24 marks)

a) Factorise and simplify;

(i) $\frac{2p+2q}{p+q}$ 2

(ii) $\frac{ac+ad+bc+bd}{a^2+ab}$ 3

b) Simplify;

(i) $\frac{x+1}{2} + \frac{x+2}{3}$ 2

(ii) $\frac{x}{x-1} - \frac{x}{x+1}$ 3

(iii) $\frac{1}{x^3-1} + \frac{x+1}{x^2+x+1}$ 3

c) Simplify;

(i) $\frac{3a^2b}{4b^3c} \times \frac{2c^2}{8a^3} \div \frac{6ac}{16b^2}$ 2

(ii) $\frac{x^2+x-2}{x+2} \times \frac{x^2-3x}{x^2-4x+3}$ 3

(iii) $\frac{m^3+1}{6m^2} \times \frac{3m}{m^2+m}$ 3

(iv) $\frac{ab-2b^2}{6a^2b} \div \frac{a^2-4ab+4b^2}{3a}$ 3

Question 5 (20 marks)

a) Solve the following equations, writing solutions in simplest exact form;

(i) $x + \frac{x}{3} = 7$ 2

(ii) $2m(2m+9) = 0$ 2

(iii) $3a^2 + 2a - 8 = 0$ 3

(iv) $2h^2 + 8h + 3 = 0$ 3

(v) $\frac{2}{x+3} + \frac{x+3}{2} = \frac{10}{3}$ 3

(vi) $|x+2| = 2x-5$ 4

b) Solve $x^2 - 6x + 3 = 0$ by completing the square. 3

Question 6 (18 marks)

a) Solve each inequation, and graph your solution on the number line;

(i) $3 - 2x < 7$ 2

(ii) $-7 \leq 5x + 3 \leq 3$ 3

(iii) $|3x - 5| > 4$ 4

b) Solve the following simultaneous equations;

(i) $2x + y = 9$ 3
 $x + y = 5$

(ii) $2x + 3y = 28$ 3
 $3x + 2y = 27$

(ii) $x^2 + y^2 = 146$ 3
 $x + 2y = 21$

Question 1 (21)

a) $\sqrt{36 \cdot 41 - 19 \cdot 57} = 11.39$ (2)
 $\sqrt{23 \cdot 62 - 11 \cdot 39} = 11.73$ (2)

- b) (i) irrational (1)
 (ii) irrational (1)
 (iii) rational (1)
 (iv) rational (1)

c) $0.000\ 000\ 060\ 7 = 6.07 \times 10^{-8}$ (2)

d) $(1/6)^{-3} = (6/1)^3 = 216$ (2)

e) $\frac{1}{2} = \frac{1}{2}$ (2)

f) $12a^7 \times 6a^9 \div 9a^5 = 8a^{11}$ (2)

g) $\frac{1}{4} = \left(\frac{1}{3}\right)^8 \times \left(\frac{8}{3}\right)^3 \times \left(\frac{3}{4}\right)^{16}$ (2)

h) $\frac{243}{32} = \frac{3^5}{2^5}$ (2)

i) $0.2i = \frac{2i}{10} = \frac{i}{5}$ (2)

j) $0.345 = \frac{345}{1000} = \frac{71}{200}$ (2)

k) $0.345 = \frac{345}{1000} = \frac{71}{200}$ (2)

Question 2 (21)

a) (i) $6\sqrt{2} \times 5\sqrt{7} = 30\sqrt{14}$ (1)

(ii) $\sqrt{8} + \sqrt{128} = 2\sqrt{2} + 8\sqrt{2} = 10\sqrt{2}$ (2)

(iii) $5\sqrt{7} \times \sqrt{3} = 5\sqrt{21}$ (2)

b) (i) $\sqrt{6}(\sqrt{3}-3) = \sqrt{18} - 3\sqrt{6} = 3\sqrt{2} - 3\sqrt{6}$ (2)

(ii) $(2\sqrt{6}+5)(2\sqrt{6}-3) = 24-9 = 15$ (2)

(iii) $(3\sqrt{2}-\sqrt{5})^2 = 18-6\sqrt{10}+5 = 23-6\sqrt{10}$ (2)

c) (i) $\frac{3}{5\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{10}$ (2)

(ii) $\frac{\sqrt{7}}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{3+3\sqrt{7}}{9-7} = \frac{3+3\sqrt{7}}{2}$ (2)

d) $\frac{1}{9-4\sqrt{5}} - \frac{2+4\sqrt{5}}{9+4\sqrt{5}} = \frac{1 - (2+4\sqrt{5})(9+4\sqrt{5})}{(9-4\sqrt{5})(9+4\sqrt{5})} = \frac{1 - (18+36\sqrt{5}+36+160)}{81-80} = \frac{-199-36\sqrt{5}}{1} = -199-36\sqrt{5}$ (2)

Question 3 (21)

a) (i) $4(2x+3) - 5(2x-3) = 8x+12-10x+15 = -2x+27$ (2)

(ii) $(2+3a)^2 = 4+12a+9a^2$ (2)

(iii) $(m^2-4m+16)(m+4) = m^3+4m^2-4m^2-16m+16m+64 = m^3+64$ (2)

(iv) $(x+1)(x-1)^2 = (x+1)(x^2-2x+1) = x^3-x^2+x+1$ (2)

b) (i) $36-25k^2 = (6+5k)(6-5k)$ (2)

(ii) $x^2+5x-36 = (x+9)(x-4)$ (2)

(iii) $a^3+8 = (a+2)(a^2-2a+4)$ (2)

(iv) $4m^2+4m-15 = (4m+10m-6m-15) = 9m(4m+5) - 3(4m+5) = (3m+5)(3m-3)$ (2)

(v) $2x^3+9x^2-4x-36 = x^2(2x+9) - 4(2x+9) = (2x+9)(x^2-4) = (2x+9)(x+2)(x-2)$ (2)

Question 4 (24)

a) (i) $\frac{2p+2q}{p+q} = \frac{2(p+q)}{p+q} = 2$ (2)

(ii) $\frac{a^2+ab}{a(a+b)} = \frac{a(a+b)}{a(a+b)} = 1$ (2)

(iii) $\frac{c+d}{a} = \frac{c+d}{a}$ (2)

b) (i) $\frac{2x+1}{2} + \frac{2x+2}{3} = \frac{3(2x+1)+2(2x+2)}{6} = \frac{6x+3+4x+4}{6} = \frac{10x+7}{6}$ (2)

(ii) $\frac{x}{x-1} - \frac{x}{x+1} = \frac{x(x+1)-x(x-1)}{(x-1)(x+1)} = \frac{x^2+x-x^2+x}{x^2-1} = \frac{2x}{x^2-1}$ (2)

(iii) $\frac{1}{x^3-1} + \frac{2x+1}{2x^2+x+1} = \frac{1+(2x+1)(x-1)}{(x-1)(x^2+x+1)} = \frac{1+2x^2-x-1}{(x-1)(x^2+x+1)} = \frac{2x^2-x}{(x-1)(x^2+x+1)}$ (2)

c) (i) $\frac{3a^2b}{4b^2c} \times \frac{2c^2}{8a^3} = \frac{6ac}{32ab^2} = \frac{3ac}{16ab^2}$ (2)

(ii) $\frac{1}{2a^2} = \frac{1}{2a^2}$ (2)

(iii) $\frac{x^2+x-2}{x+2} \times \frac{x^2-3x}{x^2-4x+3} = \frac{(x+2)(x-1)}{(x+2)} \times \frac{x(x-3)}{x(x-3)} = x-1$ (2)

(iv) $\frac{m^3+1}{6m^2} \times \frac{3m}{m^2+m} = \frac{(m+1)(m^2-m+1)}{6m^2} \times \frac{3m}{m(m+1)} = \frac{3(m^2-m+1)}{6m^2} = \frac{m^2-m+1}{2m^2}$ (2)

(v) $\frac{ab-2b^2}{6a^2b} \div \frac{a^3-4ab+4b^3}{3a} = \frac{b(a-2b)}{6a^2b} \times \frac{3a}{(a-2b)^2} = \frac{1}{2a(a-2b)}$ (2)

Question 5 (20)

(i) $x + \frac{x}{3} = 7$
 $3x + x = 21$
 $4x = 21$
 $x = \frac{21}{4}$ (2)

(ii) $2m(2m+9) = 0$
 $m = 0$ or $m = \frac{-9}{2}$ (2)

(iii) $3a^2 + 2a - 8 = 0$
 $3a^2 + 6a - 4a - 8 = 0$
 $3a(a+2) - 4(a+2) = 0$
 $(a+2)(3a-4) = 0$
 $a = -2$ or $a = \frac{4}{3}$ (3)

(iv) $2h^2 + 8h + 3 = 0$
 $h = \frac{-8 \pm \sqrt{64 - 24}}{4}$
 $= \frac{-8 \pm \sqrt{40}}{4}$
 $= \frac{-8 \pm 2\sqrt{10}}{4}$
 $= \frac{-2 \pm \sqrt{10}}{2}$ (3)

(v) $2 + \frac{2+3}{x} = \frac{10}{3}$
 $2 + 3(x+3)^2 = 20(x+3)$
 $12 + 3x^2 + 18x + 27 = 20x + 60$
 $3x^2 - 2x - 21 = 0$
 $3x^2 - 9x + 7x - 21 = 0$
 $3x(x-3) + 7(x-3) = 0$
 $(x-3)(3x+7) = 0$
 $x = 3$ or $x = -\frac{7}{3}$ (3)

(vi) $|x+2| = 2x-5$
 $x+2 = 2x-5$ or $x+2 = -(2x-5)$
 $x = 7$ or $x+2 = -2x+5$
 $3x = 3$
 $x = 1$
 Not solution (4)

(vii) $x^2 - 6x + 3 = 0$
 $x^2 - 6x = -3$
 $x^2 - 6x + 9 = 6$
 $(x-3)^2 = 6$
 $x-3 = \pm\sqrt{6}$
 $x = 3 \pm \sqrt{6}$ (3)

Question 6 (18)

(i) $3-2x < 7$
 $-2x < 4$
 $x > -2$ (2)

(ii) $-7 \leq 5x+3 \leq 3$
 $-10 \leq 5x \leq 0$
 $-2 \leq x \leq 0$ (3)

(iii) $|3x-5| > 4$
 $3x-5 > 4$ or $-(3x-5) > 4$
 $3x > 9$ or $-3x+5 > 4$
 $x > 3$ or $-3x > -1$
 $x < \frac{1}{3}$ (4)

(iv) $2x+y = 9$ (1)
 $x+y = 4$ (2)
 $\therefore x = 4, y = 1$ (3)

(v) $2x+3y = 20$ (1)
 $3x+2y = 27$ (2)
 $6x+9y = 64$ (3)
 $6x+4y = 54$ (4)
 $5y = 30$
 $y = 6$ (3)

(vi) $x^2 + y^2 = 146$
 $x + 2y = 27$
 $x = 27 - 2y$
 $\therefore (27-2y)^2 + y^2 = 146$
 $441 - 84y + 4y^2 + y^2 = 146$
 $5y^2 - 84y + 295 = 0$
 $(5y - 59)(y - 5) = 0$
 $y = \frac{59}{5}$ or $y = 5$ (3)

$\therefore x = \frac{13}{5}, y = 5$ or $x = 11, y = 5$