GOSFORD HIGH SCHOOL

Year 11

2010

Preliminary HSC

MATHEMATICS

Assessment Task 1

Time Allowed: 65 minutes

Instructions:

- Remember to start each new question on a new page.
- Students must answer questions using a blue/black pen and/or a sharpened B or HB pencil.
- Approved scientific calculators may be used.
- Students need to be aware that:
  - All necessary work must be shown in order to gain full marks.
  - Untidy and/or poorly organised solutions may not gain full marks.

QUESTION 1: (14 Marks)

Marks

1. Find the value of

   a. $\dfrac{4.5\times10^{6}}{9.8\times10^{-5}}$ *To 3 significant figures.* 1

   b. $\dfrac{5\left(\left(\frac{2}{3}\right)^{5}-1\right)}{\sqrt[5]{5}}$ *Answer in scientific notation correct to 2 significant figures.* 2

2. An antique vase is bought and sold for \$1120 making a profit of 27 ½%. What was the original cost price? 2

3. Express $2.1\dot{3}\dot{6}$ as a mixed fraction. 2

4. Convert 300 metres in 4 seconds to km/hr. 1

5. Express each of the following in simplest surd form.

   i. $\sqrt{675}$ ii. $(2\sqrt{2}-\sqrt{6})(2\sqrt{3}-1)$ 3

6. Express $\dfrac{\sqrt{3}}{2\sqrt{3}+1}$ with a rational denominator 2

7. Evaluate exactly. (Express without absolute value symbols.)

   i. $|\sqrt{3}-\sqrt{2}|$ ii. $|2\sqrt{3}-\sqrt{13}|$ 1

QUESTION 2: (19 Marks)

Marks

1. Expand and simplify 2

$$(3m-1)(2m^2+m+3)$$

2. Simplify 2

$$\frac{y}{x^2+xy}-\frac{1}{x}$$

3. Show that $x = 2\sqrt{2}-3$ is a solution to the equation $x^2 + 6x + 1$ 2

4. Write the following in index notation and without a denominator 2

i. $\frac{1}{3x+7}$ ii. $\frac{4}{(\sqrt{x-2})^3}$

5. Fully factorise

i. $(m+n)^2 + 5(m+n)$ 2

ii. $x^3 + 5x^2 - 9x - 45$ 2

iii. $x^3 - 8$ 2

iv. $26x^2 + 13x - 13$ 2

v. $3a^4 - 3$ 3

QUESTION 3: (14 Marks)

Marks

1. Solve exactly

   i. $3x - 5 = 7x + 8$ — 1

   ii. $\sqrt{5x - 2} = 3 - x$ — 2

   iii. $5 - 4x > -8$ — 2

   iv. $5 = \frac{x}{2} - \frac{4}{x}$ — 2

2. Solve simultaneously

   $y = \frac{1}{x}$ *and* $2x + 3y - 7 = 0$ — 3

3. Solve

   $9^{7x-2} = \left(\frac{1}{27}\right)^{-x}$ — 2

4. Solve

   $|4 - 3x| \leq 13$ — 2

QUESTION 4: (17 Marks)

Marks

1. i. Sketch the parabola
$y = x^2 + 6x + 14$ *(Clearly labelling the coordinates of the vertex)* 2

ii. Hence find the minimum value of
$x^2 + 6x + 14$ 1

iii. Hence determine the number of solutions to
$x^2 + 6x + 14 = 0$ 1

2. Complete the square hence find the maximum value of

$$y = 6 - x - x^2$$ 3

3. Solve

i. $(x - 3)(x + 4) \geq 0$ 2

ii. $-5x^2 + 4x - 1 < 0$ 2

4. Find the value(s) of p such that

$$x^2 + (p + 1)x + 4 = 0$$ 3

has real and different roots.

5. Find the value(s) for p for which the equation

$$x^2 - 2px + 8p - 15 = 0$$ 3

has two equal roots (ie only one solution)

END OF TEST

Solutions Assessment 1. Preliminary

Question 1.

1 (a) 4.59 (b) -1.2×10^6

2) $C \times 1.275 = 1120$

$C = \$878.43$

3) $2 + .1\dot{3}\dot{6}$

$x = .1\dot{3}\dot{6}$

$100x = 13.6\dot{3}\dot{6}$

$x = \frac{13.5}{99} = \frac{3}{22}$

Ans $2\frac{3}{22}$

4) 300m in 4sec = 75m/s

$= \frac{75 \times 60 \times 60}{1000}$

$= 270$ km/hr

5) (i) $15\sqrt{3}$

(ii) $4\sqrt{6} - 2\sqrt{2} - 2\sqrt{18} + \sqrt{6}$

$4\sqrt{6} - 2\sqrt{2} - 6\sqrt{2} + \sqrt{6}$

$5\sqrt{6} - 8\sqrt{2}$

6) $\frac{\sqrt{3}}{2\sqrt{3}+1} \frac{(2\sqrt{3}-1)}{(2\sqrt{3}-1)} = \frac{6-\sqrt{3}}{11}$

7) (i) $\sqrt{3} - \sqrt{2}$ (ii) $\sqrt{83} - 2\sqrt{3}$

Question 2

1) $6m^3 + 3m^2 + 9m - 2m^2 - m - 3$

$6m^3 + m^2 + 8m - 3$

2) $\frac{y}{x^2+xy} - \frac{1}{x}$

$\frac{y}{x(x+y)} - \frac{(x+y)}{x(x+y)}$

$\frac{-x}{x(x+y)} = \frac{-1}{x+y}$

3) sub $x = 2\sqrt{2} - 3$ into $x^2 + 6x + 1 = 0$

$(2\sqrt{2}-3)^2 + 6(2\sqrt{2}-3) - 3 = 0$

$8 - 12\sqrt{2} + 9 + 12\sqrt{2} - 18 - 3 = 0$

$0 = 0$

4) (i) $(3x+1)^{-1}$ (ii) $4(x-2)^{-\frac{3}{2}}$

5) (i) $(m+n)(m+n+5)$

(ii) $x^3 + 5x^2 - 9x - 45$

$x^2(x+5) - 9(x+5)$

$(x^2-9)(x+5)$

$(x-3)(x+3)(x+5)$

(iii) $(x-2)(x^2+2x+4)$

(iv) $26x^2 + 13x - 13$

$13(2x^2 + x - 1)$

$13(2x-1)(x+1)$

5) (a) $3a^4 - 3$

$3(a^4-1)$

$3(a^2-1)(a^2+1)$

$3(a-1)(a+1)(a^2+1)$

Question 3

1) (i) $3x - 5 = 7x + 8$

$x = -\frac{13}{4}$

(ii) $\sqrt{5x-2} = 3 - x$

$\sqrt{5x+7} = 5$

$x(\sqrt{5}+1) = 5$

$x = \frac{5}{\sqrt{5}+1}$ or $\frac{5\sqrt{5}-5}{4}$

(iii) $5 - 4x > -8$

$-4x > -13$

$x < \frac{13}{4}$

(iv) $5 = \frac{x}{2} - \frac{4}{x}$

$10x = x^2 - 8$

$x^2 - 10x - 8 = 0$

$x = \frac{10 \pm \sqrt{100+32}}{2}$

$= \frac{2(5 \pm \sqrt{33})}{2}$

$= 5 \pm \sqrt{33}$

2) $y = \frac{1}{x}$ $2x + 3y - 7 = 0$

sub $y = \frac{1}{x}$ $2x + \frac{3}{x} - 7 = 0$

$2x^2 - 7x + 3 = 0$

$(2x-1)(x-3) = 0$

$x = \frac{1}{2}$ $y = 2$ or $x = 3$ $y = \frac{1}{3}$

$(\frac{1}{2}, 2)$ or $(3, \frac{1}{3})$

3) $9^{7x-2} = \left(\frac{1}{27}\right)^{-x}$

$(3^2)^{7x-2} = (3^{-3})^{-x}$

$14x - 4 = 3x$

$11x = 4$

$x = \frac{4}{11}$

4) $|4 - 3x| \leq 13$

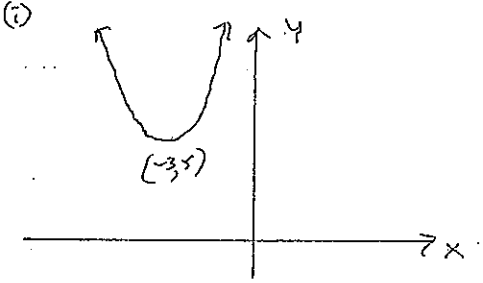
$4 - 3x \leq 13$ or $4 - 3x \geq -13$

$-3x \leq 9$ $-3x \geq -17$

$x \geq -3$ $x \leq \frac{17}{3}$

Question 4

1) (i)



$y = x^2 + 6x + 14$

Axis of symmetry $x = \frac{-b}{2a}$

$x = \frac{-6}{2}$

$x = -3 \Rightarrow y = 5$

(ii) Maximum Value 5

(iii) No solutions

Does not cut x axis

(2) $y = 6 - x - x^2$

$= -(x^2 + x - 6)$

$= -(x^2 + x + (\frac{1}{2})^2 - \frac{1}{4} - 6)$

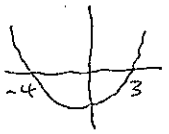
$= -[(x+\frac{1}{2})^2 - 6\frac{1}{4}]$

$= -(x+\frac{1}{2})^2 + 6\frac{1}{4}$

$\therefore$ max value $6\frac{1}{4}$ at $x = \frac{1}{2}$

(3)(i) $(x-3)(x+4) \geq 0$

consider $x = 3$ $x = -4$



From sketch

$x \geq 3$ or $x \leq -4$

OR Test $x = 0$

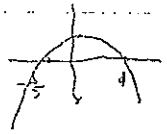
$-3 \times 4 \geq 0$ False

$\therefore x \leq -4$ or $x \geq 3$

(ii) $-5x^2 + 4x + 1 \leq 0$

$(5x+1)(-x+1)$

consider $x = -\frac{1}{5}$ or 1



From sketch

$x \leq -\frac{1}{5}$ $x \geq 1$

OR Test $x = 0$

$1 \leq 0$ False

$\therefore x \leq -\frac{1}{5}$ or $x \geq 1$

(4) $x^2 + (p+1)x + 4 = 0$

for real and different roots

$\Delta > 0$

ie $(p+1)^2 - 16 > 0$

$p^2 + 2p + 1 - 16 > 0$

$p^2 + 2p - 15 > 0$

$(p+5)(p-3) > 0$

$p < -5$ $p > 3$

(5) $x^2 - 2px + 8p - 15 = 0$

for two equal roots $\Delta = 0$

ie $(-2p)^2 - 4(8p - 15) = 0$

$4p^2 - 32p + 60 = 0$

$4(p^2 - 8p + 15) = 0$

$4(p-3)(p-5) = 0$

$p = 3$ or $p = 5$

Corrections

Question 2

~~part 3~~

part 3

show that $x = 2\sqrt{2} - 3$ is a solution to the equation $x^2 + 6x + 1 = 0$

Question 4

part 3(ii)

$-5x^2 + 4x + 1 < 0$