



Gosford High School

Year 11

2011

Preliminary Higher School Certificate

Mathematics

Assessment Task 1

Time Allowed – 70 minutes

Remember to start each new question on a new page

Students must answer questions using a blue/black pen and/or a sharpened B or HB pencil.

Approved scientific calculators may be used

Students need to be aware that

- * 'bald' answers may not gain full marks.
- * untidy and/or poorly organised solutions may not gain full marks.

QUESTION 1**BASIC ARITHMETIC****(27 marks)**

- a) Write 0.04045 correct to 2 significant figures. (1)
- b) The $\sqrt{543}$ lies between which two consecutive integers? (1)
- c) Write 12.4 billion in scientific notation. (1)
- d) What is the value of $\sqrt{(-4)^2}$? (1)
- e) A truck maintains an average speed of 65km/h. Calculate how far the truck will travel from 6am to 1.30pm on the same day. (1)
- f) Write $6\frac{1}{4}\%$ as simple fraction. (1)
- g) A bicycle is bought for \$840 and sold 3 years later for \$315. Express the loss as a percentage of the cost price. (1)
- h) Simplify $3\sqrt{27} - \sqrt{75}$ (2)
- i) Express ab in scientific notation given that $a = 1.4 \times 10^{-3}$ and $b = 0.8 \times 10^{-5}$ (1)
- j) Write $0.0\dot{5}\dot{3}$ as simple fraction. (1)
- k) Find the value of $\frac{\sqrt[5]{246-78}}{4\pi}$ correct to 3 decimal places. (2)
- l) Simplify $\frac{24 - \sqrt{48}}{4}$ (2)
- m) Express $\frac{2\sqrt{5} - \sqrt{2}}{2\sqrt{5} + \sqrt{2}}$ in simplest form and with a rational denominator (3)
- n) The hypotenuse of a right angled triangle is 17cm and the length of one of its other sides is 8cm. Find the length of the third side. (2)
- o) Factorise $2 + 2\sqrt{3} + \sqrt{5} + \sqrt{15}$ (2)
- p) If $8^{-\frac{3}{2}} = a\sqrt{2}$, find a (3)
- q) The length of the sides of a triangle are 7cm, 9cm and x cm. What is the range of possible values of x ? (2)

Start a New Page for Question 2

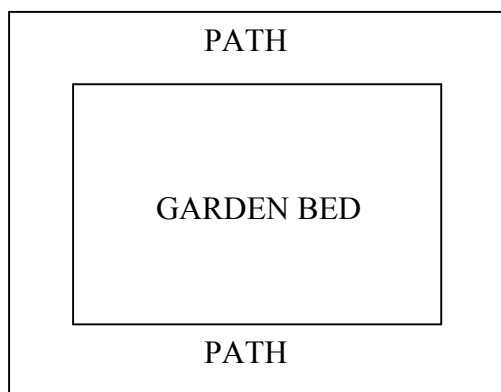
QUESTION 2**BASIC ALGEBRA****(28 marks)**

- a) Simplify (i) $(-2xm^7)^3$ (ii) $\frac{24x^{16}}{12x^8}$ (2)
- b) Expand and simplify
(i) $-44 - 4(4x - 7)$ (ii) $(3a - 5)^2$
(iii) $(2k + 3)(2k - 3) - (3k + 2)(k - 5)$ (4)
- c) If $a = 2$, $x = -3$ and $y = -4$, find the value of
(i) $ax + y$ (ii) $ay^2 - x^3$ (3)
- d) Factorise fully
(i) $1 - 16x^2$ (ii) $x^3 + 8$
(iii) $4x^2 - 13x - 12$ (iv) $m^2 - mn + 2n - 2m$ (4)
- e) Simplify $(\sqrt{1+x} - \sqrt{1-x})(\sqrt{1+x} + \sqrt{1-x})$ (2)
- f) Find all possible values of k if $9x^2 + kx + 16$ is a perfect square. (2)
- g) Simplify $\frac{a+1}{a(a-1)} - \frac{a-1}{a(a+1)}$ (3)
- h) Simplify $\frac{m^2 - 9}{m^2 - m - 12} \div \frac{m^2 - 3m}{4m}$ (3)
- i) Write $4m^{-1} + (4m)^{-1}$ as a single fraction without any negative indices (2)
- j) i) Factorise $b^2 - 2bc + c^2$ (1)
ii) Hence factorise $a^2 - b^2 + 2bc - c^2$ (2)

Start a New Page for Question 3

QUESTION 3**EQUATIONS****(30 marks)**

- a) Solve $2^x = 1$ (1)
- b) Solve $3 + a = \frac{3a - 1}{2}$ (2)
- c) Solve $|x + 3| > 1$ (2)
- d) Solve the inequality $-5 < x + 4 \leq 3$ and graph your solution on a number line (2)
- e) Find m if $x = -4$ is a solution of the equation $mx^2 + 5x - 6m = 0$ (2)
- f) Solve $7x = 2x^2$ (2)
- g) Solve $2x^2 - 5x - 1 = 0$ (2)
- h) Solve the equations $4y = x^2 - 4$ and $2y - x = 10$ (3)
- i) Solve $2x - 5 = |x + 2|$ (3)
- j) Solve $25^{m+1} = \frac{1}{5}$ (2)
- k) Make x the subject of the formula $\sqrt{x^2 + y^2} = a$ (3)
- l) Use completion of the square method to solve $a^2 - 2a\sqrt{2} = 2$ (2)
- m) A garden bed, with dimensions 20m by 24m, is surrounded by a path of uniform width x m. The total area enclosed by the path and the garden bed is 516 sq. m. Find the width of the path correct to the nearest centimetre. (4)



End of Examination

QUESTION 1

Basic Arithmetic

a) 0.040

b) 23 and 24

c) $12.4 \times 10^9 = 1.24 \times 10^{10}$

d) 4

$$\begin{aligned} e) \quad s &= \frac{d}{t} \quad \rightarrow \quad d = st \\ &= 65 \times 7.5 \\ &= 487.5 \text{ km} \end{aligned}$$

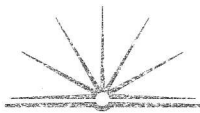
$$\begin{aligned} f) \quad 6\frac{1}{4}\% &= \frac{6\frac{1}{4}}{100} \\ &= \frac{1}{16} \end{aligned}$$

$$\begin{aligned} g) \quad \% \text{ loss} &= \frac{525}{840} \times 100\% \\ &= 62.5\% \end{aligned}$$

$$\begin{aligned} h) \quad 3\sqrt{27} - \sqrt{75} &= 9\sqrt{3} - 5\sqrt{3} \\ &= 4\sqrt{3} \end{aligned}$$

$$\begin{aligned} i) \quad ab &= 1.4 \times 10^{-3} \times 0.8 \times 10^{-5} \\ &= 1.12 \times 10^{-8} \end{aligned}$$

$$\begin{aligned} j) \quad \nearrow \therefore 100x &= 5.3\overline{53} \\ \text{Let } x &= 0.0\overline{53} \\ \therefore 99x &= 5.3 \\ x &= \frac{5.3}{99} \\ x &= \frac{53}{990} \end{aligned}$$



$$k) 0.221744058 = 0.222 \quad (\text{to } 3 \text{ d.p.})$$

$$l) \frac{24 - 4\sqrt{3}}{4} = \frac{4(6 - \sqrt{3})}{4}$$
$$= 6 - \sqrt{3}$$

$$m) \frac{(2\sqrt{5} - \sqrt{2})}{(2\sqrt{5} + \sqrt{2})} \times \frac{(2\sqrt{5} - \sqrt{2})}{(2\sqrt{5} - \sqrt{2})} = \frac{20 + 2 - 4\sqrt{10}}{18}$$
$$= \frac{22 - 4\sqrt{10}}{18}$$
$$= \frac{11 - 2\sqrt{10}}{9}$$

n) let x be the unknown side

$$\therefore x^2 = 17^2 - 8^2$$

$$x^2 = 225$$

$$x = 15$$

$$o) 2 + 2\sqrt{3} + \sqrt{5} + \sqrt{15} = 2(1 + \sqrt{3}) + \sqrt{5}(1 + \sqrt{3})$$
$$= (1 + \sqrt{3})(2 + \sqrt{5})$$

$$p) 8^{-\frac{3}{2}} = a\sqrt{2}$$

$$\frac{1}{(\sqrt{8})^3} = a\sqrt{2}$$

$$\frac{1}{(2\sqrt{2})^3} = a\sqrt{2}$$

$$\frac{1}{16\sqrt{2}} = a\sqrt{2}$$

$$\therefore \frac{\sqrt{2}}{32} = a\sqrt{2}$$

$$\frac{1}{32} \sqrt{2} = a\sqrt{2}$$

$$\therefore a = \frac{1}{32}$$

$$q) 2 < x < 16$$

Since sum of any two sides must be greater than the 3rd side.

Question 2 Basic Algebra

a) (i) $-8x^3m^{21}$ (ii) $2x^8$

b) (i) $-44 - 16x + 28 = -16 - 16x$

(ii) $9a^2 - 30a + 25$

(iii) $4k^2 - 9 - (3k^2 - 13k - 10)$

$$= 4k^2 - 9 - 3k^2 + 13k + 10$$

$$= k^2 + 13k + 1$$

c) (i) $ax + y = 2(-3) + (-4)$ (ii) $ay^2 - x^3 = 2(-4)^2 - (-3)^3$

$$= -6 - 4$$

$$= -10$$

$$= 32 + 27$$

$$= 59$$

d) (i) $1 - 16x^2 = (1 - 4x)(1 + 4x)$

(ii) $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$

(iii) $4x^2 - 13x - 12 = (4x + 3)(x - 4)$

(iv) $m^2 - mn + 2n - 2m = m(m - n) - 2(m - n)$

$$= (m - n)(m - 2)$$

e) Expression = $(1 + x) - (1 - x)$

$$= 1 + x - 1 + x$$

$$= 2x$$

f) $k = \pm 24$

g) Expression = $\frac{(a+1)^2 - (a-1)^2}{a(a-1)(a+1)}$

$$= \frac{(a+1+a-1)(a+1-a+1)}{a(a-1)(a+1)}$$

$$= \frac{2a \times 2}{a(a-1)(a+1)}$$



g) (continued) Expression = $\frac{4}{(a-1)(a+1)}$

h) Expression = $\frac{(m-3)(m+3)}{(m-4)(m+3)} \times \frac{4m}{m(m-3)}$
= $\frac{4}{m-4}$

i) $4m^{-1} + (4m)^{-1} = \frac{4}{m} + \frac{1}{4m}$
= $\frac{16+1}{4m}$
= $\frac{17}{4m}$

j) (i) $b^2 - 2bc + c^2 = (b-c)^2$
(ii) $a^2 - b^2 + 2bc - c^2 = a^2 - (b-c)^2$
= $(a-b+c)(a+b-c)$

Question 3 Equations:

a) $2^x = 1$

$\therefore x = 0$

b) $6 + 2a = 3a - 1$

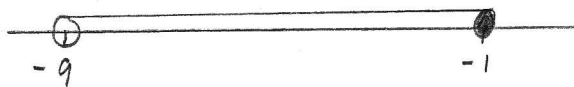
$7 = a$

c) $x + 3 < -1$ or $x + 3 > 1$

$x < -4$

$x > -2$

d) $-9 < x \leq -1$



e) $m(-4)^2 + 5(-4) - 6m = 0$

$16m - 20 - 6m = 0$

$10m = 20$

$m = 2$

f) $7x - 2x^2 = 0$

$x(7 - 2x) = 0$

$x = 0, \frac{7}{2}$

g) $x = \frac{5 \pm \sqrt{25 - 4(2)(-1)}}{4}$

$x = \frac{5 \pm \sqrt{33}}{4}$

h) $4y = x^2 - 4$ and $4y = 2x + 20$

$\therefore x^2 - 4 = 2x + 20$

$x^2 - 2x - 24 = 0$

$(x - 6)(x + 4) = 0$



h) (continued) $x = 6, -4$

when $x = 6, y = 8$

when $x = -4, y = 3$

i) $2x - 5 = x + 2$ and $-2x + 5 = x + 2$

$$x = 7$$

$$3 = 3x$$

Which satisfies

$$1 = x$$

Which does not satisfy.

$\therefore x = 7$ only

j) $(5^2)^{m+1} = 5^{-1}$

$$5^{2m+2} = 5^{-1}$$

$$2m+2 = -1$$

$$2m = -3$$

$$m = -\frac{3}{2}$$

k) $x^2 + y^2 = a^2$

$$x^2 = a^2 - y^2$$

$$x = \pm \sqrt{a^2 - y^2}$$

l) $a^2 - 2a\sqrt{2} + (\sqrt{2})^2 = 2 + (\sqrt{2})^2$

$$(a - \sqrt{2})^2 = 4$$

$$a - \sqrt{2} = \pm 2$$

$$a = \sqrt{2} \pm 2$$

m) let x be the width of the path.

$$\therefore (2x + 20)(2x + 24) = 516$$

$$(x + 10)(x + 12) = 129$$

$$x^2 + 22x + 120 = 129$$

$$x^2 + 22x - 9 = 0$$

$$x = \frac{-22 \pm \sqrt{22^2 - 4(1)(-9)}}{2}$$

$$= \frac{-22 \pm \sqrt{520}}{2}$$

$$= \frac{-22 \pm 2\sqrt{130}}{2}$$

$$= (-11 + \sqrt{130}) \text{ m since } x > 0$$

$$\doteq 0.40175 \dots \text{ m}$$

$$\doteq 40 \text{ cm}$$