HORNSBY GIRLS HIGH SCHOOL

YEAR 11 MATHEMATICS

2009

Preliminary Assessment 1

Time Allowed: 60 minutes

Instructions:

- Attempt all questions
- Start a new page for each question
- The marks for each question are indicated
- Show all necessary working
- Use blue or black pen.
- Marks may be deducted for untidy or badly arranged work
- Board approved calculators may be used

Outcomes Assessed:

- Demonstrates confidence in using mathematics to obtain realistic solutions to problems (P1)
- Provides reasoning to support conclusions which are appropriate to the context (P2)
- Performs routine arithmetic and algebraic manipulation involving surd and simple rational expressions (P3)
- Chooses and applies appropriate arithmetic and algebraic techniques (P4)
- Understands the concept of a function and the relationship between a function and its graph (P5)

Outcomes	Questions			Totals
Basic Algebra and Arithmetic	Q1:	Q2:	Q4:	/34
Real Functions of a Real Variable	Q3:	Q5:		/21
			Total	/55

This assessment task constitutes 20% of the final Preliminary Course Assessment

Question 1: (12 marks) **START A NEW PAGE** **Marks**

(a) Evaluate, correct to 2 significant figures

$$\left(\sqrt{5}-1\right)^2 \times \sqrt{\frac{1}{3\cdot 2}+\frac{3}{7}}.$$ **2**

(b) Express $0\cdot\dot{4}3\dot{8}$ as a common fraction. **2**

(c) Factorise $250x^3+16y^6$. **2**

(d) Show that $\frac{1}{3+\sqrt{3}}+\frac{1}{3-\sqrt{3}}$ is a rational number. **2**

(e) Simplify $\frac{-4a^3}{3b^7}\times\frac{a^2}{8b^2}\times\frac{-6b^4}{a^8}$. **2**

(f) Solve $|2x-3|=1$. **2**

Question 2: (11 marks) **START A NEW PAGE**

(a) Solve for x: $5x-3<3x+3$ **1**

(b) Solve for x: $\frac{x-5}{3}-\frac{x+1}{4}=5$. **3**

(c) (i) Make y the subject of $\frac{y-x}{z}=\frac{y+z}{w}$. **2**

(ii) Hence or otherwise, find the value of y when $z=2$, $w=3$ and $x=1$. **2**

(d) Solve simultaneously : **2**

$3x-4y=-3$

$3x+y=12$

(e) Find the value of a if $x^2+5x=(x+a)^2-\frac{25}{4}$. **1**

Question 3: (10 marks) **START A NEW PAGE** **Marks**

(a) For the function $f(x)=x\sqrt{x}$

(i) State the domain of $f(x)$. **1**

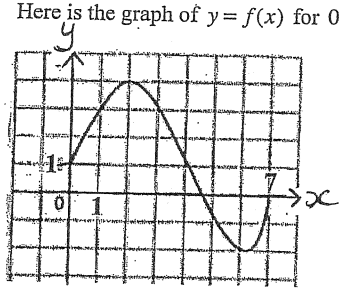
(ii) Find $f(4)$. **1**

(iii) Find an expression for $f(a)$ and $f(ab)$. **2**

(iv) Hence, show that $f(a)f(b)-f(ab)=0$. **2**

(b) If $f(x)=2x-x^2$, find $f(x+1)-f(1)$ in its simplest form. **2**

(c) Here is the graph of $y=f(x)$ for $0\le x\le 7$.



(i) Evaluate $f(2)$. **1**

(ii) Find the range of $y=f(x)$. **1**

Question 4: (11 marks) **START A NEW PAGE** **Marks**

(a) Simplify $\dfrac{c^2-9}{(c-3)^2}+\dfrac{9c+9}{(c-3)(c+1)}$. **2**

(b) (i) Show that $(x-1)^3-(x+1)^3=-2(3x^2+1)$. **3**

(ii) Hence solve $(x-1)^3-(x+1)^3=-56$. **2**

(c) Solve $|x+3|=2x-1$. **2**

(d) Solve for x: $x^2-a^2\ge 0$ for $a\ge 0$. **2**

Question 5: (11 marks) **START A NEW PAGE**

(a) Show that the function $f(x)=\dfrac{-x}{x^2-4}$ is an odd function. **2**

(b) Find the domain and range of $y=\dfrac{1}{x-3}$. **2**

(c) Consider the functions $h(x)=\sqrt{1-x}$ and $g(x)=\sqrt{1+x}$.

(i) Find the domain of $h(x)$ and $g(x)$ **2**

(ii) Hence, or otherwise, find the domain of $f(x)=\sqrt{1-x}+\sqrt{1+x}$. **1**

(iii) Determine whether $y=f(x)$ is even, odd or neither. **2**

(d) Consider the function defined by $f(x)=\begin{cases} x+2 & x<-1 \\ x^2 & |x|\le 1 \\ -x+2 & x>1 \end{cases}$.

Find the value of $f(-3)+f(3)-f\left(\dfrac{1}{2}\right)$. **2**

END OF PAPER

Year 11 Mathematics Preliminary Assessment 1 2009 - Solutions

Question 1

(a)

$$(\sqrt{5}-1)^2 \times \sqrt{\frac{1}{3.2}+\frac{3}{7}} = 1.315269...$$
$$= 1.3 (2sf)$$

(b)

$Let\ x = 0.\dot{4}3\dot{8}$

$x = 0.438438438...$

$1000x = 438.438438...$

$999x = 438$

$$x = \frac{438}{999}$$

$$x = \frac{146}{333}$$

(c)

$$250x^3 + 16y^6 = 2(125x^3 + 8y^6)$$
$$= 2\left[(5x)^3 + \left(2y^2\right)^3\right]$$
$$= 2(5x + 2y^2)(25x^2 - 10xy^2 + 4y^4)$$

(d)

$$\frac{1}{3+\sqrt{3}} + \frac{1}{3-\sqrt{3}} = \frac{3-\sqrt{3}+3+\sqrt{3}}{(3+\sqrt{3})(3-\sqrt{3})}$$
$$= \frac{6}{3^2-3}$$
$$= \frac{6}{6}$$
$$= 1$$

(e)

$$\frac{-4a^3}{3b^7} \times \frac{a^2}{8b^2} \times \frac{-6b^4}{a^8} = \frac{24a^5b^4}{24a^8b^9}$$
$$= a^{-3}b^{-5}$$
$$= \frac{1}{a^3b^5}$$

(f)

$|2x-3| = 1$

$2x - 3 = 1\ or\ 2x - 3 = -1$

$2x = 4\ or\ 2x = 2$

$x = 2\ or\ x = 1$

Question 2

(a)

$5x - 3 < 3x + 3$

$2x < 6$

$x < 3$

(b)

$$\frac{x-5}{3} - \frac{x+1}{4} = 5$$
$$\frac{4(x-5)-3(x+1)}{12} = 5$$
$4x - 20 - 3x - 3 = 60$

$x - 23 = 60$

$x = 83$

(c) (i)

$$\frac{y-x}{z}=\frac{y+z}{w}$$

$$w(y-x)=z(y+z)$$

$$wy-wx=zy+z^2$$

$$wy-zy=wx+z^2$$

$$y(w-z)=wx+z^2$$

$$y=\frac{wx+z^2}{w-z}$$

(c) (ii)

$$y=\frac{wx+z^2}{w-z}$$

$$y=\frac{3(1)+2^2}{3-2}$$

$$y=3+4$$

$$y=7$$

(d)

$$3x-4y=-3 \ldots.(1)$$

$$3x+y=12 \ldots.(2)$$

$$(2)-(1):$$

$$y+4y=12+3$$

$$5y=15$$

$$y=3$$

Sub into (1)

$$3x-4(3)=-3$$

$$3x-12=-3$$

$$3x=9$$

$$x=3$$

$$\therefore x=3, y=3$$

(e)

$$x^2+5x=\left(x+\frac{5}{2}\right)^2-\frac{25}{4}$$

$$a=\frac{5}{2}$$

Question 3

(a) (i)

$$domain: x\geq 0$$

(a) (ii)

$$f(4)=4\sqrt{4}$$

$$=8$$

(a) (iii)

$$f(a)=a\sqrt{a}$$

$$f(ab)=ab\sqrt{ab}$$

(a) (iv)

$$f(a)f(b)-f(ab)=a\sqrt{a}\times b\sqrt{b}-ab\sqrt{ab}$$

$$=ab\sqrt{ab}-ab\sqrt{ab}$$

$$=0$$

(b)

$$f(x+1)-f(1)=\left[2(x+1)-(x+1)^2\right]-\left[2(1)-(1)^2\right]$$

$$=2x+2-x^2-2x-1-2+1$$

$$=-x^2$$

(c) (i)

$$f(2)=4$$

(c) (ii)

Range : $-2 \le y \le 4$

Question 4

(a)

$$\frac{c^2-9}{(c-3)^2}+\frac{9c+9}{(c-3)(c+1)}=\frac{(c-3)(c+3)}{(c-3)^2}+\frac{9(c+1)}{(c-3)(c+1)}$$

$$=\frac{c+3}{c-3}+\frac{9}{c-3}$$

$$=\frac{c+12}{c-3}$$

(b) (i)

$(x-1)^3-(x+1)^3$

$=x^3-3x^2+3x-1-(x^3+3x^2+3x+1)$

$=-6x^2-2$

$=-2(3x^2+1)$

ALTERNATIVE METHOD

$(x-1)3-(x+1)^3=[(x-1)-(x+1)][(x-1)^2+(x-1)(x+1)+(x+1)^2]$

$=(-2)(x^2-2x+1+(x^2-1)+x^2+2x+1]$

$=(-2)(x^2+x^2+x^2-2x+2x+1-1+1)$

$=-2(3x^2+1)$

(b) (ii)

$(x-1)^3-(x+1)^3=-56$

$-2(3x^2+1)=-56$

$3x^2+1=28$

$3x^2=27$

$x^2=9$

$x=3 \text{ or } -3$

(c)

$|x+3|=2x-1$

$x+3=2x-1$

$x=4$

Test $x=4$

LHS $=7$

RHS $=1$

$\therefore x=4$

$x+3=-2x+1$

$3x=-2$

$x=\frac{-2}{3}$

Test $x=\frac{-2}{3}$

$LHS=\frac{4}{3}, \quad RHS=\frac{-1}{3}$

$\therefore x=\frac{-2}{3}$ *is NOT a solution*

(d)

$x^2-a^2 \ge 0$

$(x-a)(x+a) \ge 0$

Method 1 :

*Case*1 : $(x-a) \ge 0 \cap (x+a) \ge 0$ [*both positive or zero*]

$x \ge a \cap x \ge -a$

$\therefore x \ge a$

Case 2 : $(x-a) \le 0 \cap (x+a) \le 0$ [*both negative or zero*]

$x \le a \cap x \le -a$

$\therefore x \le -a$

Solution is $x \le -a$ *or* $x \ge a$

Method 2 :

From graph, $x \ge a$ *or* $x \le -a$

Question 5

(a)

$$f(x)=\frac{-x}{x^2-4}$$

$$f(-x)=\frac{-(-x)}{(-x)^2-4}$$

$$=\frac{x}{x^2-4}$$

$$=-f(x)$$

$\therefore f(-x)=-f(x)$

$\therefore f(x)$ *is odd function*

(b)

Domain : $x\neq -3$

Range : $y\neq 0$

(c) (i)

$h(x)=\sqrt{1-x}$

Domain : $x\leq 1$

$g(x)=\sqrt{x+1}$

Domain : $x\geq -1$

(c) (ii)

$f(x)=\sqrt{1-x}+\sqrt{1+x}$

Domain :

$x\geq -1$ *AND* $x\leq 1$

$\therefore -1\leq x\leq 1$

(c) (iii)

Let $y=f(x)$

$f(x)=\sqrt{1-x}+\sqrt{1+x}$

$f(-x)=\sqrt{1--x}+\sqrt{1-x}$

$=\sqrt{1+x}+\sqrt{1-x}$

$=\sqrt{1-x}+\sqrt{1+x}$

$=f(x)$

$\therefore y=f(x)$ *is an even function*

(d)

$f(-3)=-3+2$

$=-1$

$f(3)=-1$

$f\left(\frac{1}{2}\right)=\frac{1}{4}$

$\therefore f(-3)+f(3)-f\left(\frac{1}{2}\right)=-2\frac{1}{4}$