BAULKHAM HILLS HIGH SCHOOL

2019 | Preliminary Assessment Task 1

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black pen
- NESA approved calculators may be used
- For Questions 11 – 14, show relevant mathematical reasoning and/or calculations.

Total marks: 58

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 48 marks (pages 6 – 10)

- Attempt Questions 11 – 14
- Allow about 1 hour and 15 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer page in the writing booklet for Questions *1 – 10.*

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1 A student committee consists of 10 general members and a president. The general members are selected from 15 students in Year 11. The president is selected from 4 students in Year 12. In how many ways could the committee be selected?

(A) ${}^{15}C_{10} + {}^{4}C_{1}$ (B) ${}^{15}P_{10} + {}^{4}P_{1}$ (C) ${}^{15}C_{10} \times {}^{4}C_{1}$ (D) ${}^{15}P_{10} \times {}^{4}P_{1}$

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2 Peter and Sam are among eight people who arrange themselves at random in a straight line. What is the probability that Peter and Sam are next to each other?

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{7}$ (D) $\frac{1}{8}$

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3 There are K people in a room and each person picks a day of the year to get a free dinner at a fancy restaurant. If the year is a leap year (366 days), what is the smallest value of K such that it is guaranteed that there will be at least one group of six people who select the same day?

(A) 1830 (B) 1831 (C) 2196 (D) 2197

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4 How many numbers greater than 5000 can be formed with the digits 4, 5, 6, 7 and 8 if no digit is used more than once in a number?

(A) 96 (B) 120 (C) 196 (D) 216

5 At a dinner party, the host, hostess and their six guests sit around a circular table. In how many ways can they be arranged if the host and hostess wish to be separated?

(A) 720 (B) 1440 (C) 3600 (D) 5040

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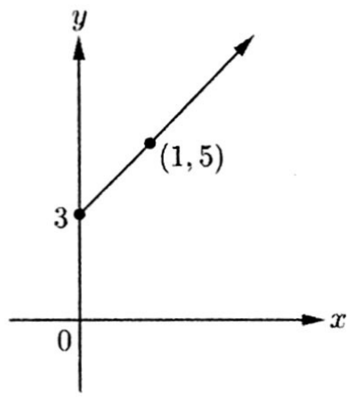
6 Consider the following ray.



Which of the following pairs of parametric equations best describes the ray above?

(A) $x=\frac{t^2}{2}, y=t^2+3$ (B) $x=2t^2, y=t^2+3$

(C) $x=\frac{t}{2}, y=t+3$ (D) $x=2t, y=t+3$

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7 Given that $f(x)$ is an odd function and $g(x)$ is an even function. Which of the following statements is **incorrect**?

(A) $f(x)g(x)$ is an odd function. (B) $f(x)+g(x)$ is an odd function.

(C) $\frac{1}{f(x)}$ is an odd function. (D) $|g(x)|$ is an even function.

8 In how many ways can a group of $2n$ people be split into two groups of n people?

(A) ${}^{2n}C_n$ (B) $\dfrac{{}^{2n}C_n}{2}$ (C) ${}^{2n}P_n$ (D) $\dfrac{{}^{2n}P_n}{2}$

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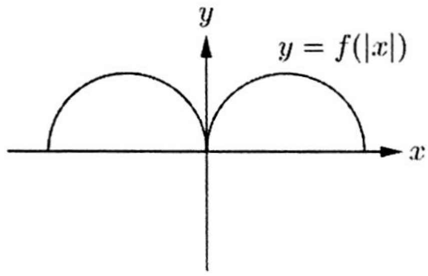
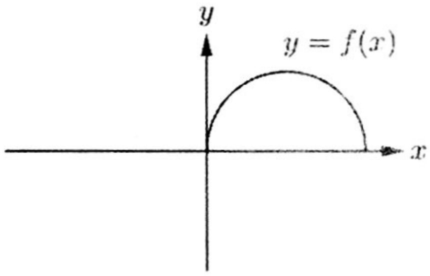
9 The curve below is the graph of $y = f\left(|x|\right)$.

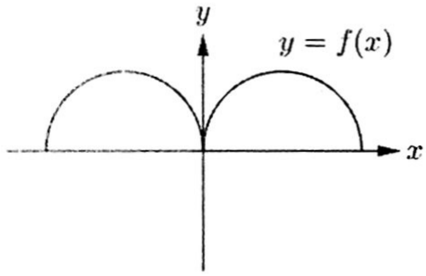


Which of the following graphs **could not** have been the graph of $y = f(x)$?

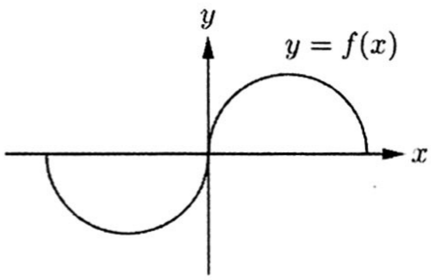
(A)



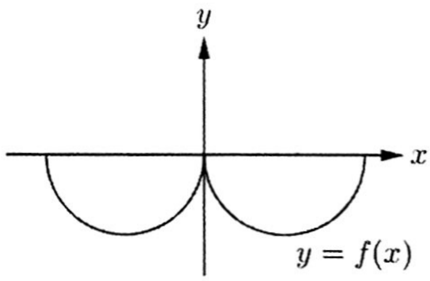
(B)



(C)

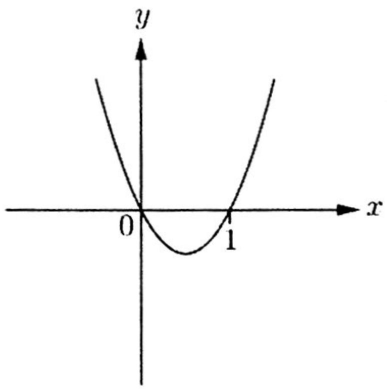


(D)

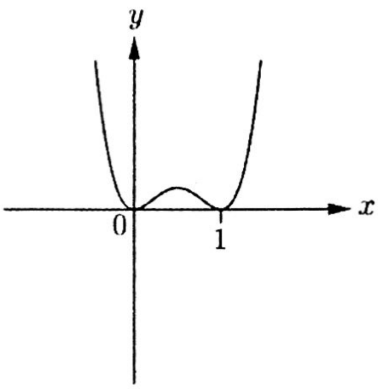


10 Which graph best represents the curve $y^2 = x^2 - x$?

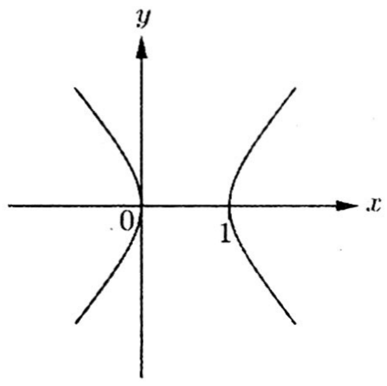
(A)



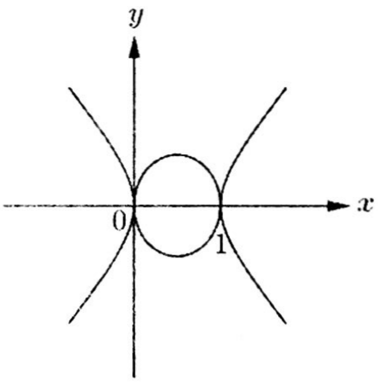
(B)



(C)



(D)



End of Section I

Section II

48 marks
Attempt Questions 11 – 14
Allow about 1 hour and 15 minutes for this section

Answer each question in the appropriate section of the writing booklet. Extra writing paper is available.

In Questions 11 – 14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (12 marks) Use the Question 11 section of the writing booklet.

(a) Solve $|1-3x|<7$ and graph the solution on a number line. **3**

(b) Solve $\dfrac{4}{x+1}<5$. **2**

(c) A function is given by $f(x)=x^2+6x+2$.

(i) Explain why the domain of the function $f(x)$ must be restricted if $f(x)$ is to have an inverse function. **1**

(ii) Give the equation for $f^{-1}(x)$ if the domain of $f(x)$ is restricted to $x \geq -3$. **2**

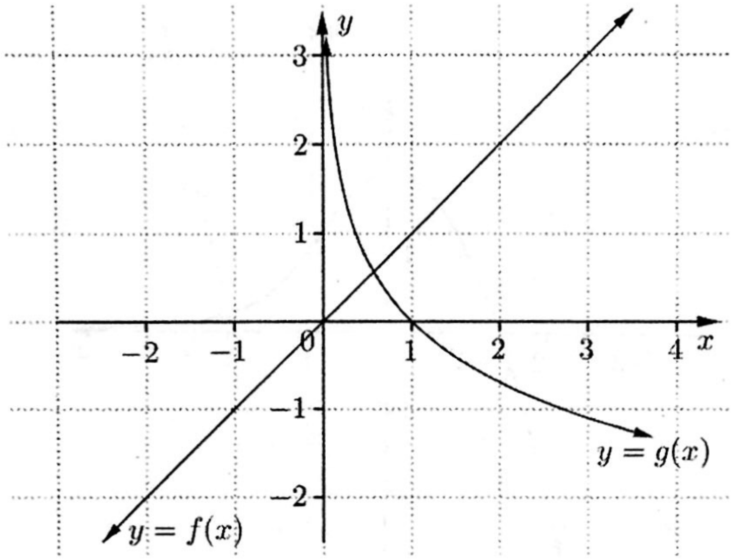
(iii) State the domain and range of $f^{-1}(x)$, given the restriction in part (ii). **2**

(iv) Sketch the curve $y=f^{-1}(x)$, showing the intercepts with the coordinate axes. **2**

End of Question 11

Question 12 (12 marks) Use the Question 12 section of the writing booklet.

(a) The diagram below shows the graphs of $y = f(x)$ and $y = g(x)$.



Draw the graph of $y = f(x) + g(x)$ on the number plane given in your answer booklet. **2**

(b) Solve $\dfrac{x^2 - 4}{x} \le 0$. **3**

(c) The eight letters of the word **ADDITION** are written on separate cards.

(i) How many different ways can the cards be arranged in a line if there are no restrictions? **2**

(ii) Five cards are chosen and placed in a line to form a 5-letter word, how many different 5-letter words are possible? **3**

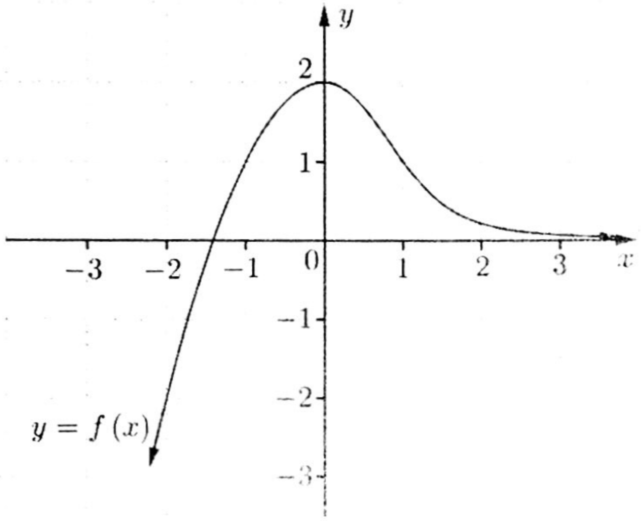
(iii) A word is palindromic if it reads the same backward as forward. For example, the words **BOB**, **PEEP** and **KAYAK** are all palindromic. **2**

If one of the 5-letter words from part (ii) is chosen at random, what is the probability that the chosen word is palindromic?

End of Question 12

Question 13 (12 marks) Use the Question 13 section of the writing booklet.

(a) The diagram below shows the graph of $y = f(x)$. The graph has a horizontal asymptote at $y = 0$.



On separate number planes given in your answer booklet, draw:

(i) $y = \dfrac{1}{f(x)}$ **2**

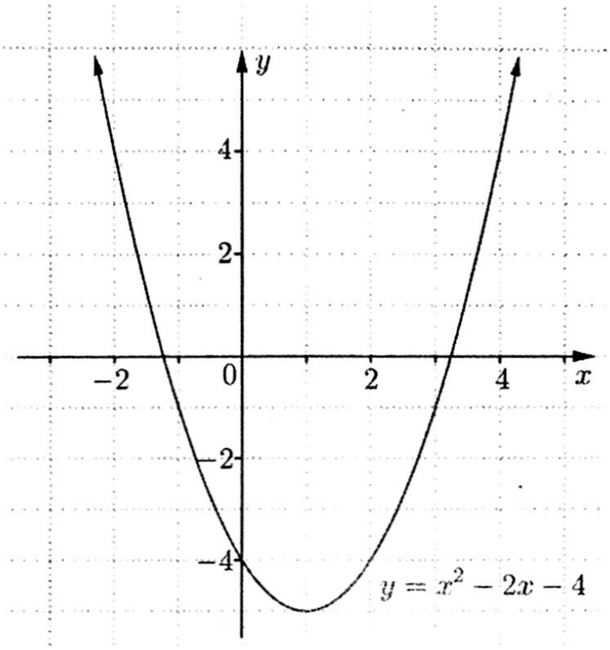
(ii) $y = \sqrt{f(x)}$ **2**

(iii) $|y| = f(x)$ **2**

Question 13 continues on page 9

Question 13 (continued)

(b) Below is the graph of $y = x^2 - 2x - 4$.



(i) On the number plane given in your answer booklet, accurately sketch the graph of $y = \left| x^2 - 2x - 4 \right|$. **2**

(ii) Hence, or otherwise, solve $\left| x^2 - 2x - 4 \right| \leq 4$. **2**

(c) In the movie "Cheaper by the Dozen", there are 12 children in the family.

(i) Show that at least two of the children were born on the same day of the week. **1**

(ii) Can we guarantee that two of the children will be born in the same month? Justify your answer. **1**

End of Question 13

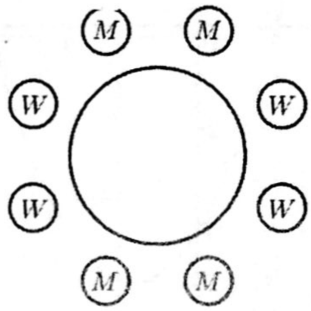
Question 14 (12 marks) Use the Question 14 section of the writing booklet.

(a) Four men and four women are to be seated around a circular table. In how many ways can this be done if:

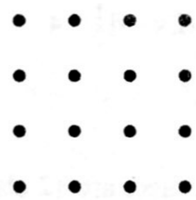
(i) there are no restrictions? **1**

(ii) all of the women want to sit together? **2**

(iii) the men (M) want to sit in separate pairs and the women (W) also want to sit in separate pairs, as shown in the diagram below? **2**



(b) Consider the 4×4 grid of dots below.



(i) How many ways can three dots be chosen such that they are collinear? **2**

(ii) Hence, or otherwise, find the number of triangles that can be formed such that each vertex of the triangle is a dot on the grid. **2**

(c) Find all possible value(s) of n such that ${}^{n+1}C_3 + 2 \times {}^{n-1}C_2 = 6 \times {}^{n}C_2$. **3**

End of Paper

BAULKHAM HILLS HS PERSEVERE

PRELIMINARY ASSESSMENT TASK 1 2019
MATHEMATICS EXTENSION 1
MARKING GUIDELINES

Section I

Multiple-choice Answer Key

Question	Answer
1	C
2	B
3	B
4	D
5	C

Question	Answer
6	A
7	B
8	B
9	D
10	C

Questions 1 – 10

Sample solution

1. Order is not important, and using the basic counting principle: ${}^{15}C_{10} \times {}^{4}C_{1}$

2. If there are no restrictions, then there are $8!$ ways of arranging the 8 people.
If Peter and Sam are to be next to each other, there are 2 ways of ordering Peter and Sam. Now treating them as one group, arrange this group and 6 others in a line. This can be done in $7!$ ways.

$$P(\text{Peter next to Sam}) = \frac{2\times 7!}{8!} = \frac{2}{8} = \frac{1}{4}$$

3. To guarantee that there will be at least one group of six people who select the same day in a 366-day year, pigeonhole principle says that a minimum of $5\times 366+1=1831$ people is needed.

4. The number of 4-digit numbers greater than 5000: $4\times {}^{4}P_{3}=96$ (4 choices for the first digit, ${}^{4}P_{3}$ choices for the rest)
The number of 5-digit numbers: $5!=120$
Total $=216$

5. Without restrictions, there are $7!=5040$ ways of arranging 8 people around a circular table.
If the host and hostess are to sit together, there are 2 ways of arranging them. Now treat them as one group, arrange this group and 6 others in a circle. This can be done in $2\times 6!=1440$ ways.
If the host and hostess wish to be separated, there are $5040-1440=3600$ ways of achieving this.

6. From the diagram, the ray is the portion of the line $y=2x+3$, where $x\ge 0$. Option A will give the correct equation and limit the x-values to non-negative values only.

$$x=\frac{t^2}{2}\Rightarrow t^2=2x$$
$$y=t^2+3$$
$$y=2x+3$$

Questions 1 – 10 (continued)

Sample solution

7. Let $h(x)=\dfrac{1}{f(x)}$.

$$h(-x)=\frac{1}{f(-x)} = \frac{1}{-f(x)} = -h(x)$$

$\therefore \dfrac{1}{f(x)}$ is an odd function.

Let $h(x)=|g(x)|$.

$$h(-x)=|g(-x)| = |g(x)| = h(x)$$

$\therefore |g(x)|$ is an even function.

Let $h(x)=f(x)+g(x)$.

$$h(-x)=f(-x)+g(-x) = -f(x)+g(x) \ne h(x) \text{ or } -h(x)$$

$\therefore f(x)+g(x)$ is neither an odd nor an even function.

Let $h(x)=f(x)g(x)$.

$$h(-x)=f(-x)g(-x) = -f(x)g(x) = -h(x)$$

$\therefore f(x)g(x)$ is an odd function.

8. Since the two groups are equally sized, there are $\dfrac{{}^{2n}C_n}{2}$ ways of splitting the $2n$ people into two groups of n people.

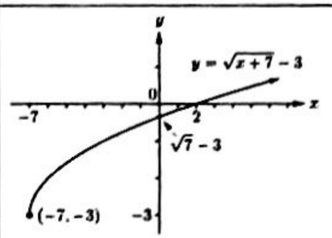
9. Graphically, $y=f(|x|)$ is drawn by taking the portion of the graph of $y=f(x)$ right of the x-axis, duplicating and reflecting it in the y-axis. Option D will not give the correct graph.

10. $y^2=x^2-x$
$y=\pm\sqrt{x(x-1)}$
Graph C is the correct graph.

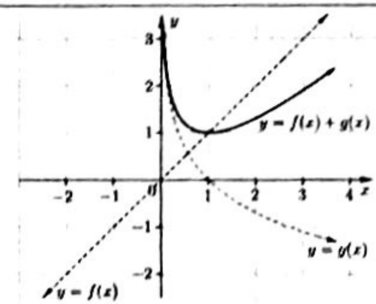
For your interest:
Graph A is the graph of $y=x^2-x$.
Graph B is the graph of $y=(x^2-x)^2$.
Graph D is the graph of $y^2=|x^2-x|$.

Section II

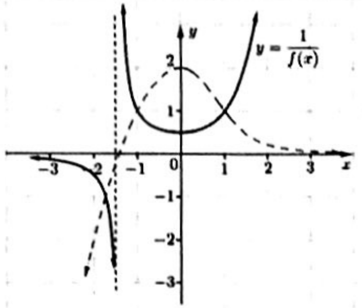
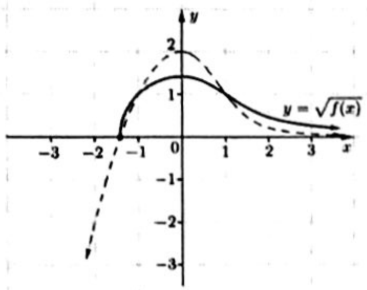
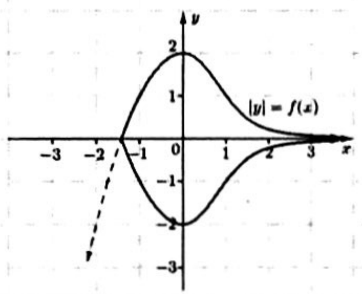
Question 11

		Sample solution	Suggested marking criteria
(a)		$\lvert 1-3x\rvert < 7$ $-7 < 1-3x < 7$ $-8 < -3x < 6$ $\frac{8}{3} > x > -2$  $-2 < x < 2\frac{2}{3}$	• 3 – correct solution • 2 – correctly solves the inequation – identifies that the inequation has two critical points and correctly plots a compound inequality on the number line • 1 – correctly plots a solution on the number line – correctly solves $1-3x<7$
(b)		$\frac{4}{x+1} < 5$ $4(x+1) < 5(x+1)^2$ $0 < (x+1)[5(x+1)-4]$ $0 < (x+1)(5x+1)$ $\therefore x < -1$ or $x > -\frac{1}{5}$	• 2 – correct solution • 1 – valid attempt at solving the inequation
(c)	(i)	$f(x)$ fails the horizontal line test, therefore its inverse will not be a function. For $f(x)$ to have an inverse that is a function, the domain must be restricted.	• 1 – correct explanation
	(ii)	$y = x^2+6x+2$ $y=(x+3)^2-7$ Equation of the inverse: $x=(y+3)^2-7$ $x+7=(y+3)^2$ $\sqrt{x+7}=y+3$ (since $y \ge -3$) $\sqrt{x+7}-3=y$ $f^{-1}(x)=\sqrt{x+7}-3$	• 2 – correct solution • 1 – attempts to find the equation of the inverse by exchanging the x's and y's
	(iii)	Domain: $x \ge -7$ Range: $y \ge -3$	• 2 – correct solution • 1 – correct domain – correct range
	(iv)	 	• 2 – correct solution • 1 – recognises the shape of a square root function

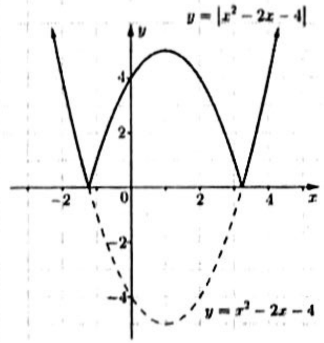
Question 12

		Sample solution	Suggested marking criteria
(a)		 	• 2 – correct solution • 1 – attempts to sketch the graph of $y=f(x)+g(x)$ with incorrect domain
(b)		$\frac{x^2-4}{x} \le 0$ $x(x^2-4) \le 0$ $x(x+2)(x-2) \le 0$ $\therefore x \le -2$ or $0 < x \le 2$	• 3 – correct algebraic or graphical solution • 2 – identifies the critical points • 1 – identifies $x=\pm 2$ as critical points – identifies that $x \ne 0$
(c)	(i)	There are $\frac{8!}{2!2!}=10080$ ways.	• 2 – correct solution • 1 – calculates the number of ways of arranging 8 letters in a line
	(ii)	<u>No double letters:</u> There are ${}^6P_5=720$ ways of choosing and arranging five letters from A, D, I, T, O or N. <u>One pair of double letters:</u> A pair of D's, and then ${}^5C_3=10$ ways of choosing the rest of the letters from A, I, T, O or N. Then there are $\frac{5!}{2!}=60$ ways of arranging the 5 letters. OR A pair of I's, and then ${}^5C_3=10$ ways of choosing the rest of the letters from A, D, T, O or N. Then there are $\frac{5!}{2!}=60$ ways of arranging the 5 letters. <u>Two pairs of double letters:</u> A pair of D's, a pair of I's, and 4 ways of choosing the remaining letter from A, T, O or N. Then there are $\frac{5!}{2!2!}=30$ ways of arranging the 5 letters. Total $=720+10\times 60+10\times 60+4\times 30$ $=2040$ different 5-letter words	• 3 – correct solution • 2 – considers a majority of the cases • 1 – considers a valid case
	(iii)	If the 5-letter word is to be palindromic, then it must have both D's and both I's in the word, in the following configurations: DI_ID or ID_DI For each of these configurations, there are 4 ways of choosing the remaining letter. Therefore, there are $2\times 4=8$ different 5-letter palindromic words. $P(\text{palidromic 5-letter word})=\frac{8}{2040}$ $=\frac{1}{255}$	• 2 – correct solution • 1 – correctly identify the cases

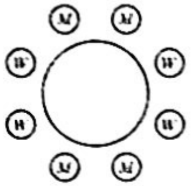
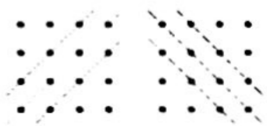
stion 13

Sample solution			Suggested marking criteria
(a)	(i)		• 2 – correct solution • 1 – vertical asymptote at the x-intercept of $y = f(x)$ – identifies $(x, \pm 1)$ as critical points on the graph of $y = \frac{1}{f(x)}$
	(ii)		• 2 – correct solution • 1 – vertical tangent at the x-intercept of $y = f(x)$ – identifies $(x, 1)$ as critical points on the graph of $y = \sqrt{f(x)}$
	(iii)		• 2 – correct solution • 1 – correct reflection in the x-axis, or equivalent merit

Question 13 (continued)

Sample solution			Suggested marking criteria
(b)	(i)		• 2 – correct solution • 1 – correct reflection in the x-axis, or equivalent merit
	(ii)	Using the graph, $-2 \le x \le 0$ or $2 \le x \le 4$.	• 2 – correct solution • 1 – identifies two of the four critical x-values
(c)	(i)	With 7 days in a week, by the pigeonhole principle, to guarantee that at least two of the children are born on the same day of the week, there needs to be a minimum of 8 children. There are 12 children, therefore we can be sure that at least two of the children were born on the same day of the week.	• 1 – correct justification
	(ii)	With 12 months in a year, by the pigeonhole principle, to guarantee that at least two of the children are born in the same month, there needs to be a minimum of 13 children. There are only 12 children, therefore we cannot be sure that at least two of the children were born on the same month. (For example, they could be born in a different month each.)	• 1 – correct justification

Question 14

Sample solution			Suggested marking criteria
(a)	(i)	With no restrictions, there are $7! = 5040$ ways of arranging 8 people around a circular table.	• 1 – correct solution
	(ii)	If the women want to sit together, there are $4! = 24$ ways of arranging the 4 women. Treating the women as one group, there are $4! = 24$ ways of arranging the group of 4 women and the remaining 4 men around a circular table. Therefore, there is a total of $24 \times 24 = 576$ seating arrangements.	• 2 – correct solution • 1 – considers the number of ways of grouping and arranging the women
	(iii)	 Number of ways this configuration can be filled: $2 \times 3! \times 4! = 288$ (2 choices for the first man (or woman), this will dictate direction around the circle, 3! ways to seat the remaining 3 men (or women), then 4! ways to seat the remaining 4 women (or men).	• 2 – correct solution • 1 – significant progress towards solution
(b)	(i)	Dots along these lines are collinear:  ${}^4C_3 \times 4 \times 2 = 32$  $\left(1 + {}^4C_3 + 1\right) \times 2 = 12$ Altogether, there are $32 + 12 = 44$ ways of choosing 3 points that are collinear.	• 2 – correct solution • 1 – considers some of the cases
	(ii)	Number of ways of forming triangles: ${}^{16}C_3 - 44 = 516$	• 2 – correct solution • 1 – recognises the number of ways of choosing 3 dots

Question 14 (continued)

Sample solution		Suggested marking criteria
(c)	${}^{n+1}C_3 + 2 \times {}^{n-1}C_2 = 6 \times {}^nC_2$ $\frac{(n+1)!}{3!(n+1-3)!} + \frac{2(n-1)!}{2!(n-1-2)!} = \frac{6n!}{2!(n-2)!}$ $\frac{(n+1)!}{6(n-2)!} + \frac{2(n-1)!}{2(n-3)!} = \frac{6n!}{2(n-2)!}$ $\frac{(n+1)n(n-1)(n-2)!}{6(n-2)!} + \frac{(n-1)(n-2)(n-3)!}{(n-3)!} = \frac{3n(n-1)(n-2)!}{(n-2)!}$ $\frac{n(n+1)(n-1)}{6} + (n-1)(n-2) = 3n(n-1)$ $n(n+1)(n-1) + 6(n-1)(n-2) = 18n(n-1)$ $(n-1)\left[n(n+1) + 6(n-2) - 18n\right] = 0$ $(n-1)\left(n^2 + n + 6n - 12 - 18n\right) = 0$ $(n-1)\left(n^2 - 11n - 12\right) = 0$ $(n-1)(n-12)(n+1) = 0$ $\therefore n = \pm 1$ or $n = 12$ Since n cannot be negative (from ${}^nC_2$) and n must be greater than or equal to 3 (from ${}^{n-1}C_2$), the only possible solution for this equation is $n = 12$. Alternatively, ${}^{n+1}C_3 + 2 \times {}^{n-1}C_2 = 6 \times {}^nC_2$ $\frac{(n+1)!}{3!(n+1-3)!} + \frac{2(n-1)!}{2!(n-1-2)!} = \frac{6n!}{2!(n-2)!}$ $\frac{(n+1)!}{6(n-2)!} + \frac{2(n-1)!}{2(n-3)!} = \frac{6n!}{2(n-2)!}$ $\frac{(n+1)n(n-1)(n-2)!}{6(n-2)!} + \frac{(n-1)(n-2)(n-3)!}{(n-3)!} = \frac{3n(n-1)(n-2)!}{(n-2)!}$ $\frac{n(n+1)(n-1)}{6} + (n-1)(n-2) = 3n(n-1)$ $\frac{n(n+1)}{6} + (n-2) = 3n$, since $n - 1 \neq 0$ $n(n+1) + 6(n-2) = 18n$ $n(n+1) + 6(n-2) - 18n = 0$ $\left(n^2 + n + 6n - 12 - 18n\right) = 0$ $\left(n^2 - 11n - 12\right) = 0$ $(n-12)(n+1) = 0$ $\therefore n = -1$ or $n = 12$ Since n cannot be negative (from ${}^nC_2$), the only possible solution for this equation is $n = 12$.	• 3 – correct solution • 2 – obtains $n = -1$ and $n = 12$ • 1 – reduces the equation to one without factorials