YEAR 11 Mathematics Ext 1
Preliminary Course
Assessment Task 1
March 2009

1. There are 3 sections.
2. Answer each question on your own paper showing all necessary working
3. Start each section on a new page
4. Calculators may be used

Topic	Mark
1. Section 1 (Algebra)	/40
2. Section 2 (Polynomials)	/16
3. Section 3 (Inequalities)	/16

TOTAL **/72**

SECTION 1 ALGEBRA

(Total for this section is 40 marks)

Marks

1. Simplify $2^x + 2^x$ (1)

2. Simplify $\dfrac{2^{n+1} - 2^{n-1}}{2^n}$ (2)

3. Simplify $(\dfrac{a+1)^3}{a^3+1}$ (2)

4. Simplify $\dfrac{4^m \times 27^{m-n}}{6^{2m}}$ (3)

5. Solve $2^{3m-1} = \dfrac{1}{4^m}$ (2)

6. Factorise $12(x^2 - y^2) - 7xy$ (3)

7. Factorise $n^4 + 4m^4$ (HINT: add and subtract $4m^2n^2$) (3)

8. Solve $x^{\frac{3}{2}} = \dfrac{1}{8}$ (2)

9. Factorise fully $x^5 - 9x^3 - 8x^2 + 72$ and hence solve

$x^5 - 9x^3 - 8x^2 + 72 = 0$ (4)

10. Express $\dfrac{1 - x^{-1}}{x^{-1} - x^{-2}}$ in simplest form (3)

11. Simplify $\dfrac{x^3 - (x-y)^3}{x^2 - (x-y)^2}$ (3)

12. If $a + \dfrac{x}{b} = b + \dfrac{x}{a}$ (and $a \neq b$), find x in simplest form (3)

13. Solve simultaneously $\begin{matrix} y = (3-x)^3 \\ y = (3-x)(3+x) \end{matrix}$ (4)

14. Write in simplest form $(x^{\frac{1}{2}} + x^{\frac{-1}{2}})^2$ (2)

15. If $a^x = b$ and $a^{2x} = b^3$, find b and x (3)

SECTION 2 Polynomials

(Total for this section is 16 marks)

Marks

1. From the following list of expressions, state which ones are polynomials (3)

   a) $x^2 + 7x - 3$

   b) $3 - \frac{1}{x}$

   c) $x\sqrt{x + 4x}$

   d) 5

   e) $9x - x^3 + x^5$

   f) $2^x + 1$

2. For the polynomial $P(x) = 2x^3 + x^2 - x + 3$ state the (3)
   a) degree
   b) leading coefficient
   c) constant term

3. If $P(x) = x^2 + 2x + 3$, find (4)
   a) $P(1)$
   b) $P(1) - P(-1)$
   c) $P(0)$
   d) State why $P(x) = 0$ has no real solutions

4. If $P(x) = x^3 + 2x^2 + 3x + 2$ and $Q(x) = x + 1$, find (6)
   a) $P(x) + Q(x)$
   b) $P(x) - Q(x)$
   c) $P(x) \times Q(x)$
   d) $P(x) \div Q(x)$

SECTION 3 Inequalities with the unknown in the denominator

(Total for this section is 16 marks)

Marks

1. Solve $\frac{4}{x-1} > 1$ (3)

2. Solve $\frac{4}{(x-1)^2} \geq 1$ (4)

3. Solve $\frac{2x}{x+1} < x$ (4)

4. Solve $-1 \leq \frac{2x}{x+1} \leq 1$ (5)

(END OF EXAMINATION)

Part A (Algebra) Solutions

1. $2 \cdot 2^x = 2^{x+1}$

2. $\dfrac{2^n(2-2^{-1})}{2^n} = 2 - \frac{1}{2}$

$= 1\frac{1}{2}$

3. $\dfrac{(a+1)^3}{(a+1)(a^2-a+1)} = \dfrac{(a+1)^2}{a^2-a+1}$

4. $\dfrac{4^m \times 27^m \times 27^{-n}}{36^m} = 3^m \times 3^{-3n}$

$= 3^{m-3n}$

5. $2^{3m-1} = \dfrac{1}{2^{2m}}$

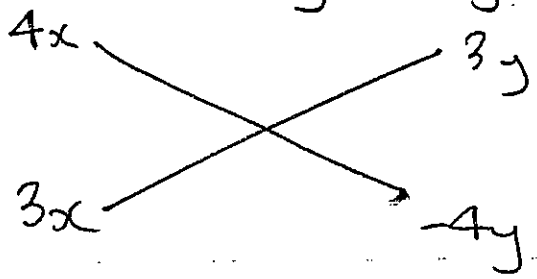
$2^{3m-1} = 2^{-2m}$

$3m-1 = -2m$

$5m = 1$

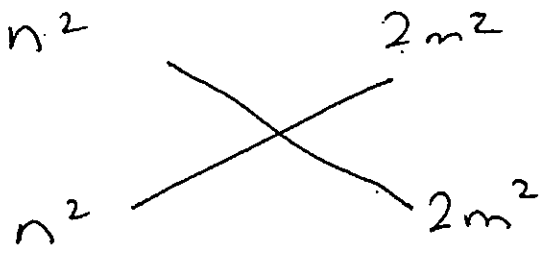
$m = \frac{1}{5}$

6. $12x^2 - 7xy - 12y^2$



$(4x+3y)(3x-4y)$

7. $n^4 + 4m^2n^2 + 4m^4 - 4m^2n^2$



$= (n^2 + 2m^2)^2 - (2mn)^2$

$= [n^2 + 2m^2 - 2mn][n^2 + 2m^2 + 2mn]$

8. $x^{\frac{3}{2}} = \frac{1}{8}$

$x = \left(\frac{1}{8}\right)^{\frac{2}{3}}$

$x = \frac{1}{4}$

9. $x^5 - 9x^3 - 8x^2 + 72$

$= x^3(x^2 - 9) - 8(x^2 - 9)$

$= (x^2 - 9)(x^3 - 8)$

$= (x-3)(x+3)(x-2)(x^2+2x+4)$

Solve $(x^2 - 9)(x^3 - 8) = 0$

$x = \pm 3$ and $x = 2$

10. $$\frac{1 - \frac{1}{x}}{\frac{1}{x} - \frac{1}{x^2}} = \frac{\frac{x-1}{x}}{\frac{x-1}{x^2}}$$

$$= \frac{x-1}{x} \times \frac{x^2}{x-1}$$

$$= x$$

11. $$\frac{[x - (x-y)][x^2 + x(x-y) + (x-y)^2]}{[x - (x-y)][x + (x-y)]}$$

$$= \frac{x^2 + x^2 - xy + x^2 - 2xy + y^2}{2x - y}$$

$$= \frac{3x^2 - 3xy + y^2}{2x - y}$$

12. $\frac{x}{b} - \frac{x}{a} = b - a$

$x\left(\frac{1}{b} - \frac{1}{a}\right) = b - a$

$x\left(\frac{a-b}{ab}\right) = b - a$

$x = (b-a) \div \frac{(a-b)}{ab}$

$= \frac{b-a}{1} \times \frac{ab}{-(b-a)}$ Since $a \neq b$

$= -ab$ ✓

13. $(3-x)^3 = (3-x)(3+x)$

$(3-x)^3 - (3-x)(3+x) = 0$

$(3-x)[(3-x)^2 - (3+x)] = 0$

$(3-x)[9 - 6x + x^2 - 3 - x] = 0$

$(3-x)[x^2 - 7x + 6] = 0$

$x = 3$ and $(x-6)(x-1) = 0$

$x = 6$ and 1

$\therefore x = 1, 3, 6$ $y = 8, 0, -27$ ✓

14. $x + 2x^0 + x^{-1} = x + \frac{1}{x} + 2$ ✓

OR

$\frac{x^2 + 2x + 1}{x} = \frac{(x+1)^2}{x}$ ✓

Part B (polynomials)

1. x^2+7x-3, 5, $9x-x^3+x^5$

2. Degree $= 3$
   leading coefficient $= 2$
   Constant term $= 3$

3. $P(x) = x^2 + 2x + 3$

(a) $P(1) = (1)^2 + 2(1) + 3$
$= 6$

(b) $P(1) - P(-1) = 6 - \left[(-1)^2 + 2(-1) + 3\right]$
$= 6 - 2$
$= 4$

(c) $P(0) = 3$

(d) $x^2 + 2x + 3 = 0$

$$x = \frac{-2 \pm \sqrt{4-4\times1\times3}}{2}$$

$$x = \frac{-2 \pm \sqrt{-8}}{2}$$

no real solutions to $\sqrt{-8}$

$\therefore$ no real solutions to $P(x) = 0$

15. $a^x = b$ $\quad a^{2x} = b^3$

$a^{2x} = b^2$

$\therefore b^3 = b^2$

$b^3 - b^2 = 0$

$b^2(b-1) = 0$

$b = 0$ and $b = 1$

when $b = 0$

$a^x = 0$

no soln

when $b = 1$

$a^x = 1$

$x = 0$

$\therefore b = 1, x = 0$ only.

✓

4. (a) $x^3 + 2x^2 + 3x + 2 + x + 1$

$= x^3 + 2x^2 + 4x + 3$

(b) $x^3 + 2x^2 + 3x + 2 - x - 1$

$= x^3 + 2x^2 + 2x + 1$

(c) $x(x^3 + 2x^2 + 3x + 2) + 1(x^3 + 2x^2 + 3x + 2)$

$= x^4 + 2x^3 + 3x^2 + 2x + x^3 + 2x^2 + 3x + 2$

$= x^4 + 3x^3 + 5x^2 + 5x + 2$

(d)
$$
\begin{array}{r|l}
x+1 & x^2 + x + 2 \\
\hline
 & x^3 + 2x^2 + 3x + 2 \\
 & \underline{x^3 + x^2} \\
 & \quad x^2 + 3x \\
 & \quad \underline{x^2 + x} \\
 & \qquad 2x + 2 \\
 & \qquad \underline{2x + 2} \\
 & \qquad\quad 0
\end{array}
$$

$\therefore P(x) \div Q(x) = x^2 + x + 2$

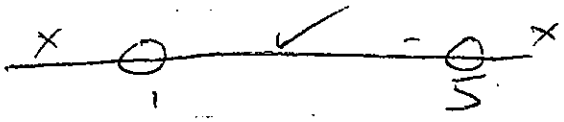
Part C

1. $\frac{4}{x-1} > 1$ CP at $x=1$, $x \neq 1$

Make equation to find other CP's

$4 = x-1$

$5 = x$ CP at $x=5$, $x \neq 5$



$\therefore \quad 1 < x < 5$

2. $\frac{4}{(x-1)^2} \geq 1$ CP at $x=1$, $x \neq 1$

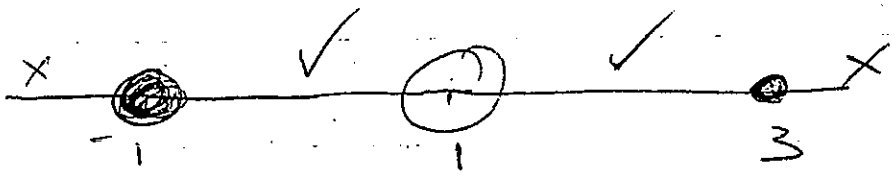
Make equation to find other CP's

$\frac{4}{(x-1)^2} = 1$

$4 = x^2 - 2x + 1$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$ CP's at $x = 3$ and -1



$-1 \leq x < 1$ and $1 < x \leq 3$

OR

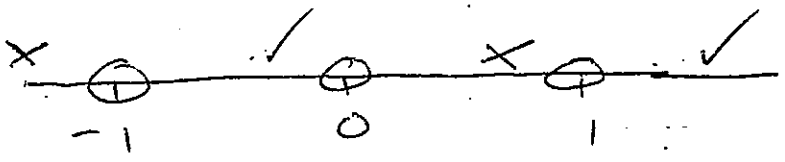
$-1 \leq x \leq 3$ but $x \neq 1$

3. $\frac{2x}{x+1} < x$ CP at $x = -1$, $x \neq -1$

Make equation to find other CP's

$$\frac{2x}{x+1} = x$$

$$2x = x^2 + x$$
$$0 = x^2 - x$$
$$0 = x(x-1)$$ CP at $x = 0$, $x \neq 0$; $x = 1$, $x \neq 1$

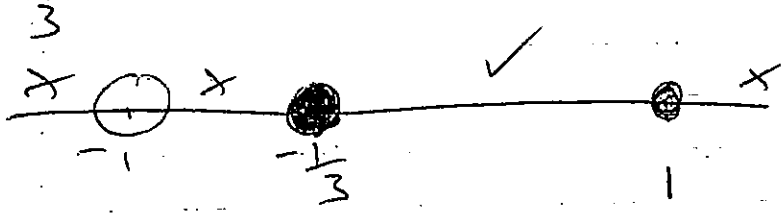


$\therefore -1 < x < 0$ and $x > 1$

4. $-1 \leq \frac{2x}{x+1} \leq 1$ CP at $x = -1$, $x \neq -1$

Consider two equations to find other CP's

$$-x - 1 = 2x \quad \text{and} \quad 2x = x + 1$$
$$-1 = 3x \quad \text{and} \quad x = 1$$
$$-\frac{1}{3} = x$$



$$-\frac{1}{3} \leq x \leq 1$$