GOSFORD HIGH SCHOOL

Year 11

MATHEMATICS EXTENSION 1

Preliminary Course

Assessment Task 1

2010

Time Allowed: 70 minutes

Instructions:

- There are 5 questions.
- Answer each question on your own paper showing all necessary working.
- Start **EACH** question on a new sheet of paper.
- Calculators may be used.

Question	Mark
1. Arithmetic / Algebra	/10
2. Polynomials	/10
3. Counting Theory	/10
4. Inequalities	/10
5. Further Algebra	/10
TOTAL	/50

QUESTION 1: (10 Marks)

Marks

a. Calculate: 2

$$\frac{76.5}{17 \times (81.76)^3} \times \frac{1851.7 - 24^2}{\sqrt{47.8}}$$

writing your answer in scientific notation, correct to 3 significant figures.

b. A builder intends to pay cash for building materials and receives a discount of 12 ½ % for doing so. As well, he receives a further 10% trade discount on this cash price. If he eventually pays $1055, find the original price of the building materials. (Answer to the nearest dollar.) 2

c. Evaluate $\left(\frac{2^x+2^x+2^x+2^x}{2^{x+1}}\right)^2$ 2

d. If $\frac{a^2b^2}{c^4} \times \frac{c^3}{a^5b^3} \times \frac{(a^2c)^3}{b^2} = a^x b^y c^z$, 2

find the value of x, y, z

e. If $\left(2\sqrt{3} - \sqrt{7}\right)^3 = m\sqrt{3} + n\sqrt{7}$, 2

find the value of m, n

QUESTION 2: (10 Marks)

Marks

a. For the polynomial $P(x) = 2x^4 - x^2 + x - 3$

State the

i. Degree
ii. Constant term

1

b. Which of the expressions: 1

A. $5 - x^2 - x^3$ B. $7x^2 + \frac{7}{x^2}$

C. 3 D. $x\sqrt{x} + \sqrt{x}$

are polynomials?

c. What is a monic polynomial? Explain **and** give an example. 1

d. Divide $P(x) = x^4 + x^3 - x^2 + x - 2$ by $x^2 - 1$ and express your answer in the form 2

$$P(x) = A(x)\,Q(x) + R(x)$$

e. Find the zeros of the polynomial 2

$$Q(x) = x^4 - 2x^3 - 8x^2$$

and use them to sketch the graph of $y = Q(x)$
(Indicate all intercepts on your sketch)

f. Show that $x - 1$ is **not** a factor of 1

$$G(x) = x^5 - 2x^4 + 7x^2 - 3x + 5$$

g. When $F(x) = ax^2 - 3x + 1$ and $H(x) = x^3 - 3x + 2$ are divided by $x + 1$ they have the same remainder. Find the value of a. 2

QUESTION 3: (10 Marks)

Marks

a. Show **how** you would calculate ${}^{6}P_{4}$. (Do **not** evaluate) 1

b. Two different numbers are chosen at random from 1, 2, 3, 4, 5

i. How many different **combinations** can be chosen? 1

ii. How many of these combinations have an even sum when the two numbers are added together? 1

c. Let each different arrangement of all the letters of INTERESTED be called a word. How many words are possible? 1

d. In how many ways can 6 girls sit at a round table if:

i. There are no restrictions 1

ii. 3 particular girls insist on sitting together 1

e. A ticket queue has 5 men and 5 women standing in a line. Find how many different arrangements of the line are possible if

i. the men and women can stand anywhere in the line 1

ii. the men and women alternate 1

iii. 2 particular women cannot stand together. 2

QUESTION 4: (10 Marks)

Marks

a. Solve $\frac{4}{5-x} \leq 1$ — 2

b. Solve $\frac{x^2-4}{x} > 0$ — 2

c. Solve $\frac{5x+9}{|x-5|} > 1$ — 3

d. Solve and graph the solution(s) to: — 3

$$|x+4| + |x-4| = 8$$

QUESTION 5: (10 Marks)

Marks

a. Express as a single fraction: 2

$(x^{-1} + y^{-1})^{-1}$

b. If $x = \frac{3-2\sqrt{2}}{3+2\sqrt{2}}$, express $x - \frac{1}{x}$ 3

in simplest form with a rational denominator.

c. Simplify: 3

$$\frac{20x + 12y - 25x^2 + 9y^2}{25x^2 - 9y^2} \div \frac{8 - 10x + 6y}{25x^2 - 30xy + 9y^2}$$

d. i. Express $x^4 + 4$ in the form 1

$$(x^2 + a)^2 - (bx)^2$$

and find the value of a and b

ii. Hence, express $x^4 + 4$ as a product of two quadratic factors 1

END OF TEST

Q1

a) $0.001519233\ldots$ (calc.)

$= 1.52 \times 10^{-3}$

b) Amount paid:

90% of $87\frac{1}{2}\%$

$= 78\frac{3}{4}\%$

$\therefore 78\frac{3}{4}\% = \$1055$

$1\% = \$13.397\ldots$

$100\% = \$1339.7\ldots$

$\therefore$ Original Price $= \$1340$

c) $\left(\frac{2^x+2^x+2^x+2^x}{2^{x+1}}\right)^2$

$= \left(\frac{4.2^x}{2^{x+1}}\right)^2$

$= \left(\frac{2^2.2^x}{2^{x+1}}\right)^2$

$= \left(\frac{2^{x+2}}{2^{x+1}}\right)^2$

$= 2^2$

$= 4$

d) $\frac{a^2b^2}{c^4} \times \frac{c^3}{a^5b^3} \times \frac{(a^2c)^3}{b^2}$

$= \frac{a^2b^2c^3}{a^5b^3c^4} \times \frac{a^6c^3}{b^2}$

$= \frac{a^8b^2c^6}{a^5b^5c^4} = \frac{a^3c^2}{b^3}$

$\therefore x = 3,\ y = -3,\ z = 2$

e) $(2\sqrt{3}-\sqrt{7})^3$

$= (2\sqrt{3}-\sqrt{7})^2(2\sqrt{3}-\sqrt{7})$

$= (12-4\sqrt{21}+7)(2\sqrt{3}-\sqrt{7})$

$= (19-4\sqrt{21})(2\sqrt{3}-\sqrt{7})$

$= 38\sqrt{3} - 19\sqrt{7} - 8\sqrt{63} + 4\sqrt{147}$

$= 38\sqrt{3} - 19\sqrt{7} - 24\sqrt{7} + 28\sqrt{3}$

$= 66\sqrt{3} - 43\sqrt{7}$

$\therefore m = 66,\ n = -43.$

Q2

a) (i) Degree $= 4$ $(\frac{1}{2})$

(ii) Constant $= -3$ $(\frac{1}{2})$

b) A and C (1)

c) A monic polynomial is a polynomial with the leading co-efficient equal to 1. $(\frac{1}{2})$

eg: $x^3 - 4x^2 + 5x - 7$ $(\frac{1}{2})$ (of course others)

d)
$$x^2-1 \;\overline{)\; x^4+x^3-x^2+x-2}$$
$$x^2+x$$
$$x^4 \quad -x^2$$
$$x^3 \quad +x$$
$$x^3 \quad -x$$
$$2x-2$$

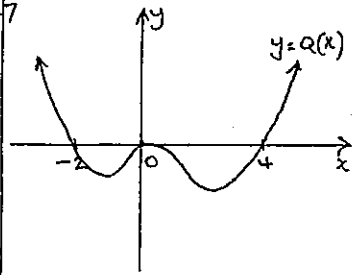
$\therefore P(x) = (x^2-1)(x^2+x)+(2x-2)$

e) $Q(x) = 0$ for zeros

i.e $x^2(x^2-2x-8) = 0$

$x^2(x-4)(x+2) = 0$

$\therefore$ zeros $= 0, 4, -2$



f) $G(1) = 1-2+7-3+5$

$= 8$

Since $G(1) \neq 0$

$x-1$ is not a factor

g) $F(-1) = a+3+1$

$= a+4$

$H(-1) = -1+3+2$

$= 4$

If the same remainder:

$\therefore a+4 = 4$

$a = 0.$

Q3

a) using ${}^nP_r = \frac{n!}{(n-r)!}$

${}^6P_4 = \frac{6!}{(6-4)!} * \frac{6!}{2!}$

$= 6\times5\times4\times3.$

Q3 b) (i) ${}^5C_2 = 10$

(11 / either 2 odds or 2 evens

i.e (2+4), (1+3), (3+5) (1+5)

Answer $= 4$

c) $\frac{10!}{2!3!} = 302\,400$

d) (i) using $(n-1)!$

$= 5!$

$= 120$ ways.

(ii) Grouping 3 girls together in a circle

$= (4-1)!$

$= 3!$ and

the 3 girls can be placed in 3! ways

total ways $= 3! \times 3!$

$= 36$

(e) (i) $10! = 3\,628\,800$

(ii) Men $\rightarrow 5!$

Women $\rightarrow 5!$

Now either

HWMWHWMWMW

or

WMWMWMWMWH

$\therefore$ Ways $= 2 \times 5! \times 5!$

$= 28800$

(iii) 2 partical women together $= 2 \times 9!$

$\therefore$ not together

$= 10! - 2 \times 9!$

$(3628\,800 - 806\,400$

$= 2\,822\,400)$

Q4 a) Consider $\frac{4}{5-x} = 1$

C.V: $x \neq 5$ $\quad 4 = 5-x$

$x = 1$

Test ✓ × ✓

1 5

$\therefore x \leq 1,\ x > 5$

b) Consider $\frac{x^2-4}{x} = 0$

C.V: $x \neq 0$

$x^2 - 4 = 0$

$x = \pm 2$

Test × ✓ × ✓

−2 0 2

$\therefore -2 < x < 0,\ x > 2$

c) Consider $\frac{5x+9}{|x-5|} = 1$

$x \neq 5$

$5x+9 = |x-5|$

$5x+9 = x-5$ or $5x+9 = -x+5$

$4x = -14$ $\quad 6x = -4$

$x = -\frac{7}{2}$ $\quad x = -\frac{2}{3}$

check × ✓

$\therefore x = -\frac{2}{3}$ is only soln to equation

Test × ✓ ✓

$-\frac{2}{3}$ 5

$-\frac{2}{3} < x < 5,\ x > 5.$

d) $|x+4| + |x-4| = 8$

4 cases: 1/ $x+4 \geq 0$ and $x-4 \geq 0$

$x+4+x-4 = 8$

$2x = 8$

$x = 4$ ✓

2/ $x+4 \leq 0$ and $x-4 < 0$

$-(x+4) - (x-4) = 8$

$-x-4-x+4 = 8$

$-2x = 8$

$x = -4$ ✓

3/ $x+4 \geq 0$ and $x-4 < 0$
$(x \geq -4)$ $\quad (x < 4)$

$x+4-(x-4) = 8$

$x+4-x+4 = 8$

$8 = 8$

i.e in this region $-4 < x < 4$ the equation is always true

4/ $x+4 < 0$ and $x-4 \geq 0$
$(x < -4)$ $\quad (x \geq 4)$

$-(x+4) + (x-4) = 8$

$-x-4+x-4 = 8$

$-8 = 8$

$\therefore$ no solutions for $x < -4$ and $x > 4$ x

$\therefore$ Solution: $-4 \leq x \leq 4$

× ✓ ✓ ✓ ×

−4 4

4/ a) $\left(x^{-1}+y^{-1}\right)^{-1}$

$= \left(\frac{1}{x}+\frac{1}{y}\right)^{-1}$

$= \left(\frac{y+x}{xy}\right)^{-1}$

$= \frac{xy}{x+y}$

Q.5 b)

$$x - \frac{1}{x} = \frac{3-2\sqrt{2}}{3+2\sqrt{2}} - \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \frac{(3-2\sqrt{2})^2 - (3+2\sqrt{2})^2}{(3+2\sqrt{2})(3-2\sqrt{2})}$$

$$= \frac{9 - 12\sqrt{2} + 8 - (9+12\sqrt{2}+8)}{9-8}$$

$$= \frac{17 - 12\sqrt{2} - 17 - 12\sqrt{2}}{1}$$

$$= -24\sqrt{2}$$

c) $$\frac{20x + 12y - 25x^2 + 9y^2}{25x^2 - 9y^2} \times \frac{25x^2 - 30xy + 9y^2}{8 - 10x + 6y}$$

$$= \frac{4(5x+3y) - (5x-3y)(5x+3y)}{(5x-3y)(5x+3y)} \div \frac{(5x-3y)(5x-3y)}{2(4-5x+3y)}$$

$$= \frac{(5x+3y)(4-5x+3y)}{(5x-3y)(5x+3y)} \times \frac{2(4-5x+3y)}{(5x-3y)(5x-3y)}$$

$$= \frac{5x-3y}{2}$$

d)(i) $x^4 + 4 = x^4 + 4x^2 + 4 - 4x^2$

$$= (x^2+2)^2 - (2x)^2$$

$\therefore a = 2, b = 2$

(ii) $x^4 + 4 =$

$$[(x^2+2) - 2x][(x^2+2) + 2x]$$

$$= (x^2 - 2x + 2)(x^2 + 2x + 2)$$