



GOSFORD HIGH SCHOOL

MATHEMATICS EXTENSION 1

2012 PRELIMINARY COURSE.

ASSESSMENT TASK 1

NAME: _____

Harder aspects of 2 unit arithmetic and algebra	/21
Further inequalities	/9
Counting theory	/17
Total	/47

General Instructions:

- Reading time – 5minutes
- Working time – 60 minutes
- Write using black or blue pen.
- Board-approved calculators may be used
- Each question should be started on a new page.
- All necessary working should be shown in every question.

Total marks: - 47

- Attempt Questions 1 - 3

Question 1.

a) Simplify:

$$\text{i) } \frac{(-3x^4)^3}{-18x^3} \quad (1)$$

$$\text{ii) } \frac{5^{n-1} + 5^{n+1}}{5^n + 5^{n+2}} \quad (2)$$

$$\text{iii) } \frac{a}{a^2 - b^2} + \frac{1}{b - a} \quad (2)$$

$$\text{iv) } \frac{x^4 - a^4}{x^2 - 2ax + a^2} \div \frac{ax^2 + a^3}{ax - a^2} \quad (3)$$

b) If $x = 3 + \sqrt{7}$, show that $x + \frac{2}{x} = 6$ and hence evaluate $x^2 + \frac{4}{x^2}$ (3)

c) Factorise fully:

$$\text{i) } 3x^2 - 27y^2 \quad (1)$$

$$\text{ii) } (x - y)^2 - 6x + 6y \quad (2)$$

$$\text{iii) } x^4 + 4y^4 \quad (2)$$

$$\text{iv) } (x^2 - 2x)^2 - 2(x^2 - 2x) - 3 \quad (2)$$

$$\text{v) } a^6 - b^6 \quad (3)$$

Question 2 (Start a new page)

Solve for x :

$$\text{a) } \frac{4}{5-x} \geq 1 \quad (2)$$

- b) $\frac{3}{|x+2|} < 5$ (2)
- c) $5 > 2x^2 + 3x$ (2)
- d) $0 < \frac{x^2 - 4}{x}$ (3)

Question 3 (Start a new page)

- a) i) How many distinct 11 letter words can be formed from the letters of the word **INDEPENDENT**? (1)
- ii) How many of these words have **D**'s at the ends? (1)
- b) A bag contains 4 white and 2 red balls. 5 balls are taken from the bag and arranged in a row. How many distinct arrangements are there? (2)
- c) In how many ways can 4 men and 2 women sit in a row if:
- i) there are no restrictions? (1)
- ii) the two women are to sit together? (2)
- iii) the two women are **not** to sit together? (1)
- iv) the two women are to be separated by two men? (2)
- d) A committee of 3 women and 4 men is to be chosen from 8 women and 7 men. In how many ways can the committee be chosen if:
- i) there are no restrictions? (1)
- ii) man A refuses to serve if man B is a member? (2)
- e) A table has 7 seats, 4 on the window side and 3 on the opposite side. In how many ways can 7 people be seated at the table if:
- i) 2 people, A and B must sit on the same side? (2)
- ii) 3 people, A,B and C must sit facing the window? (2)

Assessment Task 1 2012 Solutions

Question 1

a) i) $\frac{-27x^{12}}{-18x^3} = \frac{3x^9}{2}$

ii) $\frac{5^{n-1} + 5^{n+1}}{5^n + 5^{n+2}} = \frac{5^{n-1}(1+5^2)}{5^n(1+5^2)} = \frac{1}{5}$

iii) $\frac{a}{a^2-b^2} + \frac{1}{b-a} = \frac{a}{(a-b)(a+b)} - \frac{1}{a-b} = \frac{a - (a+b)}{(a-b)(a+b)} = \frac{-b}{(a-b)(a+b)}$

iv) $\frac{x^4 - a^4}{x^2 - 2ax + a^2} \div \frac{ax^2 + a^3}{ax - a^2} = \frac{(x-a)(x+a)(x^2+a^2)}{(x-a)^2} \times \frac{a(x-a)}{a(x^2+a^2)} = x+a$

b) $x = 3 + \sqrt{7}$
 $\frac{2}{x} = \frac{2}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}} = \frac{2(3-\sqrt{7})}{2} = 3-\sqrt{7}$
 $\therefore x + \frac{2}{x} = 3 + \sqrt{7} + 3 - \sqrt{7} = 6$

$x^2 + \frac{4}{x^2} = \left(x + \frac{2}{x}\right)^2 - 4 = 6^2 - 4 = 32$

c) i) $3(x^2 - 9y^2) = 3(x-3y)(x+3y)$

ii) $(x-y)^2 - 6(x-y) = (x-y)(x-y-6)$

iii) $x^4 + 4y^4 = (x^2 + 2y^2)^2 - 4x^2y^2 = (x^2 + 2y^2 - 2xy)(x^2 + 2y^2 + 2xy)$

iv) $(x^2 - 2x)^2 - 2(x^2 - 2x) - 3 = (x^2 - 2x - 3)(x^2 - 2x + 1) = (x-3)(x+1)(x-1)^2$

v) $a^6 - b^6 = (a^3 - b^3)(a^3 + b^3) = (a-b)(a^2 + ab + b^2)(a+b)(a^2 - ab + b^2)$

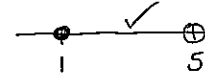
Question 4

a) $\frac{4}{5-x} \geq 1$

critical point $x \neq 5$

solve $\frac{4}{5-x} = 1$

$5-x=4$
 $x=1$



test $x=2: \frac{4}{5-2} \geq 1$

$\therefore 1 \leq x < 5$

b) $\frac{3}{|x+2|} < 5$

$x \neq -2$

$3 < 5|x+2|$

$|x+2| > \frac{3}{5}$

$x+2 > \frac{3}{5}$ or $x+2 < -\frac{3}{5}$

$x > -1\frac{2}{5}$ or $x < -2\frac{3}{5}$

c) $5 > 2x^2 + 3x$

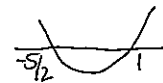
$2x^2 + 3x - 5 < 0$

Solve $2x^2 + 3x - 5 = 0$

$(2x+5)(x-1) = 0$

$x = -\frac{5}{2}, 1$

$\therefore -\frac{5}{2} < x < 1$



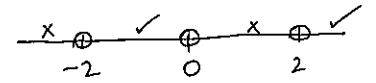
$\frac{4}{x} > 0$

critical points $x \neq 0$

Solve $\frac{x^2-4}{x} = 0$

$x^2 - 4 = 0$

$x = \pm 2$



test $x = -3: \frac{9-4}{-3} > 0$ F

$x = -1: \frac{1-4}{-1} > 0$ T

$x = 1: \frac{1-4}{1} > 0$ F

$x = 3: \frac{9-4}{3} > 0$ T

$\therefore -2 < x < 0$ or $x > 2$

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a) i) $\frac{11!}{3!3!2!} = 554400$

ii) $\frac{9!}{3!3!} = 10080$

b) 4W1R or 3W2R

$\frac{5!}{4!} + \frac{5!}{3!2!}$

$= 5 + 10$
 $= 15$

c) i) $6! = 720$

ii) $5! \times 2 = 240$

iii) $720 - 240 = 480$

iv) WMMWMM
 MWMMWM
 MMWMMW

$4! \times 2! \times 3 = 144$

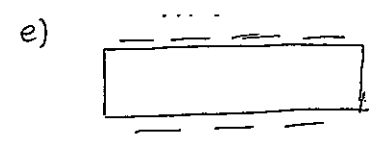
d) i) ${}^8C_3 \times {}^7C_4 = 1960$

ii) Total - both A + B

${}^8C_3 \times {}^7C_4 - {}^5C_2 \times {}^8C_3$
 $= 1400$

OR: A not B + B not A + neither

${}^5C_3 \times {}^8C_3 \times 2 + {}^5C_4 \times {}^8C_3$
 $= 1400$



i) ${}^4P_2 \times 5! + {}^3P_2 \times 5!$
 $= 2160$

ii) $3! \times 4! = 144$