

Name: _____

Teacher: _____

Gosford High School

2013

Year 11 Preliminary Mathematics Extension 1 Assessment Task #1

INSTRUCTIONS:

TIME: 60 minutes + 5 minutes reading time

- Write using blue or black pen.
- Board approved calculators may be used.
- In questions 5 – 7, show relevant mathematical reasoning and/or calculations

Questions 1 – 4	Multiple Choice	/4
Question 5	Algebra	/14
Question 6	Inequalities	/8
Question 7	Counting Techniques	/14
TOTAL		/40

Question 1:

Russel is answering an inequality and writes the answer

$$-2 \leq x < 5 \text{ and } x \geq 1$$

This statement simplifies to:

a) $1 \leq x \leq 5$

b) $1 < x \leq 5$

c) $1 \leq x < 5$

d) $1 < x < 5$

Question 2:

Evaluate $2 \times \frac{3^{n+1}}{3^{n-1}}$

a) 1

b) 18

c) 2

d) 36

Question 3:

A committee of n people is to be chosen from $n + 1$. Which of the following is **NOT EQUAL** to the number of committees possible:

a) ${}^{n+1}C_n$

b) ${}^{n+1}P_n \div n!$

c) ${}^{n+1}P_n$

d) $n + 1$

Question 4:

A bag contains 1 white and 5 black marbles. 5 will be taken from the bag and placed in a line. How many distinct arrangements are possible.

a) 6

b) 5

c) 6P_5

d) $5!$

Question 5:

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a) Fully factorise:

i) $(x + y)^2 - 2x - 2y$ (2)

ii) $a^6 - b^6$ (2)

iii) $(x^2 + xy)^2 - (xy + y^2)^2$ (2)

b) Simplify $\frac{4^m \times 27^{m-n}}{6^{2m}}$ (2)

c) By factorising, find the result of dividing $2^{n+3} + 2^n$ by 3 (2)

d) If $x + \frac{1}{x} = 3$ (Do not attempt to find x)

i) Show that $x^2 + \frac{1}{x^2} = 7$ (2)

ii) Hence find the value of $x^3 + \frac{1}{x^3}$ (2)

Question 6:

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a) Solve $\frac{2}{x+2} < 5$ (2)

b) Solve $\frac{x+2}{(x-5)(x+4)} \geq 0$ (2)

c) Find all positive values of x for which $\frac{6}{x} > x - 1$ (2)

d) Solve $\frac{x+3}{x^2+6x+9} < 0$ (2)

Question 7:

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- a) i) How many 7 letter words can be made from the letters of **EVEREST**? (1)
ii) How many have the **E**'s together? (1)
- b) 4 distinct Shrek and 3 different Toy Story movies are put on a shelf. How many arrangements are possible if:
i) there are no restrictions (1)
ii) no two movies from the same series can be adjacent (1)
- c) 6 Women and 3 men sit at a circular table with 9 seats. In how many ways can this happen if:
i) there are no restrictions (1)
ii) the men all sit together (1)
iii) 2 specific women must not sit together (2)
- d) A committee of 6 students and 3 teachers is to be selected from 25 students and 10 teachers. In how many ways can this happen if:
i) there are no restrictions (1)
ii) a certain teacher, the principal must be on the committee (1)
iii) a certain student and her boyfriend cannot both be on the committee (2)
- e) Solve for n :
$${}^n C_5 \div {}^n C_6 = \frac{2}{3}$$
 (2)

Task 1

C
B
C
A

a) i) $(x+y)^2 - 2(xy)$
 $= (x+y)(x+y-2)$
 ii) $(a^3 - b^3)(a^3 + b^3)$
 $= (a-b)(a^2 + ab + b^2)(a+b)(a^2 - ab + b^2)$
 iii) $(x^2 + xy + xy + y^2)(x^2 + xy - xy - y^2)$
 $= (x^2 + 2xy + y^2)(x^2 - y^2)$
 $= (x+y)^2(x-y)(x+y)$
 $= (x+y)^3(x-y)$

b) $\frac{2^{2n} \times 3^{3n-3n}}{2^{2n} \times 3^{2n}} = 3^{n-3n}$

c) $\frac{2^{n+3} + 2^n}{3} = \frac{2^n(2^3 + 1)}{3}$
 $= 3 \times 2^n$

d) i) $(x + \frac{1}{x})^2 = 3^2$
 $x^2 + 2 + \frac{1}{x^2} = 9$
 $x^2 + \frac{1}{x^2} = 7$
 ii) $x^3 + \frac{1}{x^3} = (x + \frac{1}{x})(x^2 - 1 + \frac{1}{x^2})$
 $= 3 \times (7-1)$
 $= 18$

b) a) CP: $x+2=0$ $2=5x+10$
 $x=-2$ $-8=5x$
 $x=-\frac{8}{5}$

 $x < -2$ or $x > -\frac{8}{5}$

b) CP: $x=-2$ $x=5$ $x=-4$

 $-4 < x \leq -2$ or $x > 5$

c) CP: $x=0$ $6 = x^2 - x$
 $x^2 - x - 6 = 0$
 $(x-3)(x+2) = 0$
 $x=3$ or -2

 $x < -2$ or $0 < x < 3$
 not positive.

d) CP: $x=3$ $x^2 + 6x + 9 = 0$
 $(x+3)^2 = 0$
 $x = -3$

 $x < -3$

$$7) a) i) \frac{7!}{3!}$$

$$ii) 5!$$

$$b) i) 7!$$

$$ii) 4! \times 3!$$

$$c) i) 8!$$

$$ii) 6! \times 3!$$

$$iii) 8! - 7! \times 2!$$

$$d) i) {}^{25}C_6 \times {}^{10}C_3$$

$$ii) {}^{25}C_6 \times {}^9C_2$$

$$iii) {}^{25}C_6 \times {}^{10}C_3 - {}^{23}C_4 \times {}^{10}C_2$$

$$e) \frac{{}^nC_5}{{}^nC_6} = \frac{2}{3}$$

$$\frac{n!}{(n-5)!5!} \div \frac{n!}{(n-6)!6!} = \frac{2}{3}$$

$$\frac{n!}{(n-5)(n-6)!5!} \times \frac{(n-6)!6 \times 5!}{n!} = \frac{2}{3}$$

$$\frac{6}{n-5} = \frac{2}{3}$$

$$18 = 2n - 10$$

$$n = 14$$