Name_____

Class_____

Teacher_____

YEAR 11
Preliminary Extension 1
ASSESSMENT Task 1
April 2014

Time allowed:-60 minutes plus 5 minutes reading

1. Answer each section separately on your own paper
2. Make sure you start a new page for each section
3. Each section will collected separately for marking
4. Show all necessary working

Section	Mark
1. Advanced Algebra	/19
2. Further Inequalities	/12
3. Counting Techniques	/15

TOTAL /46

Section 1 (Advanced algebra) Start a new page.

1. Factorise fully $x^4 - 2x^2 - 15$ 1

2. Factorise fully $x^3 - 5x^2 - x + 5$ 2

3. Factorise fully $x^9 - x^6 - x^3 + 1$ 3

4. If $x + \frac{1}{x} = 4$, use this information to find the value of $x^2 + \frac{1}{x^2}$ 2

5. Simplify $\frac{2^{x+3}+2^x}{3^{x+2}}$ 2

6. Simplify $\frac{9^n-1}{3^n-1}$ 1

7. Rationalise the denominator and simplify $\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$ 3

8. Simplify $(2x - 3y) \div (\sqrt{2x} + \sqrt{3y})$ 2

9. Solve for x: $x = \sqrt{a^2 + x\sqrt{b^2 + x^2}} - a$ 3

SECTION 2 (Inequalities) Start a new page.

1. Solve $\frac{1}{(x-1)} < 1$ 2

2. Solve $\frac{1}{x^2} \geq \frac{1}{x}$ 2

3. Solve $2 - \frac{1}{x} \leq x$ 2

4. Solve $\frac{1}{|x-4|} > 1$ 3

5. Solve $\frac{2x}{(1-x)} \geq x$ 3

SECTION 3 (Counting Techniques) Start a new page.

1. From six girls and four boys, a committee of 3 girls and 2 boys is to be chosen. How many different committees can be formed? 1

2. How many arrangements of the letters of the word ***AUSTRALIA*** are possible? 1

3. A school swim squad consists of six girls and four boys. A team consisting of four of these students is to be chosen.
   (i) In how many ways can the team be chosen? 1
   (ii) Among the squad are a brother and sister. A team of three boys and two girls is chosen at random. If the team must include the brother and sister, how many different teams are possible 2

4. Seven people are sitting around a table.
   (i) How many seating arrangements are possible? 1
   (ii) Two people, Kevin and Julia, do not sit next to each other How many seating arrangements are now possible? 2

5. Find the number of ways in which 3 boys and 3 girls can be arranged in a straight line so that the girls are all next to each other, but the boys are not all next to each other. 2

6. Ben has ten music videos saved in a folder on his laptop, which he can arrange to play in any order. Of these videos, three are by Beyonce, two are by Metallica, and the other five are all by **different** artists.

   i) How many arrangements are there of the ten videos? 1

   ii) Ben decides that when he plays the ten videos, those that are by the same artist will play together, in any order. How many arrangements of ten videos are now possible? 1

   iii) Ben then decides that he only has time to play 5 videos. The five will start with one by Beyonce and end with one by Metallica with three others by **different** artists between(ie not Beyonce or Metallica). How many arrangements of five videos are possible? 1

7. By using factorial notation, prove that ${}^{n}C_{r}={}^{n}C_{n-r}$ 2

END OF EXAMINATION

Solutions Assess Task 1 Ext 1 2014

Year 11

Advanced Algebra (Corrected #1)

1. $(x^2-5)(x^2+3)$ over rational

2. $x^2(x-5)-1(x-5) = (x-5)(x^2-1)$
$= (x-5)(x-1)(x+1)$

3. $x^6(x^3-1)-1(x^3-1)$
$= (x^3-1)(x^6-1)$
$= (x-1)(x^2+x+1)(x^3-1)(x^3+1)$
$= (x-1)^2(x^2+x+1)^2(x+1)(x^2-x+1)$

4. $\left(x+\frac{1}{x}\right)^2 - 2 = x^2 + \frac{1}{x^2}$
$16 - 2 = x^2 + \frac{1}{x^2}$
$14 = x^2 + \frac{1}{x^2}$

5. $\frac{2^x(2^3+1)}{3^{x+2}} = \frac{2^x \times 9}{3^{x+2}}$
$= \frac{2^x \times 3^2}{3^{x+2}}$
$= \frac{2^x}{3^x}$
$= \left(\frac{2}{3}\right)^x$

6. $\frac{(3^n-1)(3^n+1)}{(3^n-1)} = 3^{n+1}$

7. $\dfrac{1}{(\sqrt{2}+\sqrt{3})+\sqrt{5}} \times \dfrac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$

$= \dfrac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{(\sqrt{2}+\sqrt{3})^2-5}$

$= \dfrac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2+2\sqrt{6}+3-5}$

$= \dfrac{\sqrt{2}+\sqrt{3}-\sqrt{5}}{2\sqrt{6}} \times \dfrac{\sqrt{6}}{\sqrt{6}}$

$= \dfrac{\sqrt{12}+\sqrt{18}-\sqrt{30}}{12}$

$= \dfrac{2\sqrt{3}+3\sqrt{2}-\sqrt{30}}{12}$

9

8. $x+a = \sqrt{a^2+x\sqrt{b^2+x^2}}$

$(x+a)^2 = a^2+x\sqrt{b^2+x^2}$

$x^2+2ax+a^2 = a^2+x\sqrt{b^2+x^2}$

$x(x+2a) = x\sqrt{b^2+x^2}$

$x^2(x+2a)^2 = x^2(b^2+x^2)$

$x^2(x+2a)^2 - x^2(b^2+x^2) = 0$

$x^2[(x+2a)^2-(b^2+x^2)] = 0$

$\therefore x^2 = 0$ or $(x+2a)^2 = b^2+x^2$

$x = 0$ or $(x+2a)^2 = b^2+x^2$

$x^2+4ax+4a^2 = b^2+x^2$

$4ax = b^2-4a^2$

$x = \dfrac{b^2-4a^2}{4a}$

8. $\dfrac{2x-3y}{\sqrt{2x}+\sqrt{3y}} \times \dfrac{\sqrt{2x}-\sqrt{3y}}{\sqrt{2x}-\sqrt{3y}}$

$$= \frac{(2x-3y)(\sqrt{2x}-\sqrt{3y})}{(2x-3y)}$$

$$= \sqrt{2x}-\sqrt{3y}$$

Section 2 (Inequalities)

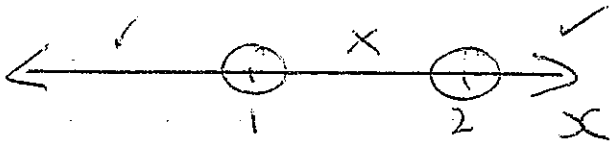
1. $\dfrac{1}{(x-1)} < 1$

CP at $x=1$, $x \neq 1$

Make equation

$1 = x-1$

$2 = x$



$\therefore \quad 1 < x < 2$

2. $\dfrac{1}{x^2} \geq \dfrac{1}{x}$

CP at $x=0$, $x \neq 0$

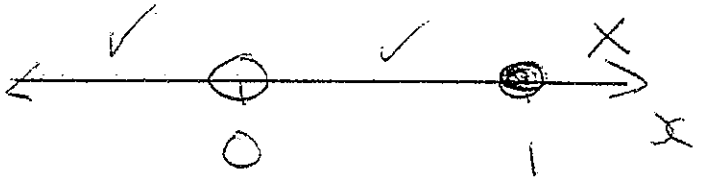
Make equation

$x^2 = x$

$x(x-1) = 0$

$x = 0$ or $x = 1$

but $x \neq 0$



$\therefore \quad x \leq 1, \; x \neq 0$

3. $2 - \frac{1}{x} \leq x$

$\frac{2x-1}{x} \leq x$

CP at $x=0$, $x \neq 0$

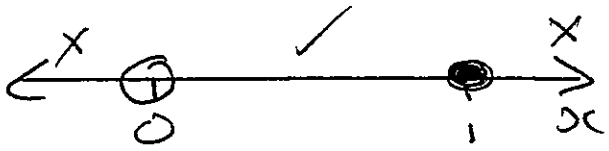
Make equation

$2x-1 = x^2$

$0 = x^2 - 2x + 1$

$0 = (x-1)^2$

$\therefore x = 1$



$\therefore 0 < x \leq 1$

4. $\frac{1}{|x-4|} > 1$

CP at $x=4$, $x \neq 4$

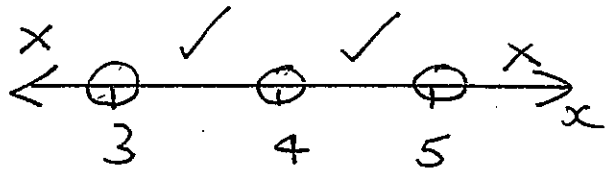
Make equation

$1 = \pm(x-4)$

$x-4=1$ or $-x+4=1$

$x = 5$ $\quad 3 = x$

But $x \neq 5$, $x \neq 3$



$\therefore 3 < x < 5$, $x \neq 4$

5. $\frac{2x}{(1-x)} \geq x$

CP at $x=1$, $x \neq 1$

Make equation

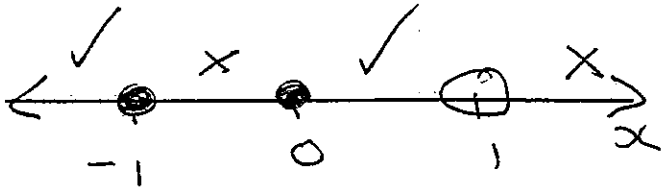
$2x = x - x^2$

$0 = -x - x^2$

$x^2 + x = 0$

$x(x+1) = 0$

$x = 0$ or $x = -1$



$0 \leq x < 1$ OR $x \leq -1$

Section 3 Counting Techniques

1/ ${}^6C_3 \times {}^4C_2 = 120$

2/ $\square\ \boxed{9}\ \boxed{9}\ \boxed{9}\ \square\ \square$ $\qquad 2 \times 3! \times 3! = 72$

3/ $\dfrac{9!}{3!} = 60480$

4/ (i) $4!$

(ii) ${}^5C_1 \times {}^3C_2 = 5 \times 3$

$= 15$

5/ (i) $6! = 720$

(ii) Sit together is $2! \times 5! = 240$

$\therefore$ apart is $720 - 240 = 480$

6/ (i) $10! = 3628800$

(ii) $7!\,3!\,2!$

(iii) $\boxed{3} \times \boxed{5 \times 4 \times 3} \times \boxed{2} = 360$

7/ ${}^nC_r = \dfrac{n!}{(n-r)!\,r!}$

$\therefore\ {}^nC_r = {}^nC_{n-r}$