Question 1   (12 Marks)

(a)   Simplify $2\sqrt{8} - \sqrt{18}$.

(b)   If $\dfrac{\sqrt{3}-4}{2+\sqrt{3}} = a + b\sqrt{3}$, find a and b.

(c)   Simplify $\dfrac{x^2 - 2x - 3}{x^2 - 4x - 5} \times \dfrac{x^2 - 25}{(x-3)(x+5)}$.

(d)   Find the simultaneous solution to the equations: $2x = 3y + 1$ and $xy + x + y = 23$.

Question 2   (12 Marks)   *Start a new page*

(a)   Factorise
        (i) $y^3 - 125$.
        (ii) $1 + x + x^2 + x^3$.

(b)   Solve
        (i) $2x^2 = 7x - 5$.
        (ii) $\dfrac{5x+3}{x+4} > 2$.
        (iii) $|4x+3| \geq 1$.

(c)   Given $f(x) = 2(x^3 - x)$
        (i)   Find $f(-x)$.
        (ii)   Show whether $f(x)$ is odd, even or neither.

Question 3   (12 Marks)   *Start a new page*

(a)   Draw a neat sketch of each of the graphs below.
        (i) $y = \dfrac{1}{x}$.
        (ii) $y = \dfrac{1}{x-3}$.
        (iii) $y = 4 + \dfrac{1}{x-3}$.

(b)   •(i)   Find the domain and range for the function $f(x) = \sqrt{9 - x^2}$.
        (iii)   Sketch the region indicated by $y \geq \sqrt{9 - x^2}$.

(c)   Solve $|x+1| - |2x-1| < 0$.

End of Paper

Question 1

(a) $2\sqrt{8} - \sqrt{18}$

$= 4\sqrt{2} - 3\sqrt{2}$

$= \sqrt{2}$

(b) $\dfrac{\sqrt{3}-4}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}}$

$= \dfrac{(\sqrt{3}-4)(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}$

$= \dfrac{\sqrt{3}-8-3+4\sqrt{3}}{4-3}$

$= 6\sqrt{3}-11 \qquad = -11 + 6\sqrt{3}$

$\therefore a = -11 \quad , \quad b = 6$

(c) $\dfrac{x^2 - 2x - 3}{x^2 - 4x - 5} \times \dfrac{x^2 - 25}{(x-3)(x+5)}$

$= \dfrac{(x-3)(x+1)}{(x-5)(x+1)} \times \dfrac{(x-5)(x+5)}{(x-3)(x+5)}$

$= 1$

d) $2x = 3y+1 \quad\text{——}\;①$

$\quad xy + x + y = 23 \text{——}②$

__rearrange ①__

$\quad x = \dfrac{3y+1}{2} \quad\text{——}\;③$

__sub ③ in ②__

$\left(\dfrac{3y+1}{2}\right) \times y + \left(\dfrac{3y+1}{2}\right) + y = 23$

$y(3y+1) + (3y+1) + 2y = 46$

$3y^2 + y + 3y + 1 + 2y = 46$

$3y^2 + 6y - 45 = 0$

$y^2 + 2y - 15 = 0$

$(y+5)(y-3) = 0$

(d) ∴ $y = -5$ or 3

 Sub y in ①

when $y = -5$ $2x = -15 + 1$

$2x = -14$

∴ $x = -7$

when $y = 3$ $2x = 9 + 1$

$2x = 10$

∴ $x = 5$

Check in ②

○ $-7 \times -5 + -7 + 5 = 23$ ✓

$5 \times 3 + 5 + 3 = 23$ ✓

∴ the solutions are when $x = -7, y = -5$

and $x = 5, y = 3$

Question 2

(a) (i) $y^3 - 125 = (y)^3 - (5)^3$

$= (y - 5)(y^2 + 5y + 25)$

(ii) $1 + x + x^2 + x^3$

$= 1(1 + x) + x^2(1 + x)$

$= (1 + x)(1 + x^2)$

(b)(i) Solve $2x^2 = 7x - 5$

$2x^2 - 7x + 5 = 0$

$2x^2 - 2x - 5x + 5 = 0$

$2x(x - 1) - 5(x - 1) = 0$

$(x - 1)(2x - 5) = 0$

∴ $x = 1$ or $\dfrac{5}{2}$

$\underline{\quad p \mid s \quad}$

$a \times b = 10 \mid a + b = -7$

$a = -5 \quad b = -2$

(ii) $\dfrac{5x + 3}{x + 4} > 2$

$(x + 4)^2 \times \dfrac{5x + 3}{x + 4} > 2(x + 4)^2$

$(x + 4)(5x + 3) > 2(x + 4)^2$

Question 2 continued
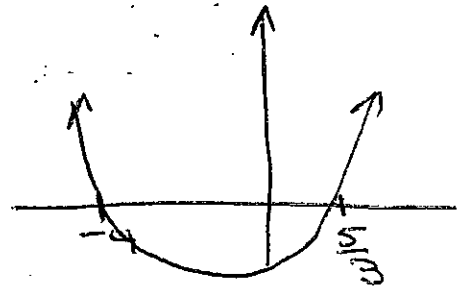
$$(x+4)(5x+3) - 2(x+4)^2 > 0$$
$$(x+4)((5x+3) - 2(x+4)) > 0$$
$$(x+4)(5x+3-2x-8) > 0$$
$$(x+4)(3x-5) > 0$$

$$x = -4, \frac{5}{3}$$

$$\therefore \underline{x < -4 \text{ or } x > \frac{5}{3}}$$

(ii) $|4x + 3| \geq 1$

$4x+3 \geq 1$	or	$4x + 3 \leq -1$
$4x \geq -2$	or	$4x \leq -4$
$\therefore x \geq -\frac{1}{2}$	or	$x \leq -1$

(c) $f(x) = 2(x^3 - x)$

(i) $f(-x) = 2((-x)^3 - (-x))$
$$= 2(-x^3 + x)$$

(ii) $f(-x) = 2(-x^3 + x)$
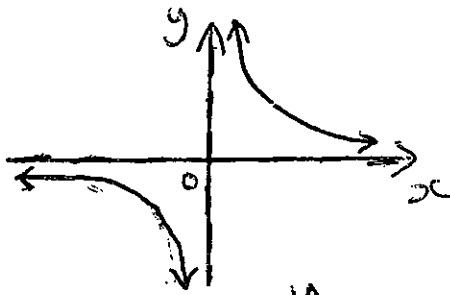$$= -2(x^3 - x)$$
$$-f(x) = -1(2(x^3-x))$$
$$= -2(x^3-x)$$
$\therefore f(-x) = -f(x)$
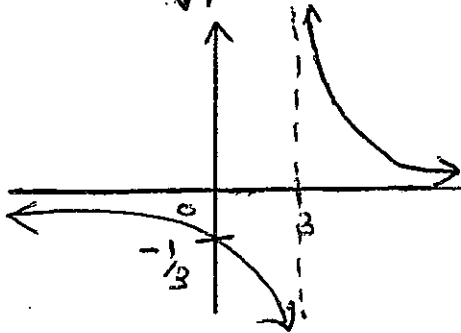$\therefore f(x)$ is an odd function.
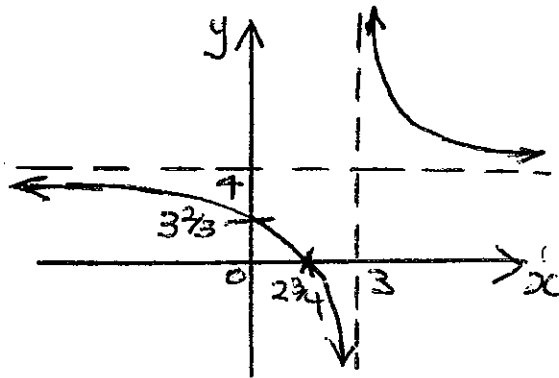
Question 3

a) (I) $y = \dfrac{1}{x}$

(II) $y = \dfrac{1}{x-3}$

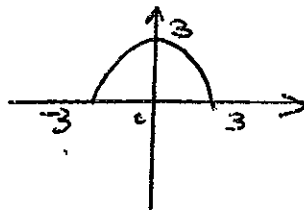(III) $y = 4 + \dfrac{1}{x-3}$

(b) $f(x) = \sqrt{9-x^2}$
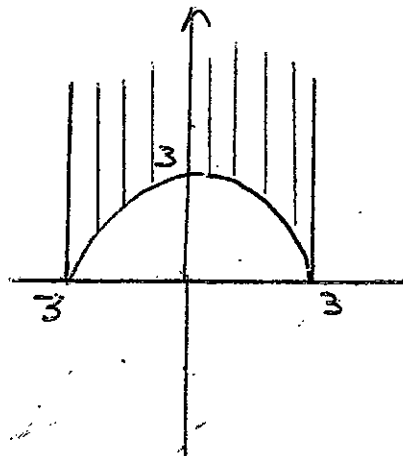
(I)

Condition $9 - x^2 \geq 0$

$(3-x)(3+x) \geq 0$

∴ Domain $: -3 \leq x \leq 3$
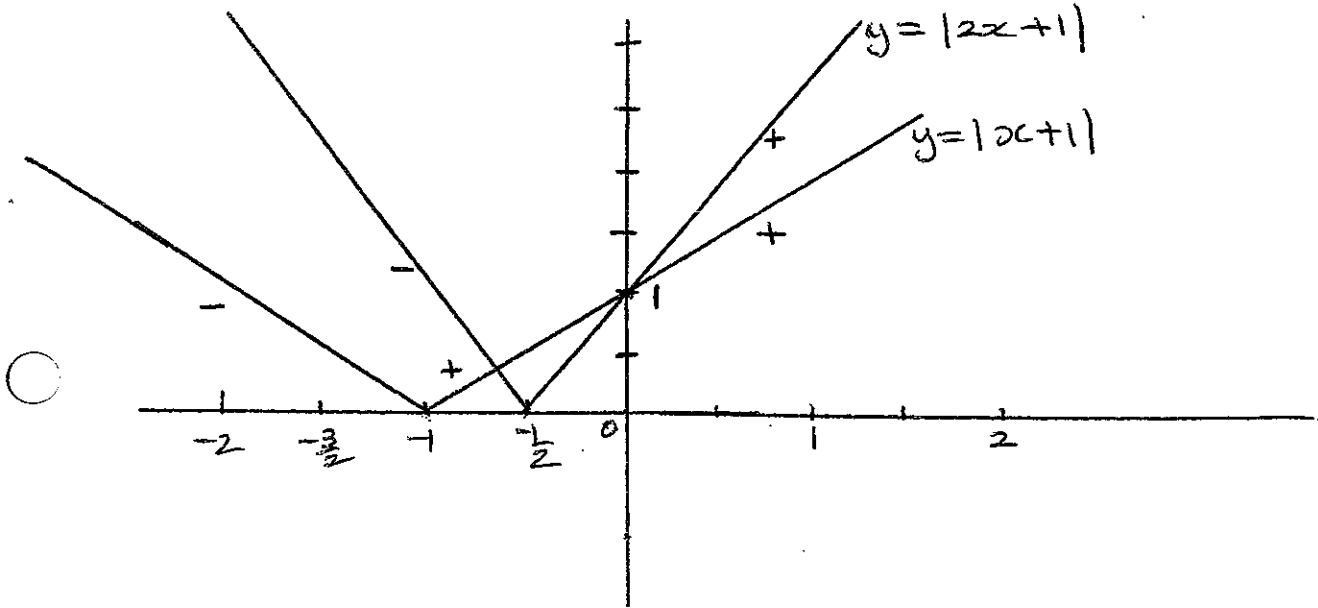
Range: $0 \leq y \leq 3$

(II) $y \geq \sqrt{9-x^2}$

Question 3 continued

(c) $\quad |x+1| - |2x-1| < 0$

$\quad\quad |x+1| < |2x+1|$

graph $\quad y = |x+1| \quad$ and $\quad y = |2x+1|$

points of intersection

$\quad x+1 = 2x+1 \quad\quad\quad\quad\quad x+1 = -(2x+1)$

$\quad\quad 0 = x \quad\quad\quad\quad\quad\quad\quad x+1 = -2x-1$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad 3x = -2$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad x = \frac{-2}{3}$

∴ the solution is

$\quad\quad x > 1 \quad$ or $\quad x < \frac{-2}{3}$