Question 1 *10 marks* **Marks**

a) Factorise i) x^3+8 **3**

ii) $5x^2+5x-60$

b) Simplify and express with a rational denominator $\dfrac{1}{2+\sqrt{3}}-\dfrac{1}{\sqrt{2}+1}$ **3**

c) Find in simplest exact form, values of x for which $2x^2-4x-7=0$ **2**

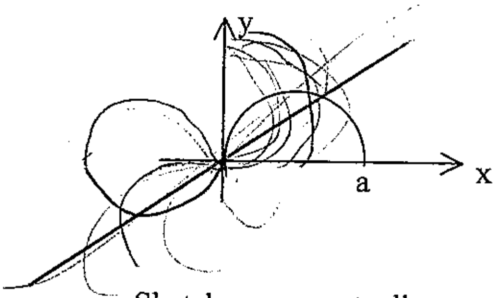
d) Solve for x and graph on a number line: $|2x-1|>5$. **2**

Question 2 *10 Marks* **Start this Question on a new page.**

a) Sketch the region $y>\sqrt{x-1}$. **3**

b) Show that $\dfrac{x-2}{y}+\dfrac{y+4}{x}=0$ is the equation of a circle with centre (1,-2) and radius $\sqrt{5}$. **3**

c) If the graph of $y=f(x)$ is as follows; **2**



Sketch on separate diagrams i) $f^{-1}(x)$

ii) $f(-x)$

d) Solve for x: $2x^2+5x-3\le 0$ **2**

Question 3 *10 Marks* **Start this Question on a new page.** **Marks**

a) Show that $\frac{x+2}{x-1}=1+\frac{3}{x-1}$. Hence or otherwise sketch $y=\frac{x+2}{x-1}$. **3**

b) i) Sketch $y=\log_2 x$ **3**
ii) State whether it is a function or relation and give reasons.
iii) State the domain.

c) Solve for x: $\frac{4x}{x-1}\geq 1$. **4**

Question 4 *10 Marks* **Start this Question on a new page.**

a) For the function $y=\frac{1}{x^2-1}$: **6**

i) State the vertical asymptotes.

ii) By considering $\lim_{x\to\pm\infty}\frac{1}{x^2-1}$ determine any horizontal asymptotes.

iii) Show that $y=\frac{1}{x^2-1}$ is an even function.

iv) Find any intercepts.

v) Graph $y=\frac{1}{x^2-1}$, showing these features.

b) Sketch , indicating any asymptotes and intercepts $y=\frac{|x-1|}{x}$ **4**

End of paper.

Yr 11 Ext 1 Assessment 1 2004

① a) (i) $x^3 + 8 = \underline{(x+2)(x^2 - 2x + 4)}$ SUM OF 2 CUBES.

(ii) $5x^2 + 5x - 60 = 5(x^2 + x - 12)$

$= \underline{5(x+4)(x-3)}$

b) $\dfrac{1}{2+\sqrt{3}} - \dfrac{1}{\sqrt{2}+1} = \dfrac{2-\sqrt{3}}{4-3} - \dfrac{\sqrt{2}-1}{2-1}$

$= 2 - \sqrt{3} - (\sqrt{2} - 1) = \underline{3 - \sqrt{3} - \sqrt{2}}$

c) $2x^2 - 4x - 7 = 0$. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

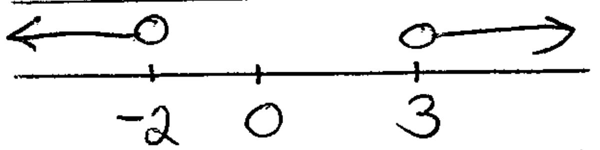
$x = \dfrac{4 \pm \sqrt{16 + 56}}{4} = \dfrac{4 \pm \sqrt{72}}{4} = \dfrac{4 \pm 2\sqrt{18}}{4} = \dfrac{4 \pm 6\sqrt{2}}{4}$

$= \underline{1 \pm \dfrac{3\sqrt{2}}{2}}$

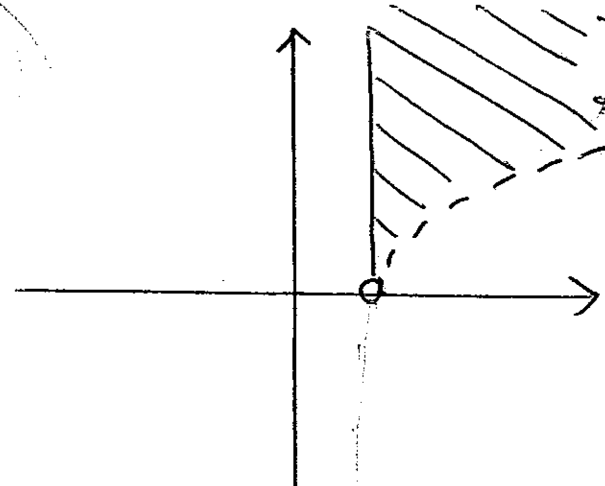
d). $|2x - 1| > 5$ ∴ $2x - 1 > 5$ or $2x - 1 < -5$

$2x > 6$ $2x < -4$

$\underline{x > 3}$ or $\underline{x < -2}$



② a) $y > \sqrt{x-1}$



Domain $x > 1$

Root Curve

$(0,0) \to (1,0)$

b) $\dfrac{x-2}{y} + \dfrac{y+4}{x} = 0$

$(\times x)$
$(\times y)$

$x(x-2) + y(y+4) = 0$

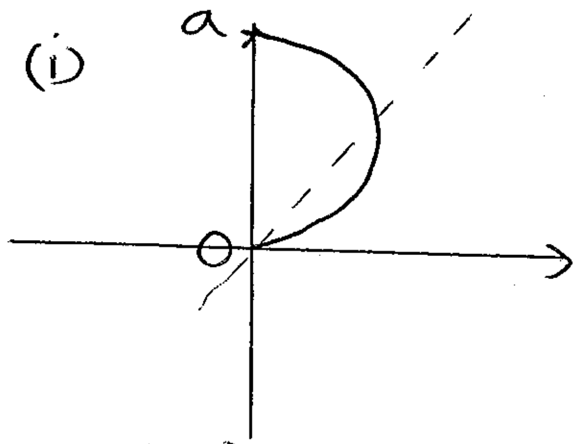
$x^2 - 2x + y^2 + 4y = 0$

$(x-1)^2 - 1 + (y+2)^2 - 4 = 0$

$(x-1)^2 + (y+2)^2 = 5.$

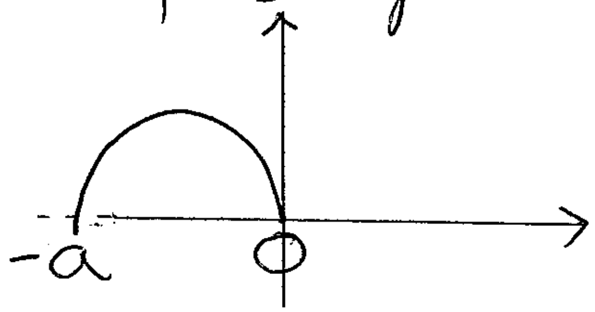
$\therefore$ Circle centre $(1,-2)$ radius $\sqrt{5}$.

c) (i)



$f^{-1}(x)$ is inverse
reflect in $y=x$
(NB Not an inverse function!)

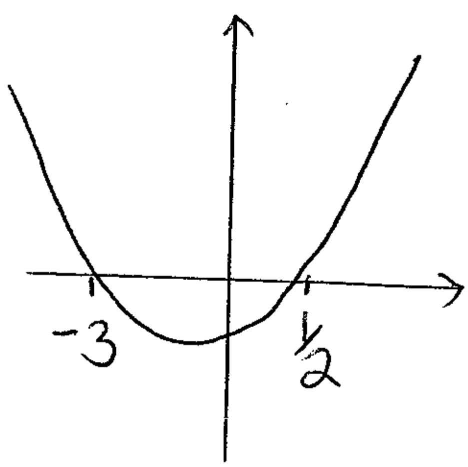
(ii) $f(-x)$ reflect in y axis $x \to -x$



d). $2x^2 + 5x - 3 \leqslant 0$

$(2x-1)(x+3) \leqslant 0$

$\therefore -3 \leqslant x \leqslant \frac{1}{2}$



③ LHS $= \dfrac{x+2}{x-1} = \dfrac{x-1+3}{x-1} = \dfrac{x-1}{x-1} + \dfrac{3}{x-1} = 1 + \dfrac{3}{x-1} =$ RHS

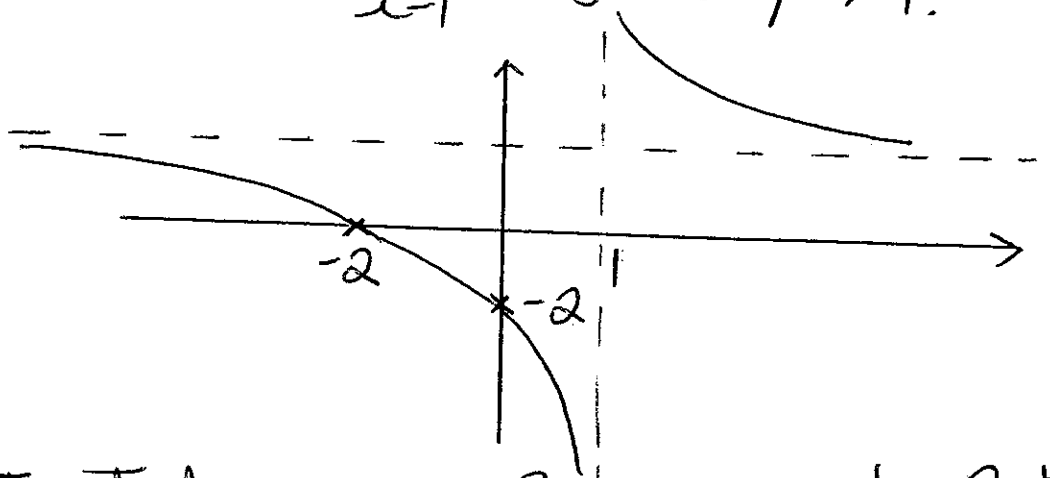
$$y = \frac{x+2}{x-1} = 1 + \frac{3}{x-1}.$$

Domain $x \in \mathbb{R}$ $x \neq 1$.

Intercepts $x = 0$ $y = \frac{2}{-1} = -2$ $(0,-2)$

$y = 0$ $x+2=0$ $x=-2$ $(-2,0)$

As $x \to \pm\infty$ $\frac{3}{x-1} \to 0$ $\therefore y \to 1$.



{ Alternatively $y - 1 = \dfrac{3}{x-1}$.

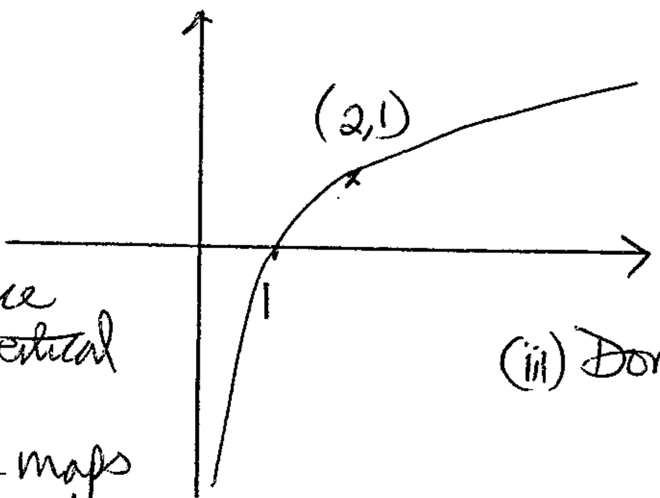
$y = \frac{1}{x}$ Rect Hyp.

$y = \frac{3}{x}$ Stretch // x

$y = \frac{3}{x-1}$ $(0,0) \to (1,0)$

$y - 1 = \frac{3}{x-1}$ $(1,0) \to (1,1)$ }

b).(i) $y = \log_2 x$



(ii) This is a function since it passes a vertical line test. Each x value maps to a unique y value

(iii) Domain $x \in \mathbb{R}$ $x > 0$

c) $\dfrac{4x}{x-1} \geq 1 \quad (x \neq 0)$

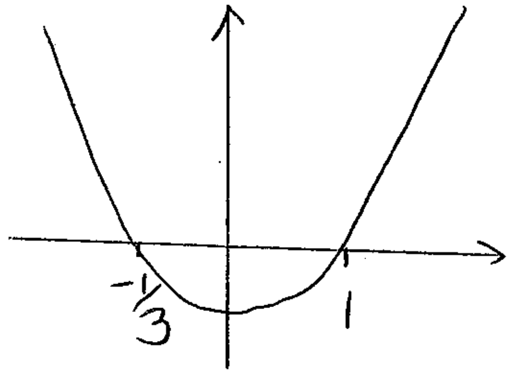
$\times(x-1)^2 \quad \times(x-1)^2$

$4x(x-1) \geq (x-1)^2$

$4x^2 - 4x \geq x^2 - 2x + 1$

$3x^2 - 2x - 1 \geq 0$

$(3x+1)(x-1) \geq 0$



$\underline{x \leq -\frac{1}{3}}$ OR $x \geq 1$ But $x \neq 1 \quad \therefore \quad \underline{x > 1}$

4 a) $y = \dfrac{1}{x^2-1} = \dfrac{1}{(x+1)(x-1)}$

i) Vertical Asymptotes $\Rightarrow$ Denominator Zero $x = \pm 1$

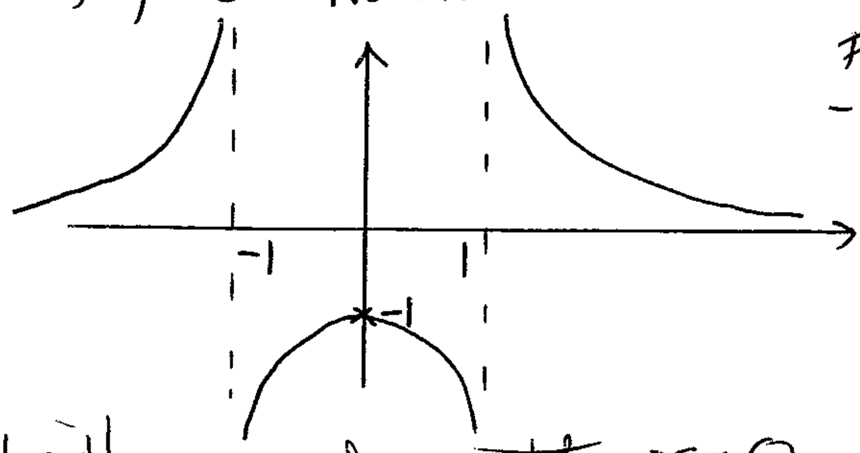
ii) $\displaystyle\lim_{x \to \pm\infty} \left(\frac{1}{x^2-1}\right) = 0 \quad \therefore$ Horizontal Asymptote $y = 0$

iii) $f(x) = \dfrac{1}{x^2-1} \qquad f(-x) = \dfrac{1}{(-x)^2-1} = \dfrac{1}{x^2-1} = f(x) \quad \therefore$ Even Fn

iv) y-int, $x = 0 \quad y = \frac{1}{-1} \qquad (0, -1)$

x-int, $y = 0 \quad$ Not Possible

v)



For $x > 1 \quad y > 0$

$-1 < x < 1 \quad y < 0$

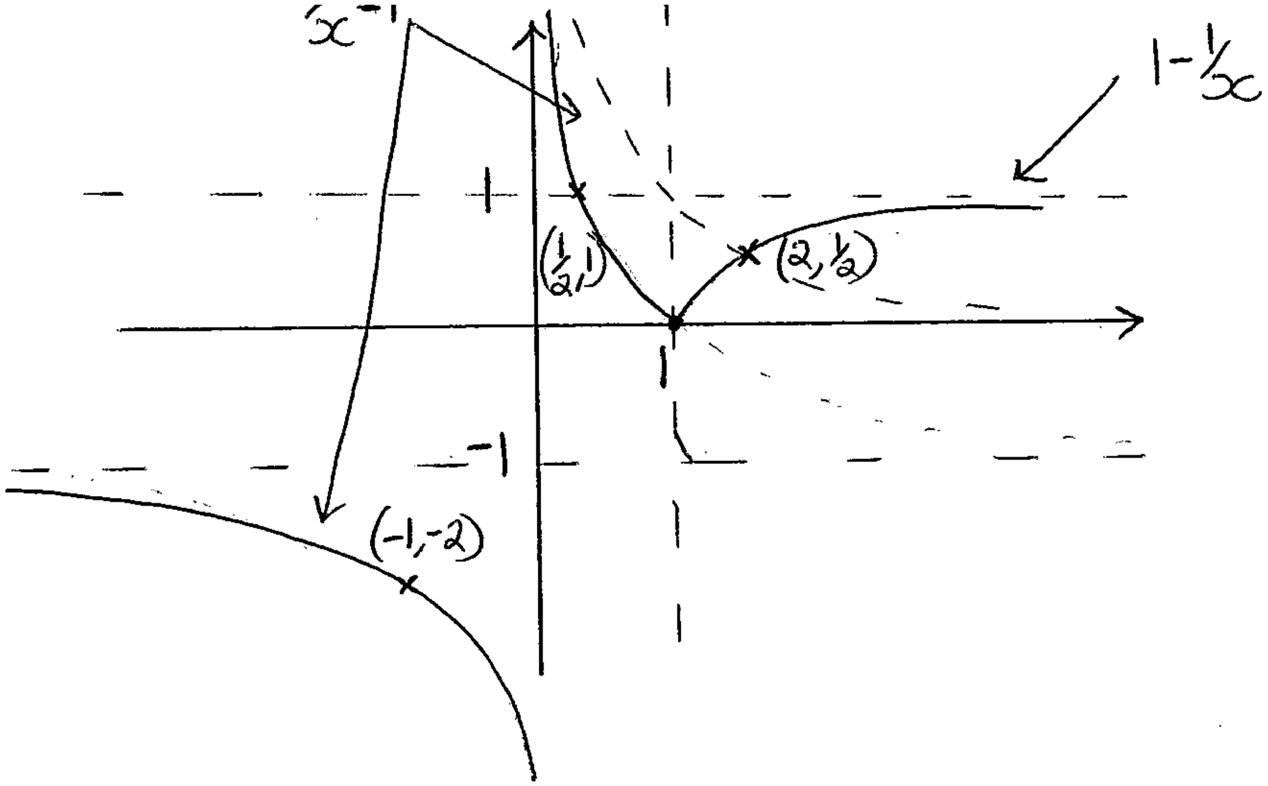
$x < -1 \quad y > 0$

b) $y = \dfrac{|x-1|}{x}$ Vertical asymptote $x = 0 \quad \therefore$ No y intercept

Domain $x > 1 \quad y = \dfrac{x-1}{x} = 1 - \frac{1}{x} \quad$ as $x \to +\infty \quad y \to 1^-$

$x < 1 \; (x \neq 0) \quad y = \dfrac{-(x-1)}{x} = \dfrac{1-x}{x} = \frac{1}{x} - 1 \quad$ as $x \to -\infty \quad y \to (-1)^-$

$y = 0 \Rightarrow x = 1 \qquad x$-intercept $(1, 0)$



x^{-1}
$1-\frac{1}{x}$
1
-1
1
$(\frac{1}{2}, 1)$
$(2, \frac{1}{2})$
$(-1, -2)$

Y11 Ext 1 Ass 1 2004

Question 1

a) i) $(x+2)(x^2-2x+4)$ ✓

ii) $5(x^2+x-12) = 5(x+4)(x-3)$ ✓

b) $\frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} - \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$ ✓

$= \frac{2-\sqrt{3}}{4-3} - \frac{\sqrt{2}-1}{2-1}$ ✓

$= 2-\sqrt{3}-\sqrt{2}+1 = 3-\sqrt{3}-\sqrt{2}$ ✓

c) $x = \frac{+4 \pm \sqrt{(-4)^2-4\times2\times-7}}{2\times2}$ ✓

$= \frac{+4 \pm \sqrt{16+56}}{4}$

$= \frac{+4 \pm \sqrt{72}}{4}$

$= \frac{+4 \pm 6\sqrt{2}}{4} = \frac{+2 \pm 3\sqrt{2}}{2}$ ✓

d) $|2x-1| > 5$

$\pm(2x-1) > 5$

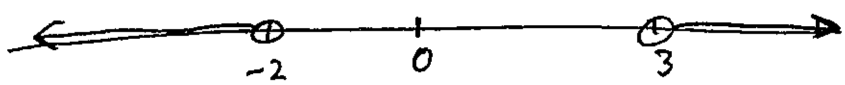
$2x-1 > 5$ $\quad\quad$ $-(2x-1) > 5$

$\quad\quad\quad\quad\quad\quad$ $2x-1 < -5$

$2x > 6$ $\quad\quad$ $2x < -4$

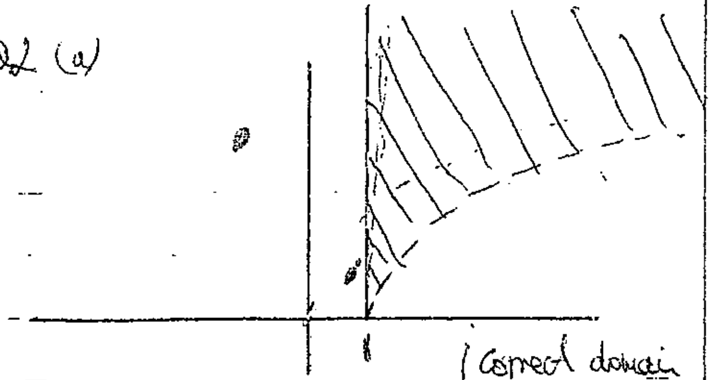
$x > 3$ $\quad\quad$ $x < -2$ ✓

$x > 3$ or $x < -2$ ✓



4U Ext 1 T1 2004 Solutions

Q2 (a)



1 correct domain
1 correct graph
1 broken line (3)

(b) $(x \neq 0 \quad y \neq 0)$.

$$\frac{x-2}{y} + \frac{y+4}{x} = 0 \quad (x \times y)$$

$x(x-2) + y(y+4) = 0$ ✓

$x^2 - 2x + y^2 + 4y = 0$

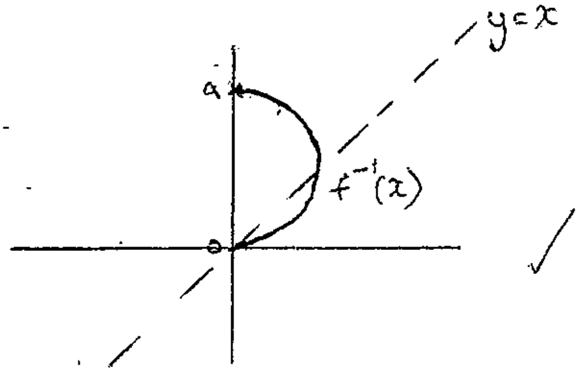
(complete the square for x and y) ✓

$x^2 - 2x + 1 + y^2 + 4y + 4 = 5$

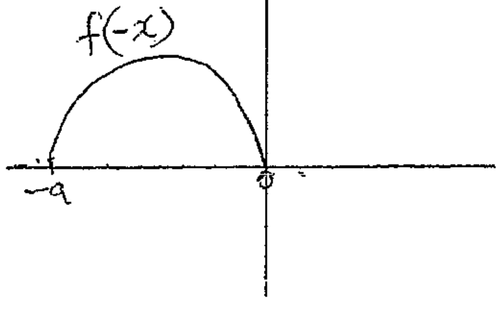
$(x-1)^2 + (y+2)^2 = (\sqrt{5})^2$ ✓

$\therefore$ this is a circle with centre $(1,-2)$ and radius $\sqrt{5}$. (3)

(c)



✓

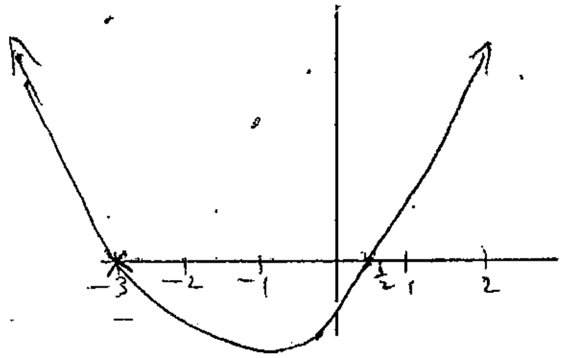


✓

(2)

(d) $2x^2 + 5x - 3 \leq 0$

$(2x-1)(x+3) \leq 0$ ✓



$\therefore -3 \leq x \leq \frac{1}{2}$ ✓ (2)

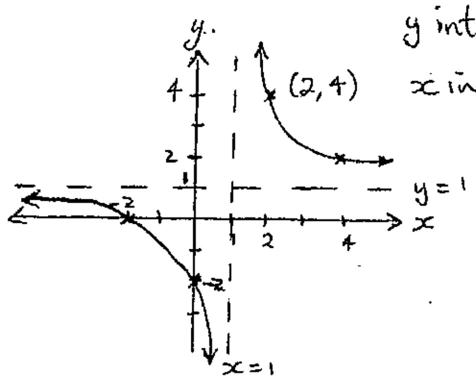
11Ext1 Assess 1 2004.

Question 3

a) $\frac{x+2}{x-1} = \frac{x+1+3}{x-1}$

$= 1 + \frac{3}{x-1}$ ✓(working shown)

∴ hyperbola asymp: $y=1, x=1$



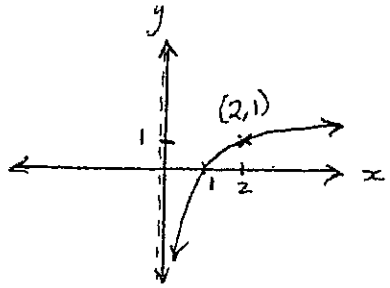
y int $(0,-2)$

x int $(-2,0)$

✓(asympt.) 1mk Shape + one asymptote

✓(intercepts) 2mk All asympt + intercepts

b) i/



✓

ii/ Function - since every relevant x-value produces only one y-value. ✓(must have reason)

- vertical line test holds.

iii/ Domain: $x > 0$ ✓

c) $\frac{4x}{x-1} \times (x-1)^2 \geq 1 \times (x-1)^2$ ✓ multiply by $(x-1)^2$

$4x(x-1) \geq (x-1)^2$

$4x(x-1) - (x-1)^2 \geq 0$

$(x-1)(4x-(x-1)) \geq 0$

$(x-1)(3x+1) \geq 0$ ✓ quadratic inequality.

$-\frac{1}{3}$ 1

∴ $x \leq -\frac{1}{3}$ or $x \geq 1$ ✓ solution of inequality.

But $x \neq 1$

∴ $x \leq -\frac{1}{3}$ or $x > 1$ ✓(restriction on domain)

NB: 1 mk only given for correct solution of linear inequality (i.e failure to multiply by square)

QUESTION FOUR

a) $y = \dfrac{1}{x^2-1} = \dfrac{1}{(x-1)(x+1)}$

(i) vertical asymptotes: $x = \pm 1$

(ii) $\lim\limits_{x \to \pm\infty} \dfrac{1}{x^2-1} = 0$

$\therefore$ horizontal asymptote: $y = 0$

Asymptotes are lines – they have equations. One mark deducted if both are not written as eqns. and a mark hasn't already been lost

(iii) If $f(x) = \dfrac{1}{x^2-1}$ then

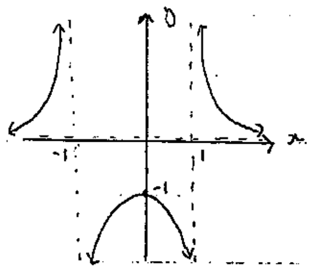
$f(-x) = \dfrac{1}{(-x)^2-1} = \dfrac{1}{x^2-1} = f(x)$

$\therefore$ function is even.

(iv) When $x = 0$, $y = -1$

NO x-intercepts.

(v)



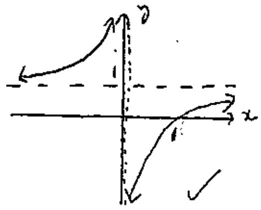
b) $|x-1| = x-1$ when $x \geq 1$

$|x-1| = 1-x$ when $x < 1$

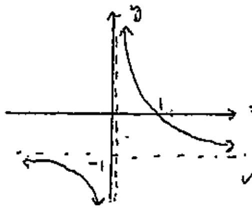
when $x \geq 1$ $\quad y = \dfrac{|x-1|}{x} = \dfrac{x-1}{x} = 1 - \dfrac{1}{x}$

when $x < 1$ $\quad y = \dfrac{|x-1|}{x} = \dfrac{1-x}{x} = \dfrac{1}{x} - 1$

$y = 1 - \dfrac{1}{x}$



$y = \dfrac{1}{x} - 1$



$y = \dfrac{|x-1|}{x}$