Question 1 (10 Marks) **Marks**

(a) Factorise fully $4x^3 - 500$. **2**

(b) Convert $0.4\dot{7}\dot{6}$ to a fraction in its simplest terms. **2**

(c) Simplify $\dfrac{6x-4y}{3x-2y}$. **1**

(d) Solve $|2x-3| > 7$ and graph your solution on the number line. **2**

(e) Express $\dfrac{5}{\sqrt{3}-1} - \dfrac{2}{2+\sqrt{3}}$ with a rational denominator. **3**

$\sqrt{3}(\sqrt{3}-1)$

$3-\sqrt{3}$

Question 2 (10 Marks) **START A NEW PAGE**

(a) A circle has the equation $x^2 - 14x + y^2 + 4y + 49 = 0$

(i) Find the co-ordinates of the centre and the length of the radius. **3**

(ii) Hence or otherwise, sketch the circle showing all essential features including intercepts. **1**

(b) Sketch the following showing all the essential features including intercepts and asymptotes.

(i) $y = (x-1)^2 + 3$ **2**

(ii) $y = \dfrac{1}{2x-1} - 2$ **2**

(c) State the domain and range of $y = \dfrac{1}{\sqrt{2x+9}}$. **2**

Question 3 (10 Marks) **START A NEW PAGE**

(a) Solve $\dfrac{x}{x-3} \le 2$. **3**

(b) A rectangle has a perimeter of 40 centimetres and a length of x centimetres.

(i) Show that the area is given by $A = 20x - x^2$. **1**

(ii) Find the length of the rectangle when the area is $96\,cm^2$. **1**

Question 3 continued Page 2.

Question 3 Continued **Marks**

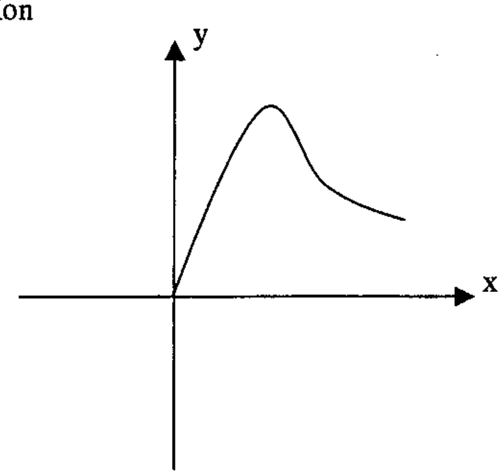
(c) Solve $|x+1| = 2+3x$ **3**

(d) The sketch of f(x) is shown below

Using separate diagrams copy and complete the sketch to make y = f(x)

(i) An odd function **1**

(ii) An even function **1**



Question 4 (10 Marks) **START A NEW PAGE**

(a) Shade the region where $y \le \sqrt{9-x^2}$ and $y > 3x$ hold simultaneously . **3**

(b) Given $f(x) = 3(2^x) - 2(3^x)$, prove that $f(x+1) = 6(2^x - 3^x)$ **2**

(c) Given $f(x) = \dfrac{2x^2}{x^2-16}$

(i) Find the vertical asymptotes. **1**

(ii) Find the horizontal asymptotes. **1**

(iii) Is the f(x) odd, even or neither? Justify your answer. **1**

(iv) Sketch the curve, showing all essential details including intercepts and asymptotes. **2**

END OF PAPER

Newington College Year 11 Ext 1 Mathematics Assess 1 2005

Question 1

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

a) $4x^3 - 500 = 4(x^3 - 125)$

$= 4(x^3 - 5^3)$

$= 4(x-5)(x^2 + 5x + 25)$

b) Let $x = 0.4\dot{7}\dot{6}$

$x = 0.47676\ldots$ ——— ①

$1000x = 476.7676\ldots$ ——— ②

$10x = 4.7676\ldots$ ——— ③

② − ③ $990x = 472$

$x = \frac{472}{990}$

$x = \frac{236}{495}$

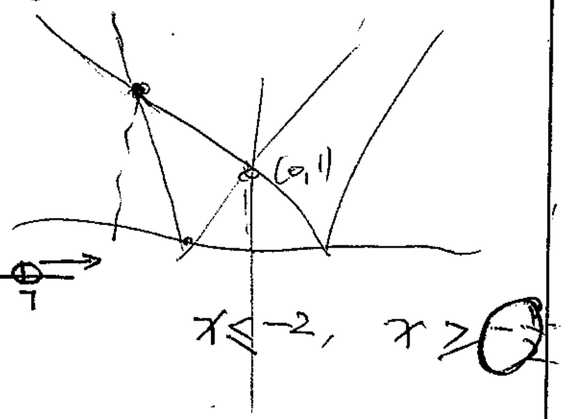
$\therefore 0.4\dot{7}\dot{6} = \frac{236}{495}$

$1 - x = |2x+1|$ $x = 0$

$2x + 1 = 1 - x$

$2x + 1 = x - 1$ $x = -2$

$|x-1| \leq |2x+1|$

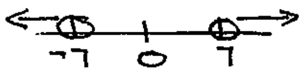


$x \leq -2,\ x \geq 0$

c) $\frac{6x - 4y}{3x - 2y} = \frac{2(3x-2y)}{(3x-2y)}$

$= 2$

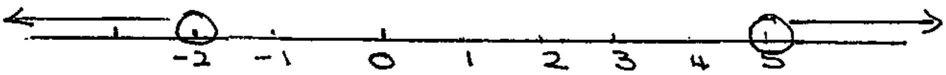
d) $|2x - 3| > 7$



$2x - 3 < -7$ or $2x - 3 > 7$

$2x < -4$ $2x > 10$

$x < -2$ $x > 5$



e) $\dfrac{5}{\sqrt{3}-1} - \dfrac{2}{2+\sqrt{3}}$

$$\frac{5}{\sqrt{3}-1} = \frac{5}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{5(\sqrt{3}+1)}{2}$$

$$\frac{2}{2+\sqrt{3}} = \frac{2}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$= 2(2-\sqrt{3})$$

$$\therefore \frac{5}{\sqrt{3}-1} - \frac{2}{2+\sqrt{3}} = \frac{5(\sqrt{3}+1) - 4(2-\sqrt{3})}{2}$$

$$= \frac{5\sqrt{3}+5-8+4\sqrt{3}}{2}$$

$$= \frac{9\sqrt{3}-3}{2}$$

Question 2

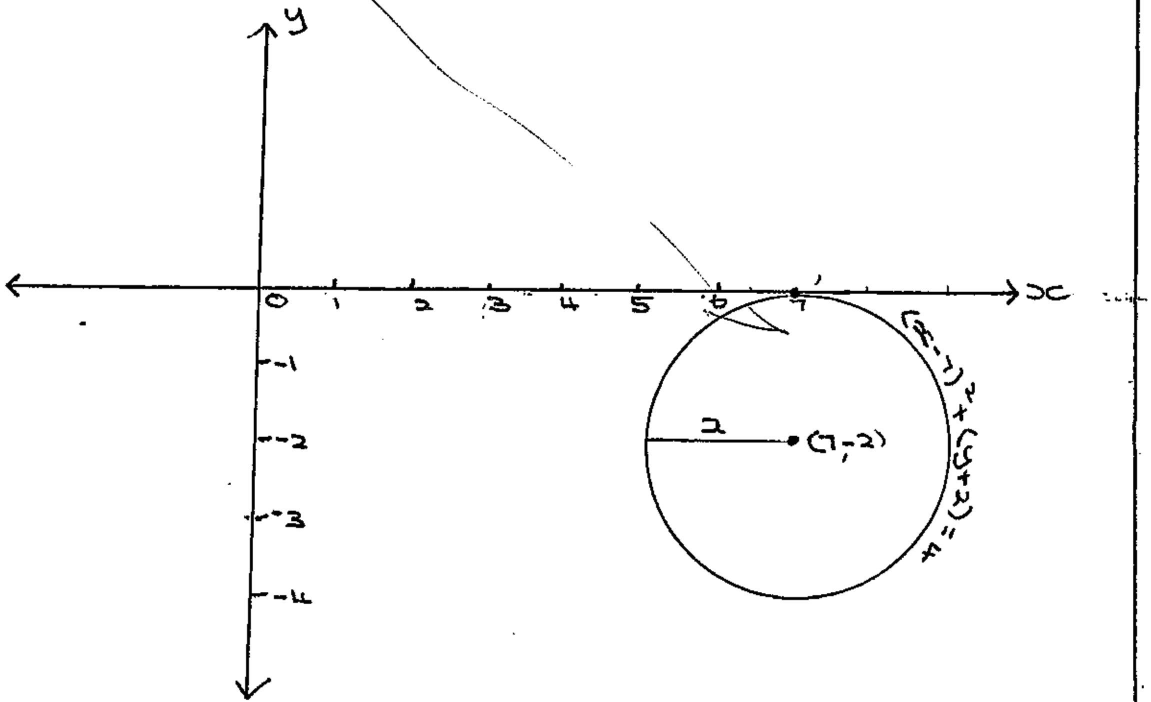
a) $x^2 - 14x + y^2 + 4y + 49 = 0$

$x^2 - 14x + 49 + y^2 + 4y + 4 = 4$

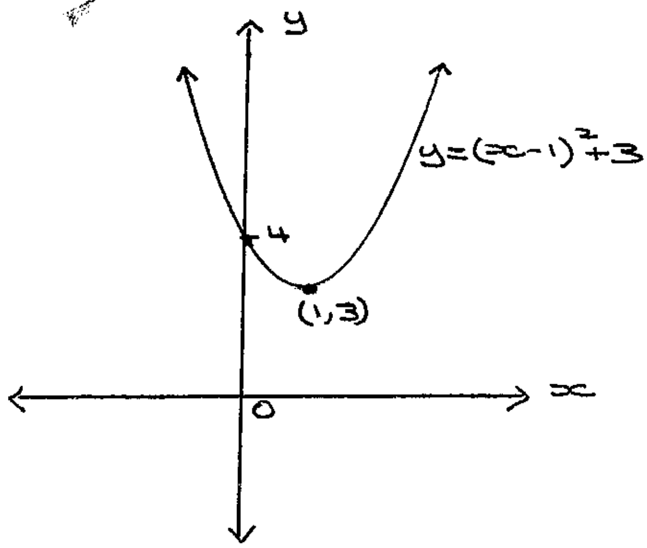
$(x-7)^2 + (y+2)^2 = 2^2$

(i) $\therefore$ The centre is $(7,-2)$

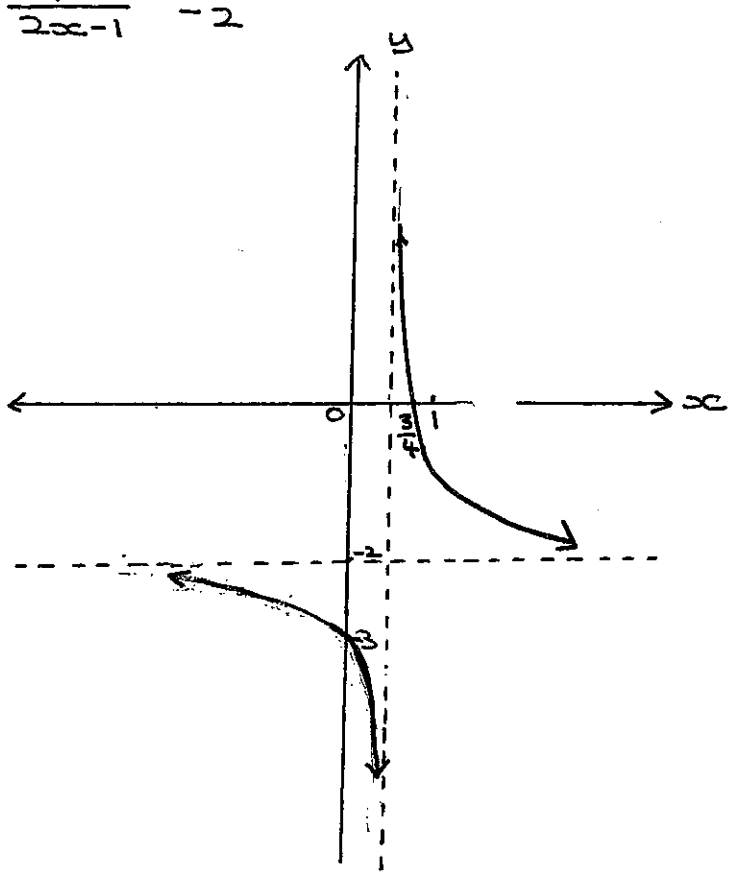
The radius is 2



b) (i) $y = (x-1)^2 + 3$



(ii) $y = \dfrac{1}{2x-1} - 2$



c) $y = \dfrac{1}{\sqrt{2x+9}}$

domain $2x + 9 > 0$

$x > -4\frac{1}{2}$

range $y > 0$

Question 3

a) $\dfrac{x}{x-3} \le 2$ NB $\boxed{x \neq 3}$

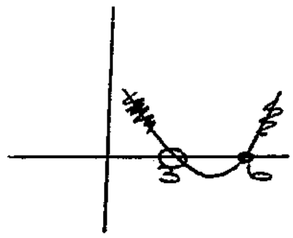
$(x-3)^2 \times \dfrac{x}{(x-3)} \le 2(x-3)^2$

$x(x-3) \le 2(x-3)^2$

$0 \le 2(x-3)^2 - x(x-3)$

$0 \le (x-3)\left[2(x-3)-x\right]$

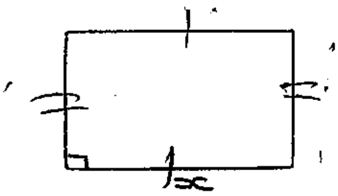
$0 \le (x-3)(x-6)$



$(x-3)(x-6) \geq 0$

$\therefore x < 3 \quad , x \geq 6$

b) Let the length be x



$2L + 2B = P$

$2x + 2B = 40$

$x + B = 20$

$B = 20 - x$

$2\cancel{B} = 40 - 2\cancel{x}$

$B = 20 - x.$

$A = LB$

$A = x(20-x)$ as required

$A = x(20-x)$

$96 = 20x - x^2$

$x^2 - 20x + 96 = 0$

$(x-12)(x-8) = 0$

$x = 8, 12$

$|x| = \pm x.$

$|x+1| = \pm(x+1)$

$\therefore$ length is 12 cm and the breadth is 8 cm

c) $|x+1| = 2+3x$

$x+1 = 2+3x$ or $x+1 = -(2+3x)$

$-2x = 1$

$x = -\frac{1}{2}$

$x+1 = -2-3x$

$4x = -3$

$x = -\frac{3}{4}$

$(x+1) = +(2+3x)$

$-2x = 1$

$x = -\frac{1}{2}$

OR $(x+1) = -(2+3x)$

Check

when $x = -\frac{1}{2}$

$$\begin{aligned} L.HS &= |x+1| \\ &= |-\tfrac{1}{2}+1| \\ &= \tfrac{1}{2} \\ R.HS &= 2+3x \\ &= 2+3(-\tfrac{1}{2}) \\ &= \tfrac{1}{2} \end{aligned}$$

$L.HS = R.HS$

$\therefore x = -\frac{1}{2}$ is a solution

when $x = -\frac{3}{4}$

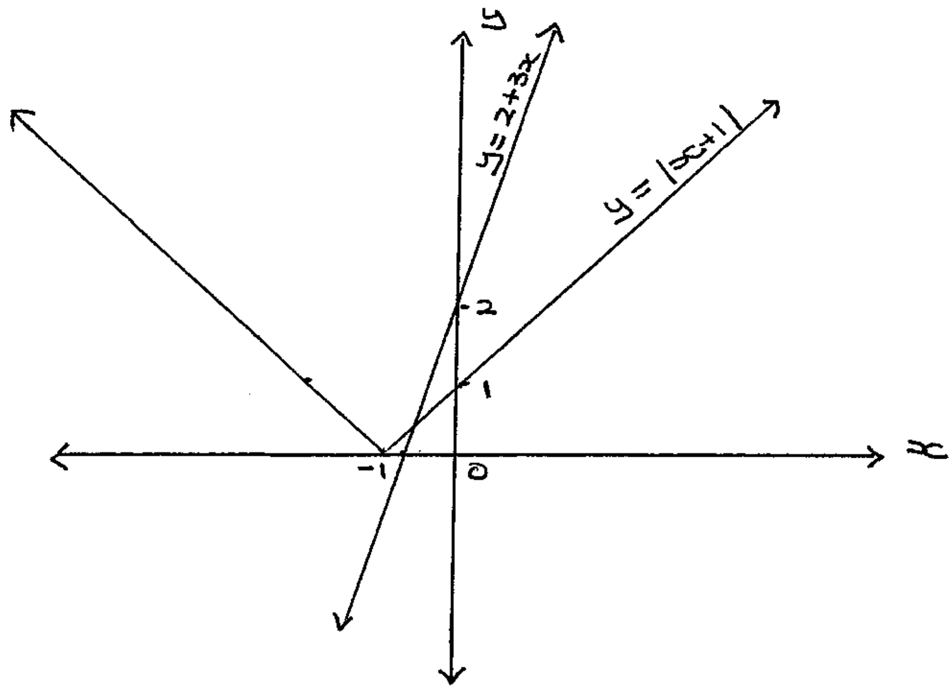
$$\begin{aligned} LHS &= |x+1| \\ &= |-\tfrac{3}{4}+1| \\ &= \tfrac{1}{4} \\ R.HS &= 2+3x \\ &= 2+3(-\tfrac{3}{4}) \\ &= -\tfrac{1}{4} \end{aligned}$$

$LHS \neq R.HS$

$\therefore x = -\frac{3}{4}$ is not a solution

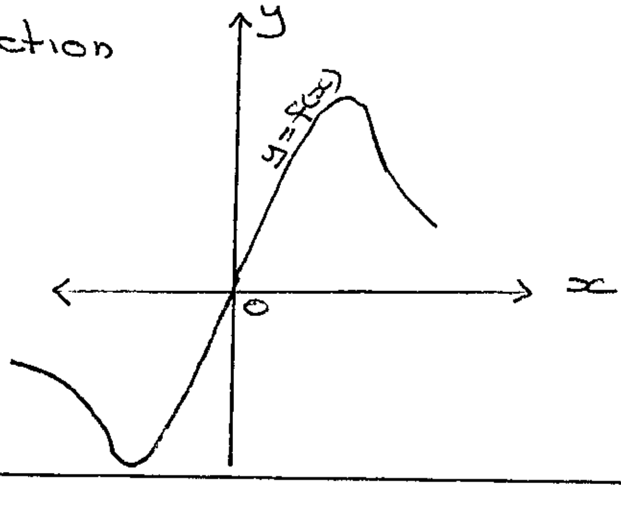
$\therefore x = -\frac{1}{2}$ is the only solution

OR

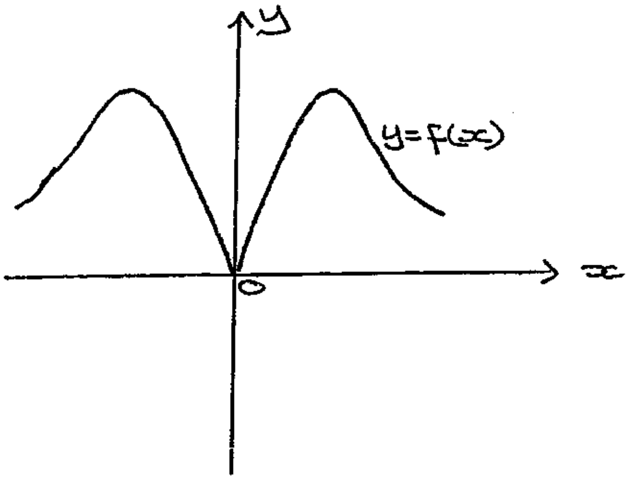


$x = -\frac{1}{2}$

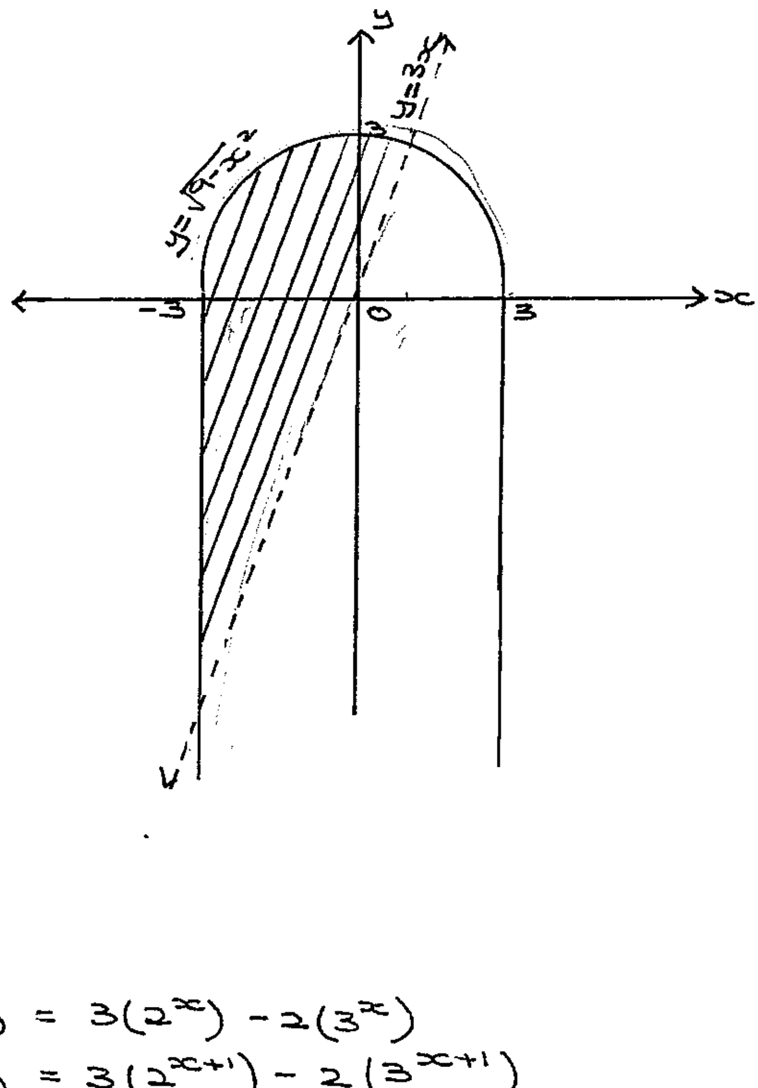
d) odd function



ii) even function



Question 4



b) $f(x) = 3(2^x) - 2(3^x)$

$f(x+1) = 3(2^{x+1}) - 2(3^{x+1})$

$= 3 \times 2^x \times 2 - 2 \times 3^x \times 3$

$= 6(2^x - 3^x)$ as required

c) $f(x) = \dfrac{2x^2}{x^2-16}$

i) $f(x) = \dfrac{2x}{(x-4)(x+4)}$

$x \neq \pm 4$

ii) $\lim\limits_{x\to\infty} \dfrac{2x^2}{x^2-16} = \lim\limits_{x\to\infty} \dfrac{\frac{2x^2}{x^2}}{\frac{x^2}{x^2} - \frac{16}{x^2}}$

$= 2$

iii) $f(x) = \dfrac{2x^2}{x^2-16}$

$f(-x) = \dfrac{2(-x)^2}{(-x)^2-16}$

$= \dfrac{2x^2}{x^2-16}$

$\therefore f(x) = f(-x)$

$\therefore f(x)$ is even