Question 1   (16 Marks)   **Marks**

(a)   Covert $1.\dot{6}\dot{0}$ to a fraction in its simplest terms.   **2**

(b)   Simplify $\dfrac{2^{-m} \times 3^{m-2}}{4^{m-1} \times 6^{2-m}}$ **3**

(c)   Factorise fully

     (i) $8x^3 - 125$ **1**

     (ii) $x^2y - 4x^2 - 9y + 36$ **2**

(d)   Simplify $\dfrac{1}{x^2 - 1} - \dfrac{2}{x^2 - x}$ **2**

(e)   Show that $\dfrac{2}{3 - \sqrt{3}} + \dfrac{11}{6 + \sqrt{3}}$ can be expressed as a rational number.   **3**

(f)   Given $6 + \sqrt{x - y} = x + y + 3\sqrt{2}$ find the values for x and y.   **3**

Question 2   **START A NEW PAGE**   (14 Marks)

(a)   Solve

     (i) $\dfrac{x+3}{2} + \dfrac{x-1}{3} = \dfrac{x+2}{6}$.   **3**

     (ii) $|x+2| = 2x - 5$.   **3**

     (iii) $2x^2 - 4x - 7 = 0$, giving your answer in the simplest exact form.   **2**

(b)   Solve the following inequalities

     (i) $x^2 - x - 2 > 0$.   **1**

     (ii) $\dfrac{3}{x-2} \geq 1$.   **3**

     (iii) $|2x - 3| \geq 7$ **2**

Question 3   START A NEW PAGE   (10 Marks)   **Marks**

(a)   Solve $x + y = 3$ and $x^2 + y^2 = 29$ simultaneously.   **3**

(b)   If $f(x) = x^2 + 3x + 5$, evaluate and simplify $\dfrac{f(x) - f(2)}{x - 2}$, $x \neq 2$.   **3**

(c)   By completing the square, find the centre and radius of the circle with   **3**

       the equation $x^2 - 10x + y^2 + 8y + 32 = 0$.

(d)   State the natural domain of the following functions

       (i) $f(x) = \dfrac{1}{2x - 7}$.   **1**

       (ii) $f(x) = \dfrac{1}{\sqrt{x + 5}}$ **1**

Question 4   START A NEW PAGE   (10 Marks)

(a)   Determine whether the function $f(x) = \dfrac{7x}{7 + x^2}$ is odd even or neither.   **2**

(b)   Sketch the following on separate number plane graphs, showing any

       asymptotes, vertices, intercepts or essential features.

       (i) $y = \sqrt{x + 3}$.   **2**

       (ii) $y = 4 - (x + 1)^2$.   **2**

       (iii) $y = |x(x - 2)(x + 2)|$.   **2**

       (iv) $y = \dfrac{x - 2}{x - 5}$.   **2**

End of Paper.

Question 1

a) Let $x = 1.\dot{6}\dot{0}$

$x = 1.606060\ldots$ —①

$100x = 160.606060\ldots$ —②

②−① $\quad 99x = 159$

$x = \dfrac{159}{99}$

$= 1\dfrac{20}{33}$

$\therefore 1.\dot{6}\dot{0} = 1\dfrac{20}{33}$

b) $\dfrac{2^{-m} \times 3^{m-2}}{4^{m-1} \times 6^{2-m}} = \dfrac{2^{-m} \times 3^{m-2}}{2^{2m-2} \times 2^{2-m} \times 3^{2-m}}$

$= \dfrac{2^{-m} \times 3^{m-2}}{2^{m} \times 3^{2-m}}$

$= 2^{-2m} \times 3^{2m-4}$

$\underline{\text{OR}}$

$\dfrac{3^{2m-4}}{2^{2m}}$

c) (i) $8x^3 - 125 = (2x)^3 - 5^3$

$= (2x-5)(4x^2+10x+25)$

(ii) $x^2y - 4x^2 - 9y + 36$

$= x^2(y-4) - 9(y-4)$

$= (x^2-9)(y-4)$

$= (x-3)(x+3)(y-4)$

d) $\dfrac{1}{x^2-1} - \dfrac{2}{x^2-x} = \dfrac{1}{(x-1)(x+1)} - \dfrac{2}{x(x-1)}$

$= \dfrac{x - 2(x+1)}{x(x-1)(x+1)}$

$= \dfrac{-x-2}{x(x-1)(x+1)}$

$= \dfrac{-(x+2)}{x(x+1)(x-1)}$

e) $\dfrac{2}{3-\sqrt{3}} = \dfrac{2}{3-\sqrt{3}} \times \dfrac{3+\sqrt{3}}{3+\sqrt{3}}$

$= \dfrac{2(3+\sqrt{3})}{9-3}$

$= \dfrac{2(3+\sqrt{3})}{6}$

$= \dfrac{3+\sqrt{3}}{3}$

$\dfrac{11}{6+\sqrt{3}} = \dfrac{11}{6+\sqrt{3}} \times \dfrac{6-\sqrt{3}}{6-\sqrt{3}}$

$= \dfrac{11(6-\sqrt{3})}{36-3}$

$= \dfrac{11(6-\sqrt{3})}{33}$

$= \dfrac{6-\sqrt{3}}{3}$

$\therefore \dfrac{2}{3-\sqrt{3}} + \dfrac{11}{6+\sqrt{3}} = \dfrac{3+\sqrt{3}+6-\sqrt{3}}{3}$

$= \dfrac{9}{3}$

$= 3$

which is rational.

f) $6 + \sqrt{x-y} = x+y + 3\sqrt{2}$

$= x+y + \sqrt{18}$

$\therefore \quad x+y = 6$ —①

$x-y = 18$ —②

①+② $\quad 2x = 24$

$x = 12$

sub ① $\quad x+y = 6$

$12 + y = 6$

$y = -6$

$\therefore x = 12 , y = -6$

Question 2

a) i) $\dfrac{x+3}{2} + \dfrac{x-1}{3} = \dfrac{x+2}{6}$

$3(x+3) + 2(x-1) = x+2$

$3x+9 + 2x-2 = x+2$

$5x+7 = x+2$

$4x = -5$

$x = -\dfrac{5}{4}$

ii) $|x+2| = 2x-5$

$x+2 = 2x-5$ $x+2 = -(2x-5)$

$-x = -7$ $= -2x+5$

$x = 7$ $x = 1$

CHECK

$LHS = |x+2|$ $LHS = |x+2|$

$= |7+2|$ $= |1+2|$

$= 9$ $= 3$

$RHS = 2x-5$ $RHS = 2x-5$

$= 2(7) = -5$ $= 2(1)-5$

$= 9$ $= -3$

$= LHS$ $LHS \neq RHS$

$\therefore x = 7$ is a $\therefore x = 1$ is

solution   not a solution

$\therefore x = 7$ is the only solution

iii) $2x^2 - 4x - 7 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{4 \pm \sqrt{(-4)^2 - 4 \times 2 \times -7}}{2(2)}$

$= \dfrac{4 \pm \sqrt{72}}{4}$

$= \dfrac{4 \pm 6\sqrt{2}}{4}$

$= \dfrac{2(2 \pm 3\sqrt{2})}{4}$

$\therefore x = \dfrac{2 \pm 3\sqrt{2}}{2}$

b) i) $x^2 - x - 2 > 0$

$(x-2)(x+1) > 0$

$x < -1$ $x > 2$
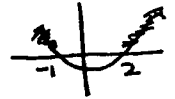
ii) $\dfrac{3}{x-2} \geq 1$, $x \neq 2$
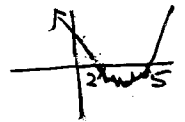
$(x-2)^2 \times \dfrac{3}{(x-2)} \geq 1 \cdot (x-2)^2$

$3(x-2) \geq (x-2)^2$

$(x-2)^2 - 3(x-2) \leq 0$

$(x-2)[(x-2)-3] \leq 0$

$(x-2)(x-5) \leq 0$

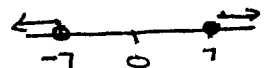$2 < x \leq 5$

iii) $|2x-3| \geq 7$

$2x-3 \leq -7$ $2x-3 \geq 7$

$2x \leq -4$ $2x \geq 10$

$x \leq -2$ $x \geq 5$

Question 3

a) $x + y = 3$ —— ①

$x^2 + y^2 = 29$ —— ②

rearrange ①

$y = 3 - x$ —— ③

sub ③ in ②

$x^2 + y^2 = 29$

$x^2 + (3-x)^2 = 29$

$x^2 + 9 - 6x + x^2 = 29$

$2x^2 - 6x - 20 = 0$

$x^2 - 3x - 10 = 0$

$$(x-5)(x+2) = 0$$
$$x = 5, -2$$

when $x = 5$ when $x = -2$

sub ① $x + y = 3$ sub① $x + y = 3$

$5 + y = 3$ $-2 + y = 3$

$y = -2$ $y = 5$

$\therefore$ $\left. \begin{matrix} x = 5 \\ y = -2 \end{matrix} \right\}$ $\left. \begin{matrix} x = -2 \\ y = 5 \end{matrix} \right\}$

b) $f(x) = x^2 + 3x + 5$

$f(2) = 2^2 + 3(2) + 5$
$\qquad = 15$

$\therefore \dfrac{f(x) - f(2)}{x - 2} = \dfrac{x^2 + 3x + 5 - 15}{x - 2}$

$\qquad\qquad = \dfrac{x^2 + 3x - 10}{x - 2}$

$\qquad\qquad = \dfrac{(x + 5)(x - 2)}{(x - 2)}$

$\therefore \dfrac{f(x) - f(2)}{x - 2} = x + 5$

c) $x^2 - 10x + y^2 + 8y + 32 = 0$

$x^2 - 10x + \left(\dfrac{-10}{2}\right)^2 + y^2 + 8y + \left(\dfrac{8}{2}\right)^2 = -32 + 25 + 16$

$(x - 5)^2 + (y + 4)^2 = 9$

$(x - 5)^2 + (y + 4)^2 = 3^2$

$(x - h)^2 + (y - k)^2 = r^2$

$\therefore$ centre is $(5, -4)$

    radius is 3

d) ı) $f(x) = \dfrac{1}{2x - 7}$

$2x - 7 \neq 0$

$2x \neq 7$

$x \neq 3\tfrac{1}{2}$

$\therefore$ domain $x \neq 3\tfrac{1}{2}$

ıı) $f(x) = \dfrac{1}{\sqrt{x + 5}}$

$x + 5 > 0$

$x > -5$

$\therefore$ domain $x > -5$

Question 4

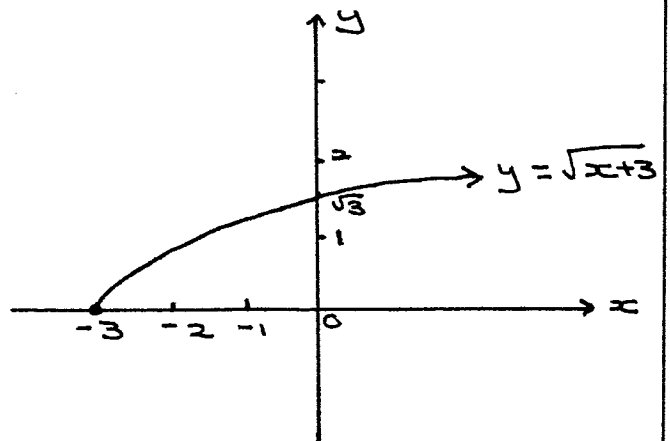a) $f(x) = \dfrac{7x}{7 + x^2}$

$f(-x) = \dfrac{7(-x)}{7 + (-x)^2}$

$\qquad = \dfrac{-7x}{7 + x^2}$
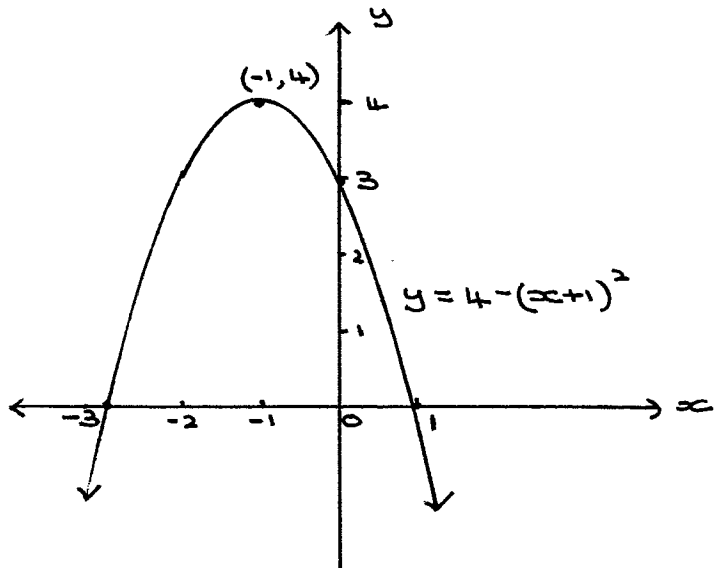
$-f(x) = -\left(\dfrac{7x}{7 + x^2}\right)$

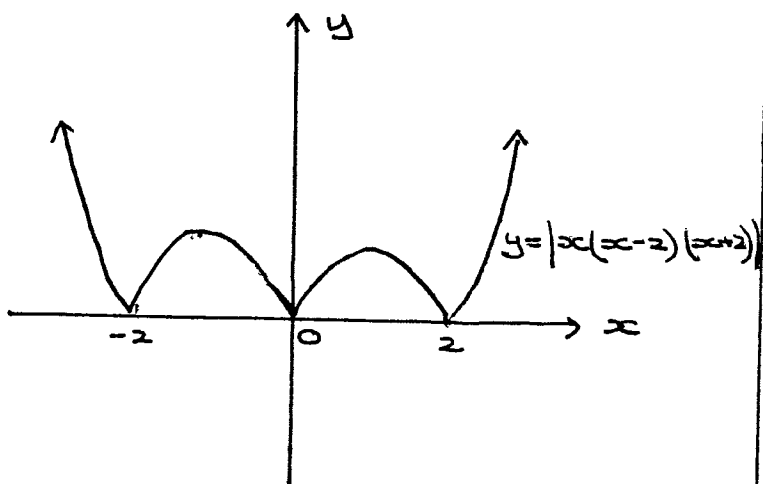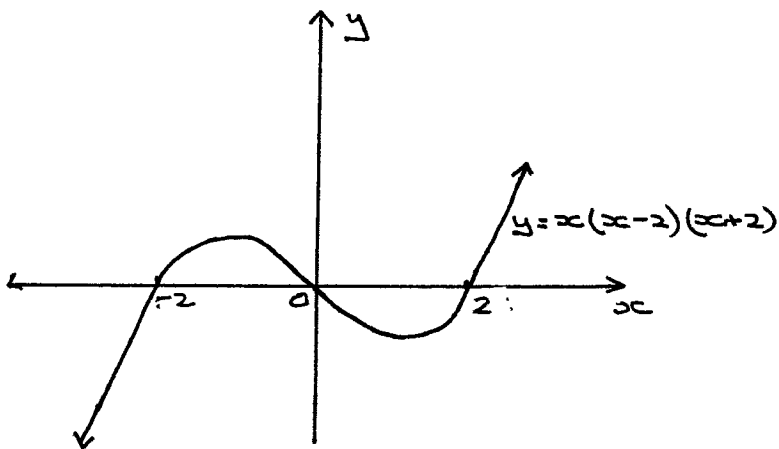$\qquad = \dfrac{-7x}{7 + x^2}$

As $f(-x) = -f(x)$ the function is odd.

b) ı) $y = \sqrt{x + 3}$

ii) $y = 4 - (x+1)^2$

(-1, 4)

$y = 4 - (x+1)^2$

iii) $y = |x(x-2)(x+2)|$

$y = x(x-2)(x+2)$

$y = |x(x-2)(x+2)|$

(iv) $y = \dfrac{x-2}{x-5}$

$= \dfrac{x-5}{x-5} + \dfrac{3}{x-5}$

$= 1 + \dfrac{3}{x-5}$

when $x = 0$

$y = \dfrac{x-2}{x-5}$

$= \dfrac{2}{5}$

$\therefore$ y intercept $\left(0, \dfrac{2}{5}\right)$

when $y = 0$

$y = \dfrac{x-2}{x-5}$

$0 = \dfrac{x-2}{x-5}$

$x - 2 = 0$

$x = 2$

$\therefore$ x intercept $(2, 0)$

$y = \dfrac{x-2}{x-5}$