Name: ______________________________________ **Teacher:** __________________________

YEAR 11 MATHEMATICS ASSESSMENT TASK – JUNE 2008

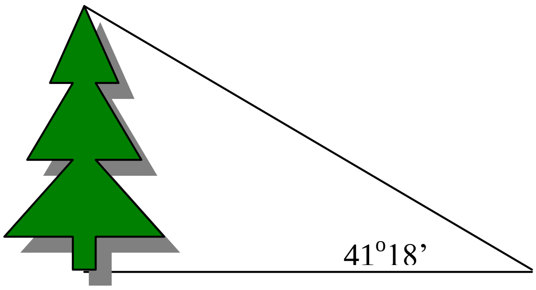
Time Allowed: 60 minutes
Full working should be shown in every question..
Marks may be deducted for careless or badly arranged work.
No liquid paper is to be used.
If a correction is to be made, one line is to be ruled through the incorrect answer.

Marks

1. Find the exact value of

 a) tan 300° **1**
 b) cosec (-225^o) **1**

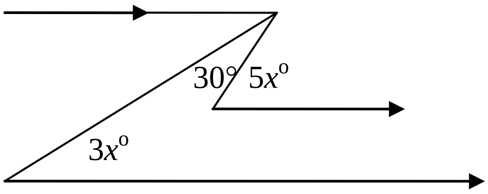
2. **2**



A tree casts a 15m shadow when the elevation of the sun is $41^o\ 18'$.
Find the height of the tree.

3. The interior angles of a regular polygon are 150^o **2**
How many sides does it have?

4. Find x (no reasons required) **1**



5. If $\sin\ \theta = \frac{4}{7}$ and $\cos\ \theta < 0$ find the exact value of $\tan\ \theta$ **2**

6. If $\sin\ (2x + 20)^o = \cos\ (3x - 80)^o$ find x **2**

7. Show that $3x + 4y + 25 = 0$ is a tangent to the circle $x^2 + y^2 = 25$ **3**

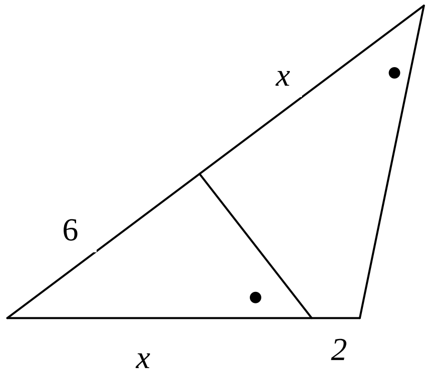
8. Solve for $0 \leq \theta \leq 360^{o}$ **Marks**

a) $\text{Sin } \theta = \dfrac{\sqrt{3}}{2}$ **2**

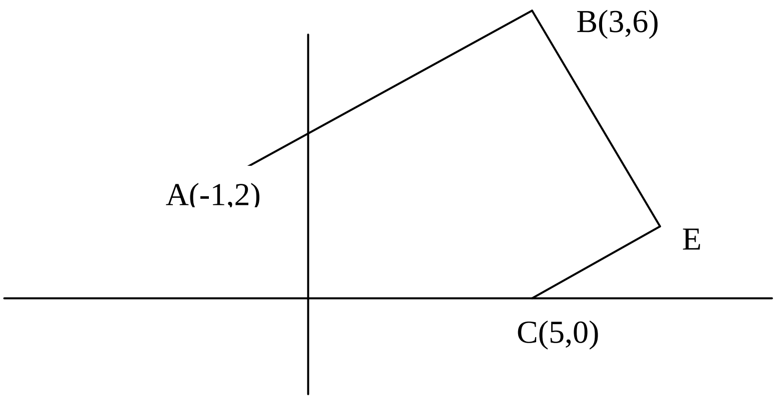
b) $2\cos^2 \theta = 2 + \sin \theta$ **3**

c) $\sec 2\theta = \cos 2\theta$ **3**

9. Find the exact value of x in the following, if the triangles below are similar. **4**



10.



i) Find the gradient of AB **1**

ii) Find M, the midpoint of AB **1**

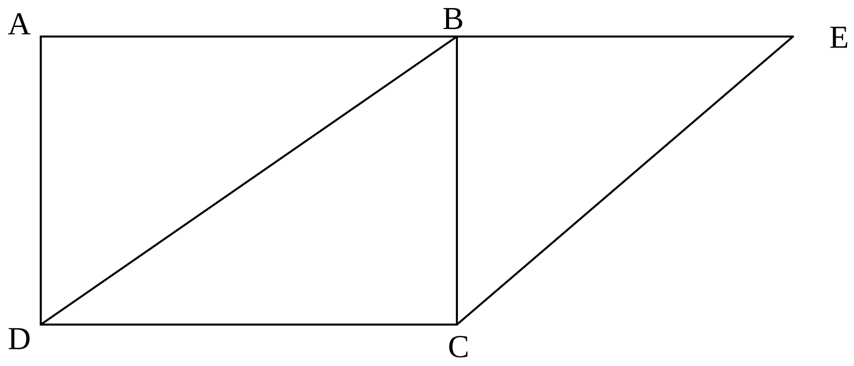
iii) Find the equation of the line ℓ through M perpendicular to AB **2**

iv) Show that C(5,0) lies on the line ℓ **1**

v) If AB has equation $x - y + 3 = 0$ find the perpendicular distance from C to AB **2**

vi) If MCEB is a rectangle find the area of the trapezium ABEC **3**

11. **Marks**



ABCD is a rectangle. AB is produced to E such that BD || CE

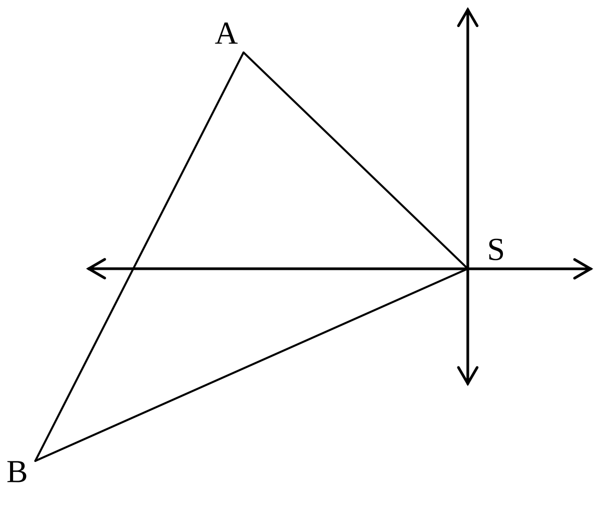
i) Prove Δ ABD $\equiv$ Δ BCE **3**

ii) If AB : BC = 2 : 1 find $\angle$ BEC **2**

12. a) Sketch the graphs of y = cos x for $0 \leq x \leq 360^{o}$ and y = ½ on the same set of axes **2**

b) For what values of x in the domain $0 \leq x \leq 360°$ is cos $x \geq$ ½. **2**

13. Two planes leave Sydney at the same time. One flies 300nm northwest to Point A. The other flies 420nm on a bearing of 251^{o} to Point B.



i) Show $\angle$ ASB = 64^{o} **1**

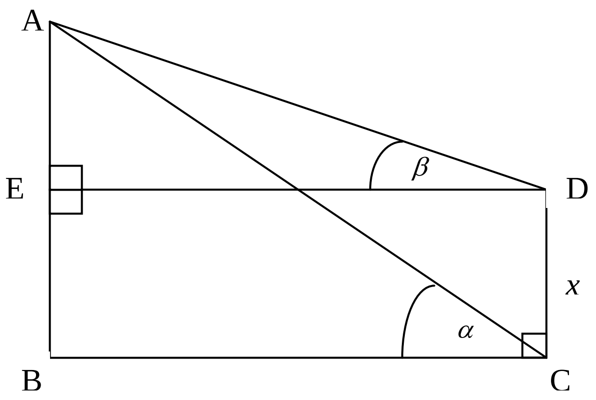
ii) What is the distance AB? **2**

iii) What is the bearing of B from A? **3**

Marks

14. Show $\dfrac{\cot\theta - \tan\theta}{\cos\theta - \sin\theta}$ = cosec θ + sec θ **3**

15.



i) Show AD = $\dfrac{BC}{\cos\beta}$ **2**

ii) Prove BC = $\dfrac{x\cos\alpha\cos\beta}{Sin(\alpha-\beta)}$ **3**

END OF EXAMINATION

1\. a) $\tan 300 = -\tan 60°$

$= -\sqrt{3}$ 1

b) $\operatorname{cosec}(-225)$

$= \dfrac{1}{\sin 135°}$

$= \dfrac{1}{\frac{1}{\sqrt{2}}}$

$= \sqrt{2}$ 1

2\. $\tan 41°18' = \dfrac{x}{15}$

$x = 15 \tan 41°18'$

$= 13{\cdot}18$ 2

3\. $\dfrac{180(n-2)}{n} = 150°$

$180n - 360 = 150n$

$30n = 360$

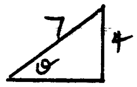
$n = 12$

$\therefore$ 12 sides.   2

4\. $3x + 30 = 5x$

$x = 15°$ 1

Solutions.

5\. 

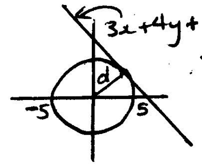
$\tan\theta = -\dfrac{4}{\sqrt{33}}$

$\sqrt{33} \rightarrow$ by Pyth.   2

6\. $2x+20 + 3x-80 = 90°$

$5x - 60 = 90$

$x = 30°$ 2

7\. 

$3x+4y+25=0$ (for algebraic solution see back page)

If $3x+4y+25=0$ is a tangent to $x^2+y^2=25$

then $d = 5$ units.

ie $\dfrac{|3(0)+4(0)+25|}{\sqrt{3^2+4^2}} = d$

$d = \dfrac{25}{5}$

$d = 5$ 3

$\therefore$ $3x+4y+25=0$ is a tangent

8\. a) $\sin\theta = \dfrac{\sqrt{3}}{2}$

$\therefore \theta = 60°, 120°$ 2

b) $2\cos^2\theta = 2 + \sin\theta$

$2(1-\sin^2\theta) = 2 + \sin\theta$

$\therefore 2\sin^2\theta + \sin\theta = 0$

$\sin\theta(2\sin\theta + 1) = 0$ 3

$\sin\theta = 0$ $\sin\theta = -\frac{1}{2}$

$\therefore \theta = 0, 180°, 360°, 210°, 330°$

8c) See bottom of last page

9\. $\dfrac{x}{x+6} = \dfrac{6}{x+2}$

$x(x+2) = 6x+36$

$x^2 + 2x - 6x - 36 = 0$

$x^2 - 4x - 36 = 0$

$x = \dfrac{4 \pm \sqrt{16 - 4.1.-36}}{2}$

$= \dfrac{4 \pm \sqrt{160}}{2}$

$= \dfrac{4 \pm 4\sqrt{10}}{2}$

$= 2 \pm 2\sqrt{10}$ but $x > 0$

$\therefore x = 2 + 2\sqrt{10}$

(accept $\dfrac{4+4\sqrt{10}}{2}$)   4

10 (i) $m_{AB} = \dfrac{6-2}{3--1} = 1$ 1

(ii) $M\left(\dfrac{3+-1}{2}, \dfrac{6+2}{2}\right)$

$= M(1, 4)$ 1

(iii) $y-4=-1(x-1)$

$y=-x+5$ 2

(iv) sub in $(5,0)$

$0=-5+5$

$0=0$

$\therefore (5,0)$ lies on $y=-x+5$ 1

(v) $d=\frac{|1(5)+-1(0)+3|}{\sqrt{1^2+(-1)^2}}$

$d=\frac{8}{\sqrt{2}}$ 2

(vi) Area of trapezium

$=3\times\Delta AMC$

$AM=\sqrt{(1--1)^2+(4-2)^2}$

$=\sqrt{8}$

$\therefore \Delta AMC=\frac{1}{2}\sqrt{8}\cdot\frac{8}{\sqrt{2}}$

$=\frac{1}{\cancel{2}}\cdot\cancel{2}\sqrt{2}\cdot\frac{8}{\cancel{\sqrt{2}}}$

$=8$ 3

$\therefore$ Area Trapezium $=24$ unit^2

Alternative. $E(7,2)$

$AB=\sqrt{32}$ $CE=\sqrt{8}$

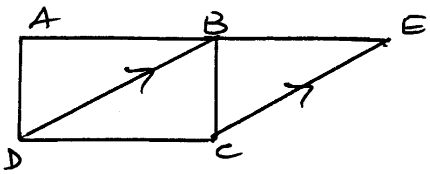
$\therefore$ Trap. $=\frac{1}{2}\cdot\frac{8}{\sqrt{2}}(\sqrt{32}+\sqrt{8})$

$=\frac{8}{2\sqrt{2}}(4\sqrt{2}+2\sqrt{2})$

$=\frac{48\cancel{\sqrt{2}}}{2\cancel{\sqrt{2}}}$

$=24$ units^2 (3)

11.



(i) $\angle DAB=90°=\angle ABC$ (Angles of a rect.)

$\angle CBE=90°$ (straight $\angle$)

$\therefore \angle DAB=\angle CBE$

$AD=BC$ (Opposite sides of a rectangle)

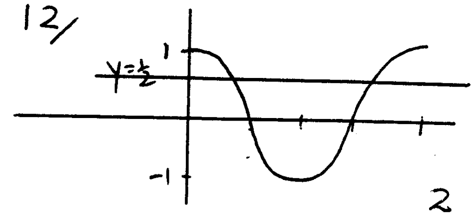
$\angle ABD=\angle AEC$ (corresponding $\angle$s on $\parallel$ lines)

$\therefore \Delta ABD\equiv\Delta BEC$ (AAS) 3

(ii) $\tan\angle BEC=\frac{1}{2}$

$\therefore \angle BEC=26°34'$ 2

12/



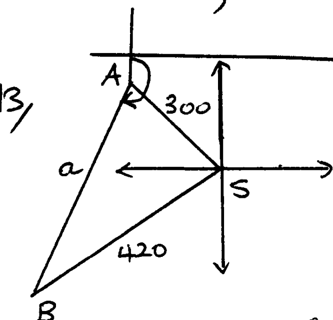
2

$\cos x=\frac{1}{2}$ when $x=60°, 300°$

$\cos x\geq\frac{1}{2}$ for

$0\leq x\leq 60°$, $300°\leq x\leq 360°$ 2

13/



(i) $\angle ASB=360-(251+45°)$

$=64°$ 1

(ii) $a^2=300^2+420^2-2.(300)(420)\cos 64°$

$a=394.88$ n.m. 2

(iii) $\frac{\sin\angle BAS}{420} = \frac{\sin 64}{394.88}$

$\sin\angle BAS = \frac{420\sin 64}{394.88}$

$\angle BAS = 72°56'$

$\therefore$ Bearing $= 116° + 72°56'$

$= 188°56'$ 3

14/ $\frac{\cot\theta - \tan\theta}{\cos\theta - \sin\theta} = \operatorname{cosec}\theta + \sec\theta$

LHS =

$\frac{\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}}{\cos\theta - \sin\theta}$

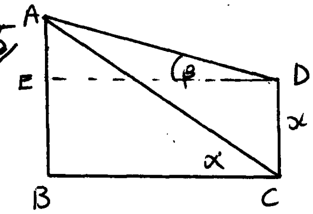
$\frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} \div \frac{1}{\cos\theta - \sin\theta}$

$= \frac{(\cos\theta + \sin\theta)\cancel{(\cos\theta - \sin\theta)}}{(\sin\theta\cos\theta)\cancel{(\cos\theta - \sin\theta)}}$

$= \frac{1\cancel{\cos\theta}}{\sin\theta\cancel{\cos\theta}} + \frac{\cancel{\sin\theta}\,1}{\cancel{\sin\theta}\cos\theta}$

$= \operatorname{cosec}\theta + \sec\theta$ 3

15/



(i) $\cos\beta = \frac{DE}{AD}$ but $DE = BC$

$\therefore \cos\beta = \frac{BC}{AD}$

$\therefore AD = \frac{BC}{\cos\beta}$ --(1) 2

(ii) $\angle CAD = \alpha - \beta$

$\therefore \frac{x}{\sin(\alpha-\beta)} = \frac{AD}{\sin(90-\alpha)}$

$\therefore AD = \frac{x\sin(90-\alpha)}{\sin(\alpha-\beta)}$

$= \frac{x\cos\alpha}{\sin(\alpha-\beta)}$

from (1) $BC = AD\cos\beta$

$\therefore BC = \frac{x\cos\alpha\cos\beta}{\sin(\alpha-\beta)}$ 3

Question 7. (Algebraically)

$3x+4y+25=0 \Rightarrow y = \frac{-25-3x}{4}$

$x^2+y^2=25$

$\therefore x^2 + \left(\frac{-25-3x}{16}\right)^2 = 25$

$x^2 + \frac{625+150x+9x^2}{16} = 25$

$16x^2 + 625 + 150x + 9x^2 = 400$

$25x^2 - 150x + 225 = 0$

$x^2 - 6x + 9 = 0$

$(x-3)^2 = 9$

$\therefore x = 3$

$y = 4$ $(3,4)$

$\therefore$ Only 1 pt of intersection

$\therefore 3x+4y+25=0$ is a tangent to $x^2+y^2=25$.

8c) $\sec 2\theta = \cos 2\theta$

$\frac{1}{\cos 2\theta} = \cos 2\theta$

$\cos^2 2\theta = 1$

$\cos 2\theta = \pm 1$ 3

$\therefore 2\theta = 0, 180, 360, 540, 720$

$\theta = 0, 90, 180, 270, 360$