BAULKHAM HILLS HIGH SCHOOL
YEAR 11 MATHEMATICS TASK 2
JUNE 2009

Time Allowed: 70 minutes

Instructions:
a) Write all your answers on your own paper.
b) Show all necessary working.
c) Marks may be deducted for careless or badly arranged work.
d) No liquid paper is to be used. If a correction is to be made, one line is to be ruled through the incorrect answer.

Marks

1. Find the exact value of;

   a) $\tan 330^\circ$ *1*

   b) $\sec 120^\circ$ *1*

2. If $\tan\theta = \frac{3}{5}$ and θ is a reflex angle, find the exact value of $\cos\theta$ *2*

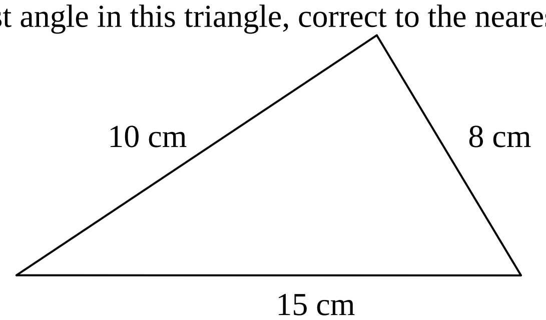
3. A ship leaves port and sails on a bearing of 120°T for 50 km.

   a) Draw a sketch to show this information. *1*

   b) How far east of the port is it at this time, correct to the nearest km? *2*

4. Sketch $y = \tan x$ for $0^\circ \le x \le 360^\circ$ *2*

5. Find the largest angle in this triangle, correct to the nearest minute. *3*



6. Prove that the points $A(1,2)$, $B(-1,6)$ and $C(2,0)$ are collinear. *3*

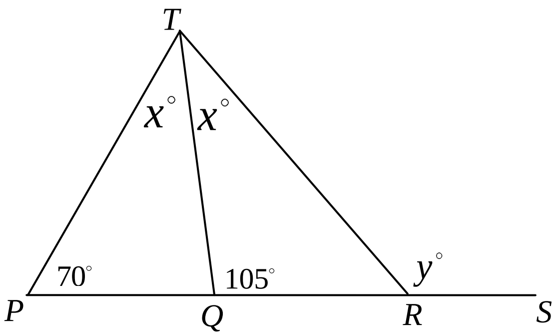
7. If $(2,3)$ is the midpoint of the two points $A(x,y)$ and $B(4,4)$, find the coordinates of *A*. *3*

8. A line is represented by the equation $x + 2y - 6 = 0$. Find the equation of the line that is perpendicular to this line and passes through the point $(1,3)$. *3*

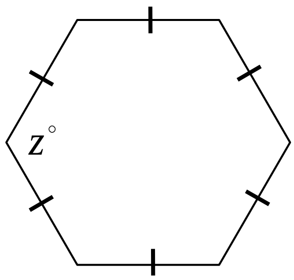
Marks

9. Find the value of each pronumeral, (no reasons required);

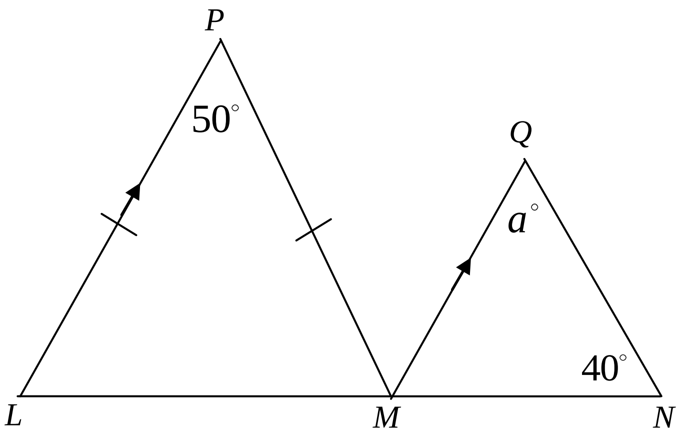
a) 2



b) 1



c) 2



10. Solve for $0^\circ \le \theta \le 360^\circ$

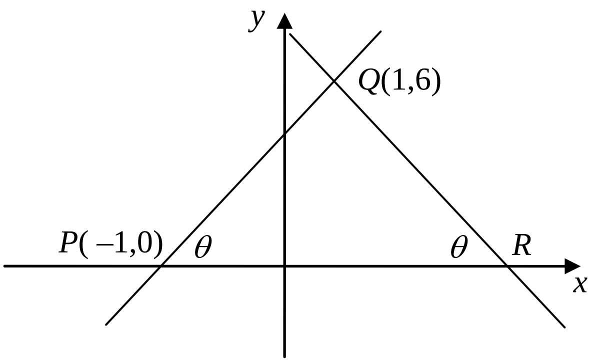
a) $4\cos^2\theta = 3$ 3

b) $2\sin\theta - 4\cos\theta = 0$ 3

c) $\tan 2\theta = \dfrac{1}{\sqrt{3}}$ 3

Marks

11.



In the diagram, P and Q have coordinates (–1,0) and (1,6) and $\angle QPR = \angle QRP = \theta$

a) Find the length of PQ as a surd. *2*

b) Show that $\tan\theta = 3$ *2*

c) Show that the gradient of QR is –3. Give reasons. *2*

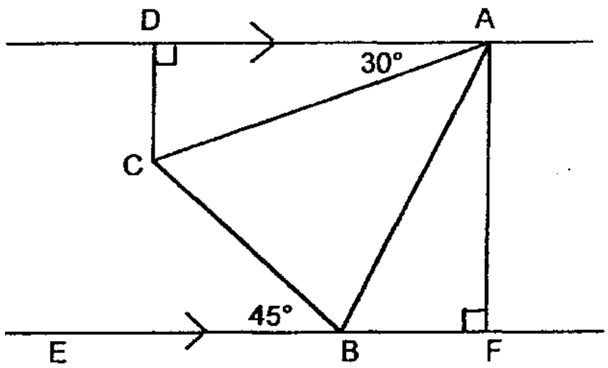
d) Show that the equation of QR is $3x + y - 9 = 0$. *2*

e) Find the coordinates of R. *1*

f) Find the perpendicular distance from P to QR. *2*

g) Hence calculate the area of ΔPQR *2*

12.



In the diagram above $AC = AB$ and $DA \parallel EF$. $\angle DAC = 30^\circ$ and $\angle CBE = 45^\circ$. $CD \perp DA$ and $DA \perp AF$.

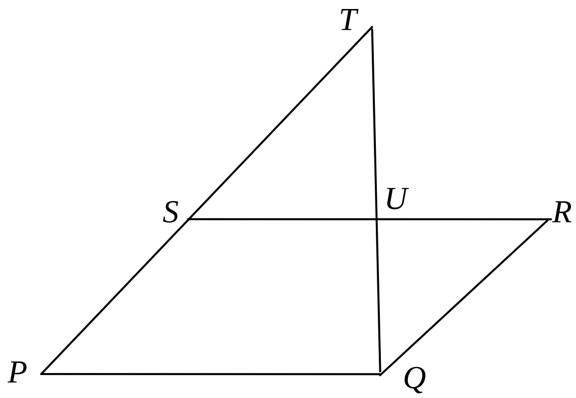
a) Find the size of $\angle ACB$, giving reasons. *3*

b) Hence find the size of $\angle CAB$, giving reasons *2*

c) Prove that $\Delta ACD \equiv \Delta ABF$ *3*

Marks

13.



PQRS is a parallelogram and *PT* is a straight line through *S*. $TQ \perp PQ$

a) Prove $\Delta URQ \,|||\, \Delta QPT$ *3*

b) If *QR* = 5, *UR*= 4, *ST*= 8, find *TP* and *SU* and hence *TQ*. *5*

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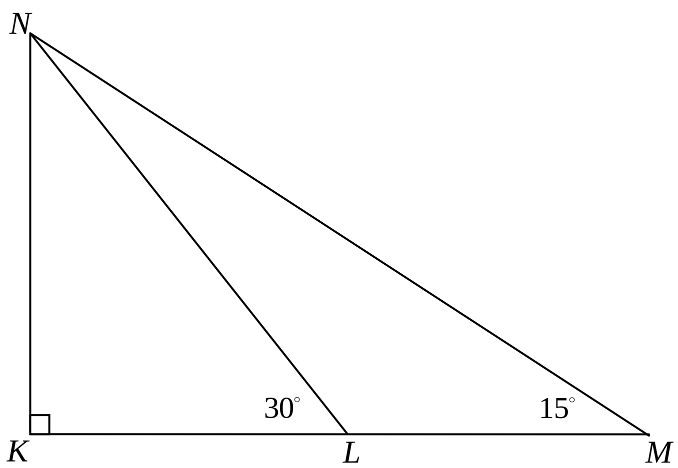
14. Prove that $\dfrac{1}{\operatorname{cosec}\theta - \cot\theta} - \dfrac{1}{\operatorname{cosec}\theta + \cot\theta} = 2\cot\theta$ *2*

15.



a) If *LM* = 1 metre, explain why *LN* also equals 1 metre. ***2***

b) Use the sketch to deduce that $\tan 15^\circ = 2 - \sqrt{3}$ ***3***

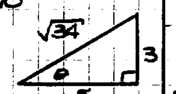
Year 11 Mathematics Task 3 Solutions /71

1/ a) $\tan 330^\circ = -\tan 30^\circ$
$= -\frac{1}{\sqrt{3}}$ (1)

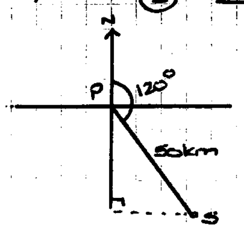
b) $\sec 120^\circ = \frac{1}{\cos 120^\circ}$
$= \frac{1}{-\cos 60^\circ}$
$= -2$ (1)

2/ $\tan\theta = \frac{3}{5}$, θ reflex
$\therefore 180^\circ < \theta < 270^\circ$

$\cos\theta = -\frac{5}{\sqrt{34}}$ (2)



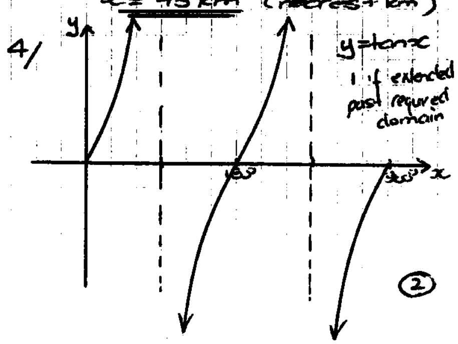
3/ a)



(1)

b) $\frac{x}{50} = \sin 60^\circ$ −1
$x = 50\sin 60^\circ$
$x = 43.30127019$
$x = 43$ km (nearest km) (2)

4/



1 if extended past required domain (2)

5/ largest angle is opposite the largest side.

$\cos\theta = \frac{8^2 + 10^2 - 15^2}{2\times 8\times 10}$ −1

$\theta = 112^\circ 25'$ −2. (3)

6/ $m_{AB} = \frac{6-2}{-1-1}$ $m_{BC} = \frac{0-6}{2+1}$
$= \frac{4}{-2}$ $= \frac{-6}{3}$ −1 for slopes
$= -2$ $= -2$ −1 for conclusion.
$\therefore m_{AB} = m_{BC}$
Thus A, B, C are collinear (3)

7/ $\frac{x+4}{2} = 2$ $\frac{y+4}{2} = 3$ −1 for interpretation
$x + 4 = 4$ $y + 4 = 6$ −1 if only one value evaluated
$x = 0$ $y = 2$
$\therefore$ A is the point (0, 2) (3)

8/ $x + 2y - 6 = 0 \Rightarrow m = -\frac{1}{2}$
$\therefore$ required $m = 2$ −1

$y - 3 = 2(x - 1)$ −1
$y - 3 = 2x - 2$
$2x - y + 1 = 0$ −1

OR
equation is in the form
$2x - y + k = 0$ −1
(1, 3): $2 - 3 + k = 0$
$k = 1$ −1
$\therefore 2x - y + 1 = 0$ (3)

9/ a) $x + 70 = 105$
$x = 35$ (1)

$y = x + 105$
$y = 140$ (1)

b) $6z = 180(6-2)$
$= 720$
$z = 120$ (1)

c) $2\angle PML + 50 = 180$
$2\angle PML = 130$
$\angle PML = 65$
$\angle PMQ = 50$
$a + 40 = 65 + 50$
$= 115$
$a = 75$ (2)

subtract 1 for each error.

10/ a) $4\cos^2\theta = 3$
$\cos^2\theta = \frac{3}{4}$
$\cos\theta = \pm\frac{\sqrt{3}}{2}$ −1
Q1, 2, 3, 4
$\cos\alpha = \frac{\sqrt{3}}{2}$
$\alpha = 30^\circ$ −1
$\theta = 30^\circ, 150^\circ, 210^\circ, 330^\circ$ −1 (3)

b) $2\sin\theta - 4\cos\theta = 0$
$2\sin\theta = 4\cos\theta$
$\tan\theta = 2$ −1
Q1, 3
$\tan\alpha = 2$
$\alpha = 63^\circ 26'$ −1
$\theta = 63^\circ 26', 243^\circ 26'$ −1 (3)

c) $\tan 2\theta = \frac{1}{\sqrt{3}}$ $0^\circ \le 2\theta \le 720^\circ$
Q1, 3
$\tan\alpha = \frac{1}{\sqrt{3}}$
$\alpha = 30^\circ$ −1
$2\theta = 30^\circ, 210^\circ, 390^\circ, 570^\circ$ −1
$\theta = 15^\circ, 105^\circ, 195^\circ, 285^\circ$ −1 (3)
OR (2) if only two correct solutions

11/ a) $PQ = \sqrt{6^2 + 2^2}$ −1
$= \sqrt{40}$
$= 2\sqrt{10}$ units (2)

b) $m = \tan\theta$ −1
$\tan\theta = \frac{6-0}{1+1}$
$= \frac{6}{2}$
$= 3$ −1 (2)

c) $m_{QR} = \tan(180 - \theta)$
$= -\tan\theta$
$= -3$ (1)

d) $y - 6 = -3(x - 1)$ −1
$y - 6 = -3x + 3$
$3x + y - 9 = 0$ (2)

e) $y = 0$: $3x = 9$
$x = 3$
$\therefore$ R is (3, 0) (1)

f) $d = \frac{|3(-1) + (0) - 9|}{\sqrt{3^2 + 1^2}}$ −1
$= \frac{12}{\sqrt{10}}$ units (2)

g) $\triangle PQR$ is isosceles
$\therefore QR = 2\sqrt{10}$ −1
Area $= \frac{1}{2}\times 2\sqrt{10}\times\frac{12}{\sqrt{10}}$ must use in calculation
$= 12$ units^2 −1 (2)

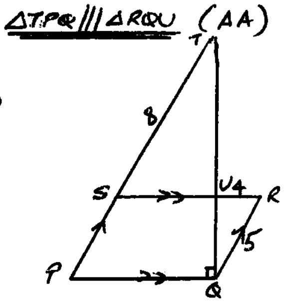
12/ a) Construct XY ∥ DA passing through C
$\angle ACY = 30^\circ$ (alternate ∠'s =, DA ∥ XY)
$\angle YCB = 45^\circ$ (alternate ∠'s =, XY ∥ EF)
$\angle ACB = \angle ACY + \angle YCB$ (common ∠)
$\angle ACB = 75^\circ$ (3)
2 + 1 for reasoning

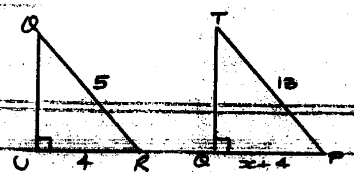
b) $AC = AB$ (given)
$\triangle ABC$ is isosceles (2 = sides)
$\angle ABC = \angle ACB$ (base ∠'s isosceles △ =)
$\angle ABC = 75^\circ$
$\angle CAB + \angle ABC + \angle BAC = 180^\circ$ (∠ sum △ABC)
$\angle CAB = 30^\circ$ (2)
1 + 1 for reasoning.

c) $\angle DAF + \angle BFA = 180^\circ$ (co-interior ∠'s = 180°, DA ∥ BF)
$\angle BFA = 90^\circ$ (given)
$\therefore \angle DAF = 90^\circ$
$\angle DAF = \angle DAC + \angle CAB + \angle BAF$ (common ∠)
$90 = 30 + 30 + \angle BAF$
$\angle BAF = 30$
$\therefore \angle BAF = \angle CAD$ (A)
$\angle AFB = \angle ADC = 90^\circ$ (given) (A)
$AB = AC$ (given) (S)
$\therefore \triangle ABF \equiv \triangle ACD$ (AAS) (3)
1 + 2 for reasoning.

13/ PT ∥ QR (opposite sides in ∥gram)
$\angle PQT = \angle RUQ$ (alternate ∠'s =, PT ∥ QR)
$\angle TPQ = \angle RQU$ (opposite ∠'s in ∥gram =)
$\therefore \triangle TPQ \,|||\, \triangle RQU$ (AA) (3)
1 + 2 for reasoning

b)





$PS = QR$ (opposite sides //gram)

$PS = 5$

$PT = ST + PS = \underline{13}$ -1

let $SU = x$,

$PQ = RS$ (opposite sides //gram)

$PQ = x + 4$

$\frac{x+4}{4} = \frac{13}{5}$ (ratio sides in ||| Δs)

$5x + 20 = 52$

$x = \frac{32}{5}$

$\therefore \underline{SU = \frac{32}{5}}$ -1

$QT^2 = PT^2 - PQ^2$

$= 13^2 - \left(\frac{52}{5}\right)^2$

$= \frac{1521}{25}$ -1 for reasoning.

$\underline{QT = \frac{39}{5}}$ -1

14/ $\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

$= \frac{\operatorname{cosec}\theta + \cot\theta - (\operatorname{cosec}\theta - \cot\theta)}{\operatorname{cosec}^2\theta - \cot^2\theta}$ -1 -1

$= \frac{2\cot\theta}{1}$ -1

$= \underline{2\cot\theta}$ (2)

15/ $\angle KLN = \angle LNM + \angle NML$ (exterior ∠, ΔLMN)

$30 = \angle LNM + 15$

$\angle LNM = 15°$

$\therefore$ ΔLNM is isosceles (2 = ∠s) -1

$LN = LM$ -1 (= sides in isosceles Δ)

$\underline{LN = 1}$. (2)

b) $\frac{KL}{1} = \cos 30°$

$KL = \frac{\sqrt{3}}{2}$

$\frac{KN}{1} = \sin 30°$

$KN = \frac{1}{2}$ -1

$\tan 15° = \frac{KN}{KM}$

$= \frac{KN}{KL + ML}$

$= \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2} + 1}$ -1

$= \frac{1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2}$

$= \frac{\sqrt{3}-2}{3-4}$

$= \underline{2 - \sqrt{3}}$ (3)