Year 11 Mathematics

June 2010

Time: 65 Minutes

DIRECTIONS

- Full working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Use black or blue pen only (*not pencils*) to write your solutions.
- No liquid paper is to be used. If a correction is to be made, one line is to be ruled through the incorrect answer.

1. Find the exact value of

a) $\cos 225^{o}$ **1**

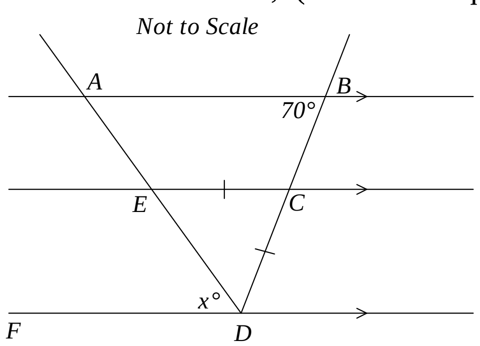
b) $\cot(-120^{o})$ **1**

2. Solve for $0 \leq \theta \leq 360^{0}$ **2**

$2\cos\theta + 1 = 0$

3. a)

i) Find the value of x^{o}, (no reason required) **1**

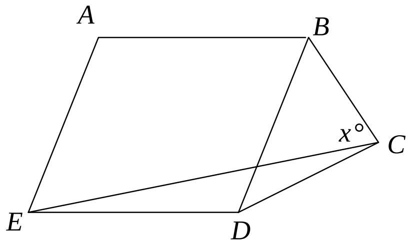


Given :
AB || *CE* || *DF*
and *EC* = *CD*

ii) If $CD = a$, $DE = 5$, $BC = 3$ and $AE = a + 1$, find the exact value of a **3**

b)

Not to Scale



ABDE is a rhombus
BCD is an equilateral triangle $\angle BAE = 110^{o}$
Find without reasons, the value of *BCE* **1**

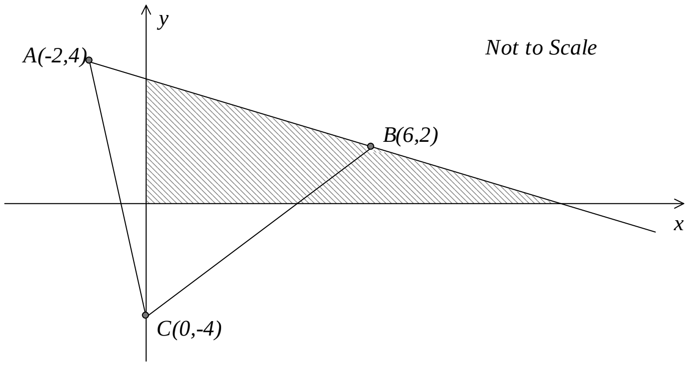
4. The ratio of the interior to the exterior angles of a regular polygon is 4 **:** 1

a) Find the size of each exterior angle **1**

b) How many sides does the polygon have? **1**

5. If $\tan\theta = \frac{2}{3}$ and $\sin\theta < 0$, find the exact value of $\cos\theta$ **2**

6.



a) Find the gradient of AB **1**

b) Find the distance of AB **1**

c) Show the equation of the line AB is $x + 4y - 14 = 0$ **1**

d) Find the perpendicular distance from C to the line AB **1**

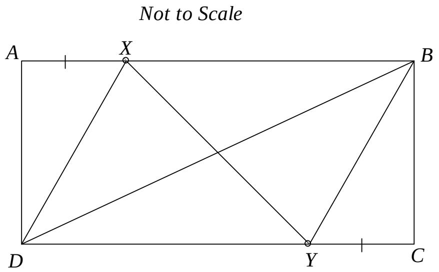
e) Find the point D such that $ADBC$ is a parallelogram **1**

f) Find the area of the parallelogram $ADBC$ **1**

g) Using inequalities, describe the shaded region above **1**

7. Joe walks 6 kms due east then 8 km on a bearing of 142^{o}. **3**
Draw a diagram representing this information and find how far Joe is from his starting point

8.



$ABCD$ is a rectangle
X and Y lie on AB and CD respectively such that
$AX = YC$

a) Prove $\Delta\ AXD \equiv \Delta BCY$ **3**

b) Prove that BD and XY bisect each other **2**

9. Hayden and Harry leave a point A. Hayden travels northwest and Harry travels southeast.
Hayden and Harry are then 20 kms apart.

a) If Hayden travelled twice as far as Harry, how far did Harry walk? **2**

b) What is the bearing of Harry's position from Hayden's? **2**

10.

a) Prove $\dfrac{1+\tan^2 x}{1+\cot^2 x} = \tan^2 x$ **2**

b) Hence solve $\dfrac{1+\tan^2 x}{1+\cot^2 x} = 2\tan x$ for $0 \le x \le 360^0$ **3**

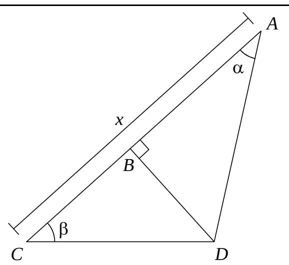
11.

a) On the same set of axes, sketch the graphs of $y = \sin x$ and $y = \cos x$ for $0 \le x \le 360^0$ **2**

b) Hence, find the values of x in this domain for which $\sin x \ge \cos x$ **2**

12. For $0 \le x \le 360^0$ solve $2\tan^2\theta - 5\sec\theta + 5 = 0$ **4**

13.



Not to Scale

$AC = x$ and $DB \perp AC$ **3**

Prove that $BD = \dfrac{x\sin\alpha\ \sin\beta}{\sin(\alpha+\beta)}$

~ END OF EXAM ~

1. a) $\cos 225^\circ = -\cos 45 = -\frac{1}{\sqrt{2}}$ (1) | $\frac{}{47}$

b) $\cot(-120^\circ) = \cot 240^\circ = \cot 60^\circ = \frac{1}{\sqrt{3}}$ (1)

2. $2\cos\theta + 1 = 0$

$\cos\theta = -\frac{1}{2}$

$\theta = 180 - 60^\circ,\ 180 + 60^\circ = 120^\circ,\ 240^\circ$ (2)

3 a) (i) $x = 55^\circ$ (1)

(ii) $\frac{a}{3} = \frac{5}{a+1}$, $a^2 + a - 15 = 0$ (1)

$a = \frac{-1 \pm \sqrt{1^2 - 4.1.-15}}{2}$ (1)

$= \frac{-1 \pm \sqrt{61}}{2}$

$a > 0 \therefore a = \frac{-1 + \sqrt{61}}{2}$ (1)

b) $x = 55^\circ$ (1)

4. Exterior $\angle = 180 - 156 = 24^\circ$

$\therefore$ No of sides $= 360 \div 24$

$\therefore$ No of sides $= 15$ (1)

6. a) $m_{AB} = \frac{4-2}{-2-6} = -\frac{1}{4}$ (1)

b) $d = \sqrt{(4-2)^2 + (-2-6)^2}$ (1)

$= \sqrt{4 + 64}$

$= 2\sqrt{17}$ (accept $\sqrt{68}$)

c) $y - 2 = -\frac{1}{4}(x - 6)$ (1)

$4y - 8 = -x + 6$

$x + 4y - 14 = 0$

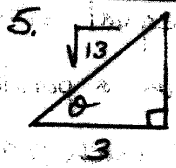
d) $d = \frac{|1(0) + 4(-4) + -14|}{\sqrt{1^2 + 4^2}} = \frac{30}{\sqrt{17}}$ (1)

e) $(4, 10)$ (1)

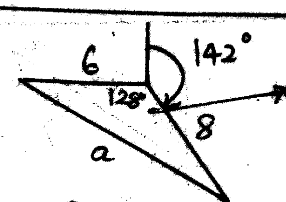
f) Area $= 2\left(\frac{1}{2} \times \frac{30}{\sqrt{17}} . 2\sqrt{17}\right)$

$= 60$ units^2 (1)

g) $x \geq 0$, $y \geq 0$ and $x + 4y - 14 \leq 0$ (1)

5. 

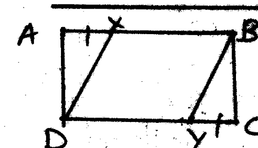
$\cos\theta = -\frac{3}{\sqrt{13}}$ (1) (1)

7. 

(1)

$a^2 = 6^2 + 8^2 - 2.6.8.\cos 128^\circ$ (1)

$= 12.9$ (1 d.p.) (1)

8. a) 

(1) AD = BC (opp. sides of a rect.)
AX = YC given

(1) $\angle A = \angle C = 90^\circ$ (Angles in a rectangle)

(1) $\Delta AXD \equiv \Delta BYC$ (SAS).

b) AB = CD (opp. sides of a rect.)

~~AX = YC~~

$\therefore$ BX = DY.

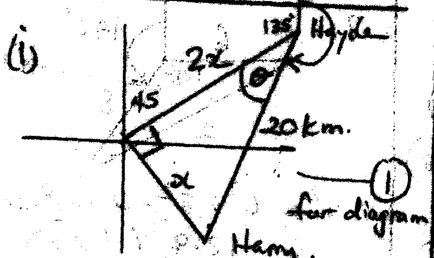
Since BX ∥ DY (on the opp. sides of a rectangle)

then XBYD is a parallelogram (1 pair of ∥ equal sides) (1)

since XBYD is a parallelogram

XY + BD bisect each other (1)

9. (i)



(1) for diagram

$5x^2 = 20^2$

$x^2 = 80$

$x = 4\sqrt{5}$ } (1)

$\therefore$ Harry walked $4\sqrt{5}$ km

(ii) $\tan\theta = \frac{1}{2}$

$\theta = 26°34'$ (1)

Bearing $= 360 - (135 + 26°34')$

$= 198°26'$ (1)

10a) LHS $= \frac{\cos\theta}{1-\sin\theta} - \frac{\cos\theta}{1+\sin\theta}$

$= \frac{\cos\theta + \cos\theta\sin\theta - \cos\theta + \cos\theta\sin\theta}{1-\sin^2\theta}$ (1)

$= \frac{2\sin\theta\cos\theta}{\cos^2\theta}$ (1)

$= 2\tan\theta$

b) $\therefore$ solve

$2\tan\theta = \cot\theta$

$2\tan\theta = \frac{1}{\tan\theta}$

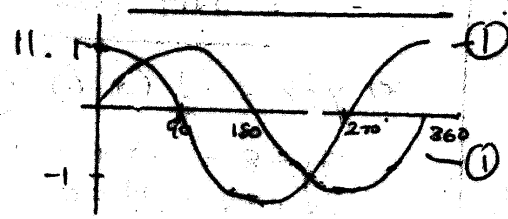
$\therefore \tan^2\theta = \frac{1}{2}$ (1)

$\tan\theta = \pm\frac{1}{\sqrt{2}}$

(1)

$\therefore \theta = 35°16', 144°44', 215°16', 324°44'$ (1)

11.



(1)

(1)

$\sin x = \cos x$

When $\tan x = 1$ ie $x = 45°, 225°$ (1)

$\sin x \geq \cos x$

$45° \leq x \leq 225°$ (1)

12. Solve for $0 \leq \theta \leq 360°$

$2\tan^2\theta - 5\sec\theta + 5 = 0$

$2(\sec^2\theta - 1) - 5\sec\theta + 5 = 0$

$2\sec^2\theta - 5\sec\theta + 3 = 0$

$(2\sec\theta - 3)(\sec\theta - 1) =$

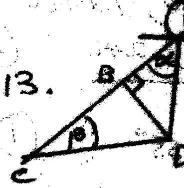
$\sec\theta = \frac{3}{2}$ $\sec\theta = 1$

$\therefore \cos\theta = \frac{2}{3}$ $\cos\theta = 1$ (1)

$\theta = 48°11', 311°49', 0, 360$

(1) (1)

13.



AC = x.

In ΔABD

$\sin\alpha = \frac{BD}{AD}$

$\therefore AD = \frac{BD}{\sin\alpha}$ (1) – (A)

In ΔACD $\frac{AD}{\sin\beta} = \frac{x}{\sin(180 -}$

$\therefore AD = \frac{x\sin\beta}{\sin(\alpha+\beta)}$ (1) – (B)

(A) – (B)

$\therefore \frac{BD}{\sin\alpha} = \frac{x\sin\beta}{\sin(\alpha+\beta)}$ (1)

$\therefore BD = \frac{x\sin\beta\sin\alpha}{\sin(\alpha+\beta)}$