Year 11 Mathematics

June 2011

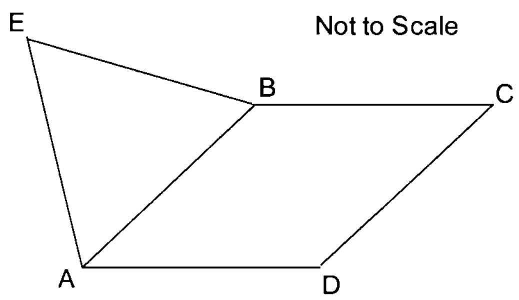
Time: 70 Minutes

DIRECTIONS

- Full working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Use black or blue pen only (*not pencils*) to write your solutions.
- No liquid paper is to be used. If a correction is to be made, one line is to be ruled through the incorrect answer.

1.	a) Find the domain and range of $y = 3\sqrt{2x-5}$.	**2**
	b) If $f(x) = x^2 + x$, determine if $f(x)$ is odd, even, or neither. Show all working.	**2**
2.	a) Sketch the region which satisfies $x^2 + y^2 < 1$	**3**
	b) State the domain and range of the region.	**3**

3.

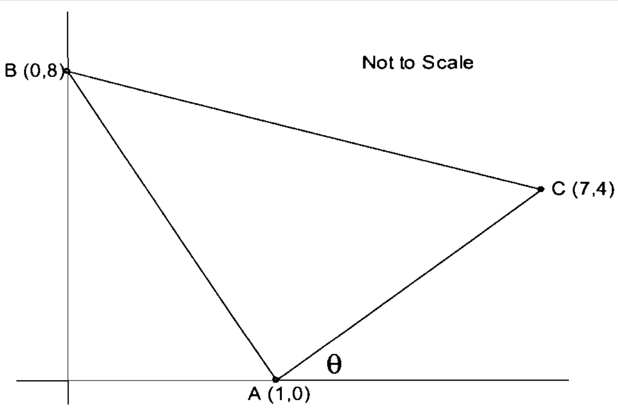


ABCD is a rhombus.
ABE is an equilateral triangle.
Given $\angle BCD = 48°$.

Find:

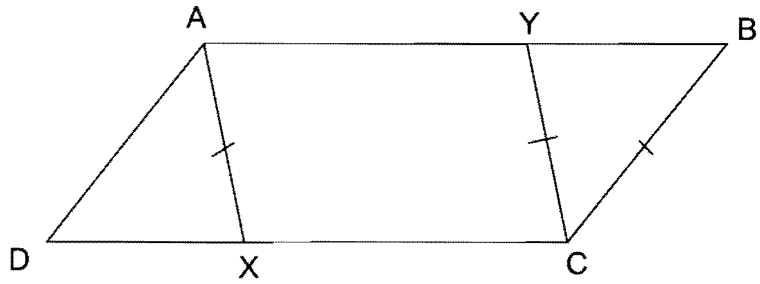
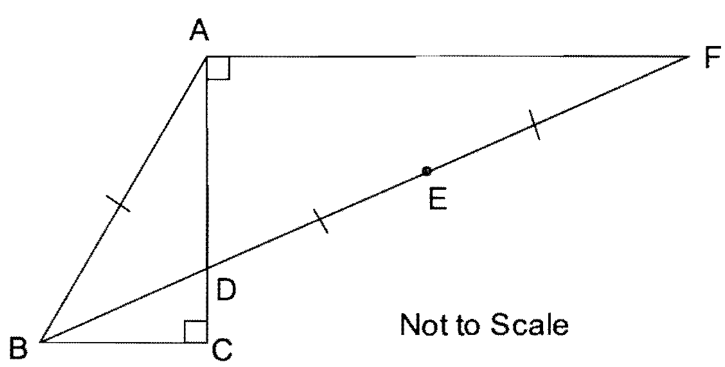
i)	$\angle EAD$, giving reasons for your answer.	**2**
ii)	$\angle EDA$, giving reasons for your answer.	**2**

4.



The points A, B, C have co-ordinates (1,0), (0,8), and (7,4) as shown.
The angle between line AC and the x-axis is θ.

i)	Find the size of angle θ in degrees.	**1**
ii)	Find the equation of line AC.	**1**
iii)	Find co-ordinates of D, the midpoint of AC.	**1**
iv)	Show that AC is perpendicular to BD.	**2**
v)	What does part (iv) show about ΔABC ?	**1**
vi)	Find the area of ΔABC .	**2**
vii)	Write down the co-ordinates of E such that ABCE is a rhombus.	**1**

5.	 $ABCD$ is a parallelogram $AX = CY = BC$ i) Prove $\Delta AXD \equiv \Delta BYC$ ii) Hence, prove $AYCX$ is a parallelogram.	3 2
6.	A function is defined as $$f(x) = \begin{cases} x & for \quad x < -1 \\ x^2 - 2 & for \quad -1 \leq x < 1 \\ x^2 & for \quad x \geq 1 \end{cases}$$ Evaluate a) $f(-3) + f(0) + f(1)$ b) $f(a^2 + 1)$	1 1
7.	a) Sketch the region $y \leq \sqrt{9 - x^2}$ b) Find the domain and range of $y = \sqrt{x^2 - 25}$	2 2
8.	a) Show that the line $3x + 4y + 25 = 0$ is a tangent to the circle $x^2 + y^2 = 25$ b) Find the equation of the line through $(a\cos\theta, a\sin\theta)$ perpendicular to $x\cos\theta + y\sin\theta = a$	2 3
9.	 In the diagram $AC \perp BC$, $AC \perp AF$ and $AB = DE = EF$ i) Show $\angle DBC = \angle DFA$ ii) G is on the line AF such that $EG \parallel AC$ Show $\Delta AGE \equiv \Delta FGE$ iii) Prove $\angle ABD = 2\angle DBC$	2 3 2

~ END OF EXAM ~

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Question 1.

a) $y = 3\sqrt{2x-5}$

D: $x \geq 2\frac{1}{2}$ ① R: $y \geq 0$ ①

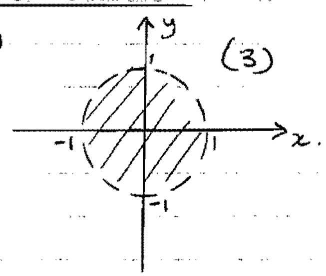
b) $f(x) = x^2 + x$ $f(-x) = (-x)^2 + (-x)$
$= x^2 - x$ ①

test even: $f(x) = f(-x)$
$x^2 + x \neq x^2 - x$ NOT EVEN

test odd: $f(x) = -f(-x)$ ①
$x^2 + x = -(x^2 - x)$
$\neq -x^2 + x$ NOT ODD

$\therefore$ $f(x)$ is neither

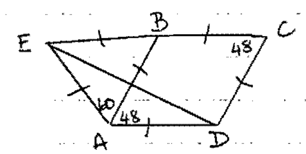
Question 2

a) 

(3)

b) D: $-1 < x < 1$ ③
R: $-1 < y < 1$

Question 3.



(a possible response)

i) $\angle BAD = \angle BCD$ (opposite $\angle$s equal in rhombus)
$= 48°$ ①

$\angle EAB = 60°$ ($\triangle ABE$ equilateral given)

$\therefore \angle EAD = 48 + 60$
$= 108°$ ①

ii) since $EA = AB$ (sides of equilateral $\triangle$)
and $AB = AD$ (sides of rhombus) ①
then $EA = AD$

$\angle EDA = \angle AED$ (equal $\angle$s opposite equal sides)
$= \frac{180 - 108}{2} = 36°$ ①

Question 4

i) $m = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{4-0}{7-1}$
$= \frac{2}{3}$

$\tan\theta = m$
$= \frac{2}{3}$
$\theta \doteqdot 34°$ ①

ii) $y - y_1 = m(x - x_1)$
$y - 0 = \frac{2}{3}(x - 1)$
$3y = 2x - 2$
$2x - 3y - 2 = 0$ ①

iii) D: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
$= \left(\frac{1+7}{2}, \frac{0+4}{2}\right)$
$= (4, 2)$ ①

iv) $m_1 = \frac{2}{3}$
$m_2 = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{8-2}{0-4}$
$= -\frac{3}{2}$ ①

$\therefore m_1 \times m_2 = \frac{2}{3} \times -\frac{3}{2}$ ①
$= -1$

$AC \perp BD$

(could use perp dis formula.

v) isosceles $\triangle$ ①

vi) $\text{length}_{AC} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$= \sqrt{(7-1)^2 + (4-0)^2}$
$= \sqrt{52}$ units ①

$D_{BD} = \sqrt{(0-4)^2 + (8-2)^2}$
$= \sqrt{52}$

Area $\triangle_{ABC} = \frac{1}{2} \times \sqrt{52} \times \sqrt{52}$
$= 26\ u^2$ ①

vii) $\text{Mid pt}_{AC} = (4, 2)$

Mid pt BE
$4 = \frac{x+0}{2}$, $2 = \frac{8+y}{2}$
$(8, -4)$ ①

$\boxed{9}$

Question 5.

i) (a possible response) (3) marks.

In $\triangle AXB$ and $\triangle BYC$

$A\hat{D}X = Y\hat{B}C$ (opposite $\angle$s in parallelogram equal)

$AD = BC$ (opposite sides in parallelogram equal)

$AX = CY$ (given)

$\therefore AD = AX = BC = CY$

$\triangle ADX$ is isosceles (2 sides equal)

then $\angle AXD = \angle ADX$ (base $\angle$s, isosceles $\triangle$)

Similarly $\triangle BYC$ is isosceles

and $\angle CYB = \angle CBY$ (base $\angle$s, isosceles $\triangle$)

$= \angle ADX$ (from above)

$\therefore \triangle AXB \equiv \triangle BYC$ (AAS)

ii) Since $AB = DC$ (opposite sides in parallelogram)

and $DX = BY$ (matching sides in congruent $\triangle$'s)

then $AY = AB - YB$ } (subtraction of sides)

and $CX = CD - DX$ }

(2)

$\therefore AY = CX$

and $AX = CY$ (given)

$\therefore AYCX$ is a parallelogram (2 pairs of equal sides)

Question 6.

a) $f(-3) = -3 \quad f(0) = -2 \quad f(1) = 1$

$\therefore f(-3) + f(0) + f(1) = -3 - 2 + 1$

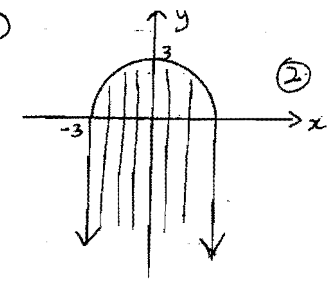
$= -4$ (1)

b) $f(a^2+1) = (a^2+1)^2$ (1)

$= a^4 + 2a^2 + 1$

Question 7.

a)



(2)

b) $y = \sqrt{x^2 - 25}$

$x^2 - 25 \geq 0$

D: $x \leq -5, \; x \geq 5$ (1)

R: $y \geq 0$ (1)

Question 8

a) tangent to circle – perp dist = radius = 5

centre = (0,0)

$$\text{perp } d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2+b^2}} \right|$$

$$= \left| \frac{3\times 0 + 4\times 0 + 25}{\sqrt{9+16}} \right|$$

$$= \frac{25}{5} = 5 \quad (1)$$

$\therefore$ since line is perpendicular to radius it is a tangent (1)

b) $x\cos\theta + y\sin\theta = a$

$y\sin\theta = a - x\cos\theta$

$y = \frac{a}{\sin\theta} - \frac{\cos\theta}{\sin\theta}x$ (1)

$m_1 = \frac{-\cos\theta}{\sin\theta}$ (1) now $m_2 = -\frac{1}{m_1} = \frac{\sin\theta}{\cos\theta}$

$y - y_1 = m(x - x_1)$

$y - a\sin\theta = \frac{\sin\theta}{\cos\theta}(x - a\cos\theta)$

$\cos\theta\, y - \cancel{a\sin\theta\cos\theta} = \sin\theta\, x - \cancel{a\sin\theta\cos\theta}$

$y = \frac{\sin\theta}{\cos\theta}x$ }

$y = \tan\theta\, x$ } (1)

Question 9. (possible solutions)

i) aim: show $\angle DBC = \angle DFA$.

Method: $\angle ADF = \angle CDB$ (vertically opposite $\angle$s)

$\angle CAF = \angle ACB$ (given – ⊾)

$= 90°$

$\therefore \angle AFD = \angle DBC$ ($\angle$ sum of $\triangle$s) (2)

ii) aim: to prove $\triangle AGE \equiv \triangle FGE$

method: $\angle CAG = \angle EGF$ (corresponding $\angle$s, $AC \parallel GE$)

$= 90°$

$\angle AGE = 180 - \angle EGF$ ($\angle$ sum straight line)

$= 90°$

since $EF = ED$ (given) (3)

then $EF : ED = 1 : 1$

$FG : GA = 1 : 1$ (Intercepts on $\parallel$ lines)

$FG = GA$

EG is common

$\therefore \triangle AGE \equiv \triangle FGE$ (SAS)

iii) aim: prove $\angle ABD = 2 \times \angle DBC$

method: let $\angle DBC = x$

$\therefore \angle EFG = x = \angle GAE$ (matching $\angle$s in congruent $\triangle$s)

$\angle AED = 2x$ (External $\angle$ $\triangle$ equals sum of interior opposite $\angle$s)

$EF = AE$ (matching sides $\equiv \triangle$s) (2)

$\therefore AB = AE$

$\angle ABE = 2x$ (base $\angle$s in isosceles $\triangle$ equal)

$= 2 \times \angle DBC$.