BAULKHAM HILLS HIGH SCHOOL

JUNE 2012
YEAR 11 TASK 2

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 60 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in questions 6 to 14
- Marks may be deducted for careless or badly arranged work

Total marks – 39
Exam consists of 4 pages.

This paper consists of TWO sections.

<u>Section 1</u> – Page 2
Questions 1-5 (5 marks)

- Attempt Question 1-5

<u>Section II</u> – Pages 3-4 (34 marks)

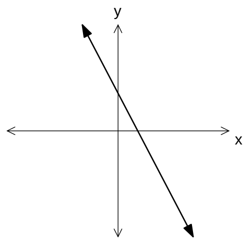
- Attempt questions 6 to 14

Section I - 5 marks
Attempt questions 1-5

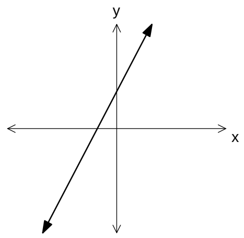
Use the multiple choice answer sheet for question 1-5

1. Which line could have the equation $y = -2x + 4$?

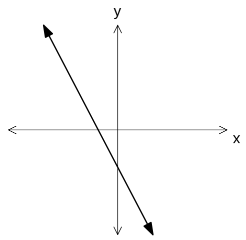
(A)



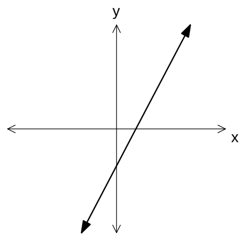
(B)



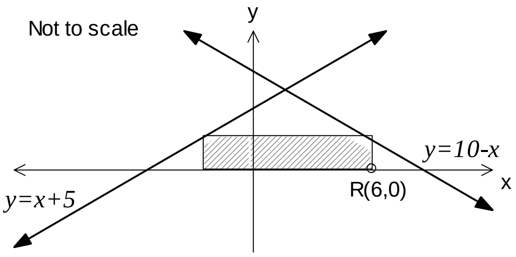
(C)



(D)



2. If R is the point with co-ordinates (6,0)



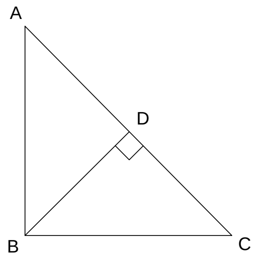
The area of the shaded rectangle is

(A) 20 (B) 24 (C) 28 (D) 60

3. If $(2, k)$ lies on the line $x + 2y = 8$ then the value of k is

(A) 2 (B) 3 (C) 4 (D) 5

4.



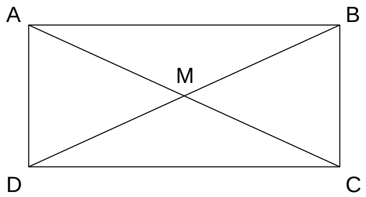
In the diagram, ΔABD is similar to ΔCBD

Which of the following is not necessarily true?

(A) $AB = BC$ (B) $AD = DC$

(C) $AB \perp BC$ (D) $BD \perp AC$

5.



In the diagram, $\Delta AMB \equiv \Delta BMC \equiv \Delta CMD \equiv \Delta DMA$.

$ABCD$

(A) is a parallelogram (B) is a rectangle

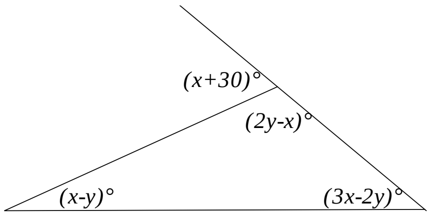
(C) is a rhombus (D) cannot be determined

End of Section 1

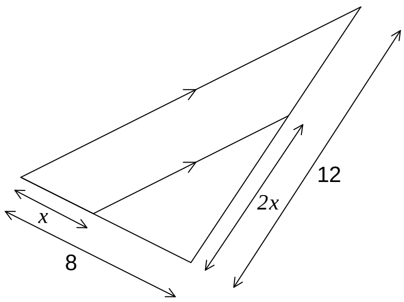
Section II – Extended Response

Attempt questions 6-14.
Answer each question on a SEPARATE PAGE. Clearly indicate question number.
Each piece of paper must show your name.
All necessary working should be shown in every question.

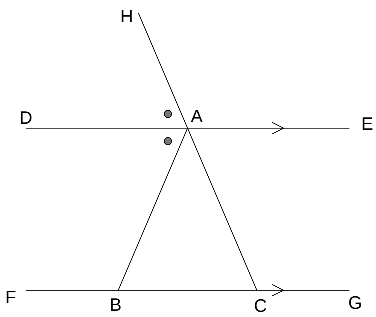
6. Find the values of x and y in the figure below. (no reasons required) **2**



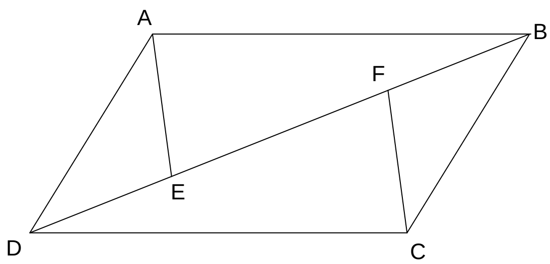
7. In the diagram below, find the value of .
(no reasons required) **2**



8. Prove that ΔABC is an isosceles triangle. **3**



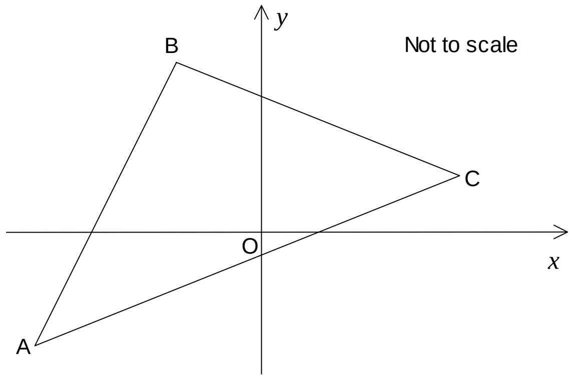
9. $ABCD$ is a parallelogram with $DE = BF$



a) Show that $\Delta AED \equiv \Delta BCF$ **2**

b) Prove that AE is parallel to CF **2**

10.



The diagram shows the points $A(-3,-2)$, $B(-1,4)$ and $C(5,2)$

Copy this diagram into your answer booklet.

a) Find the gradient of AC — 1

b) Point P is the midpoint of AC. Show that the coordinates of P are $(1,0)$. Mark P on your diagram. — 1

c) Show that the equation of the line perpendicular to AC and passing through the point P is $2x + y - 2 = 0$ — 2

d) Show that B lies on the line $2x + y - 2 = 0$ — 1

e) Show that the length of BP is $2\sqrt{5}$ units — 1

f) Point P is the midpoint of BD. Find the coordinates of D — 1

g) Explain why $ABCD$ is a rhombus — 1

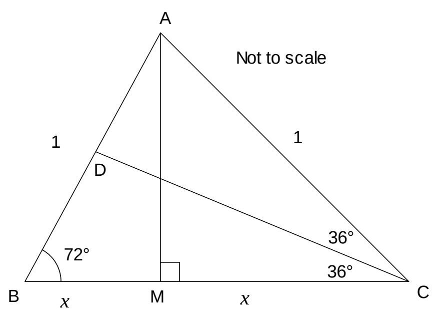
h) Find the area of the rhombus ABCD — 2

11. Show that the line $7x - 4y + 5 = 0$ passes through the point of intersection of the lines $3x - 2y + 3 = 0$ and $2x - y + 1 = 0$ without finding the point of intersection of these lines. — 3

12. Show that the perpendicular distance of the point $(2p, -p)$ from the line $4px + 3py + 5 = 0$ is $\frac{p^2+1}{|p|}$ — 2

13. The sides of a triangle are $(2x-1)\ cm$, $(x+1)\ cm$ and $(11-x)\ cm$. Show that $2\frac{3}{4} < x < 6\frac{1}{2}$ — 2

14.



In the diagram, ABC is an isosceles triangle where $\angle ABC = \angle BCA = 72°$, $AB = AC = 1$ and $BC = 2x$.

Angle BCA is bisected by CD and angle BAC is bisected by AM which is also the perpendicular bisector of BC.
Copy the diagram in your answer booklet.

a) Show that $AD = 2x$ — 2

b) Show that triangles ABC and CBD are similar. — 1

c) Using (b), find the exact value of x — 2

d) Hence find the exact value of $\sin 18°$ — 1

~ End of Exam ~

JUNE YR 11 MATHEMATICS 2012 60 mins

M-C 1 A
2 C
3 D
4 C
5 C

6. $x+30+2y-x=180$
$2y=150$
$y=75$ ✓
$x+30=x-y+3x-2y$
$30+3y=3x$
$3x=255$
$x=85$ ✓
$\therefore x=85,\ y=75$

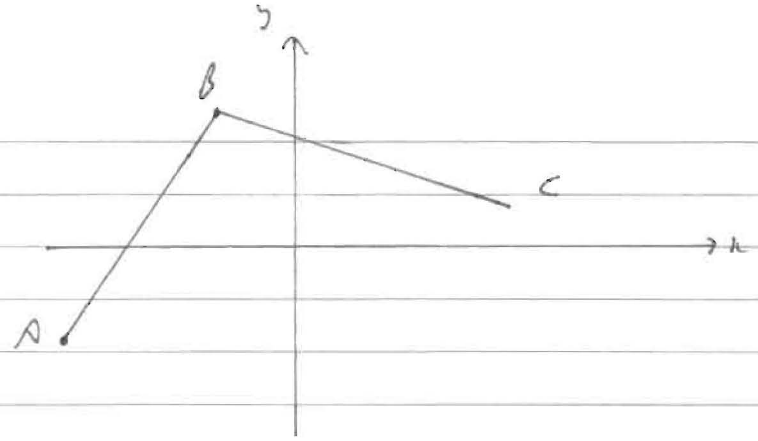
7. $\frac{2x}{12}=\frac{8-x}{8}$ ✓
$16x=12(8-x)$
$4x=3(8-x)$
$4x=24-3x$
$7x=24$
$x=3\frac{3}{7}$ ✓

8. $\angle ACB=\angle HAD$ (corresponding angles, $DE \parallel FG$) ✓
$\angle ABC=\angle DAB$ (alternate angles, $DE \parallel FG$) ✓
$\therefore \angle ABC=\angle ACB$ ($\angle HAD=\angle DAD$) ✓

9.a) In Δ's AED and CFB
$AD=BC$ (opposite sides of a parallelogram) ✓
$DE=BF$ (given) ✓
$\angle ADE=\angle FBC$ (alternate angles, $AD \parallel BC$) ✓
$\therefore \Delta AED \equiv \Delta BCF$ (SAS) ✓

b) $\angle BFC=\angle AED$ (matching angles in congruent Δ's AED, CFB)
$\angle CFE=180°-\angle CFB$ (angle sum on straight line EB) ✓
$\angle AEF=180°-\angle AED$ (angle sum on straight line EB) ✓
$\therefore \angle AEF=\angle EFC$
$\therefore AE \parallel FC$ (alternate angles are equal) ✓

10.



a) $M_{AC} = \frac{2 - -2}{5 - -3}$

$= \frac{4}{8}$

$= \frac{1}{2}$ ✓

b) Midpoint $= \left(\frac{-3+5}{2}, \frac{-2+2}{2}\right)$ ✓

$= (1, 0)$

c) $M_{\perp AC} = -2$ ✓

$y - 0 = -2(x - 1)$

$y = -2x + 2$

$\therefore 2x + y - 2 = 0$ as required ✓

d) $LHS = 2x + y - 2$

$= 2 \times -1 + 4 - 2$

$= 0$ ✓

$= RHS$

$\therefore B$ lies on $2x + y - 2 = 0$

e) $BP = \sqrt{(-1-1)^2 + (4-0)^2}$

$= \sqrt{(-2)^2 + 4^2}$

$= \sqrt{20}$ ✓

$= 2\sqrt{5}$

f) $1 = \frac{-1+x}{2}$ $\quad 0 = \frac{4+y}{2}$

$2 = -1 + x$ $\quad 0 = 4 + y$

$x = 3$ $\quad y = -4$

$\therefore D$ is $(3, -4)$ ✓

g) Diagonals bisect each other at 90° ✓
(ie P is midpoint of diagonals BD and AC).

10 h) $CP = \sqrt{(5-1)^2 + (2-0)^2}$

$= \sqrt{4^2 + 2^2}$

$= 2\sqrt{5}$ ✓

Area $= \left(\frac{2\sqrt{5} + 2\sqrt{5}}{2}\right) \times 4$ ✓

$\therefore$ Area $= 40$ units^2

11. let $3x - 2y + 3 + k(2x - y + 1) = 0$ where k is a real no.

$(3+2k)x - (2+k)y + 3 + k = 0$

gradient $= \frac{3+2k}{2+k}$ ✓

Now gradient of $7x - 4y + 5 = 0$ is $m = \frac{7}{4}$

$\therefore \frac{3+2k}{2+k} = \frac{7}{4}$

$12 + 8k = 14 + 7k$ ✓

$k = 2$

$\therefore$ Equation is

$(3+4)x - (2+2)y + 3 + 2 = 0$ ✓

$7x - 4y + 5 = 0$ as req'd

12. Perp. dist $= \left| \frac{4p \times 2p + 3p \times -p + 5}{\sqrt{(4p)^2 + (3p)^2}} \right|$ ✓

$= \left| \frac{8p^2 - 3p^2 + 5}{\sqrt{16p^2 + 9p^2}} \right|$

$= \frac{|5p^2 + 5|}{\sqrt{25p^2}}$

$= \frac{5|p^2 + 1|}{5|p|}$ ✓

$\therefore$ Perp distance $= \frac{p^2 + 1}{|p|}$

13. Since the sum of the two shorter sides of a triangle are more than the longest side, then

$2x-1+x+1 > 11-x$ and $11-x+x+1 > 2x-1$

$3x > 11-x \qquad 12 > 2x-1$

$4x > 11 \qquad 2x < 13$

$x > \frac{11}{4} \qquad x < 6\frac{1}{2}$

$x > 2\frac{3}{4}$ ✓

[Note $11-x+2x-1 > x+1$
* $x+10 > x+1$
$10 > 1$] * This is since if $x > 2\frac{3}{4}$ then $x+1$ is not the longest side

$\therefore\ 2\frac{3}{4} < x < 6\frac{1}{2}$ ✓

14. a) $\angle CDB = 72°$ (angle sum of ΔBCD)

$CD = BC$ (equal sides opposite equal angles in ΔBCD) ✓

$\therefore CD = 2x$ ($BC = 2x$)

$\angle DAC = 36°$ (angles sum of ΔBAC)

$\therefore\ CD = AD$ (equal sides opposite equal angles in ΔACD) ✓

$\therefore\ AD = 2x$ ($CD = 2x$)

b) In Δ's ABC and CBD

$\angle ABC$ is common

$\angle BDC = \angle ACB = 72°$ (from (a)) ✓

$\therefore\ \Delta ABC \,|||\, \Delta CBD$ (two pairs of angles are equal)

c) $\frac{CB}{AC} = \frac{BD}{BC}$ (matching sides in similar Δ's, ABC & CBD)

$\frac{2x}{1} = \frac{1-2x}{2x}$ ✓

$4x^2 = 1-2x$

$4x^2 + 2x - 1 = 0$

$x = \frac{-2 \pm \sqrt{4 - 4 \times 4 \times -1}}{2 \times 4}$

$x = \frac{-2 \pm \sqrt{20}}{8}$

$x = \frac{-2 \pm 2\sqrt{5}}{8}$

$x = \frac{-1 \pm \sqrt{5}}{4}$

But $x > 0$, $\therefore\ x = \frac{\sqrt{5}-1}{4}$ ✓

(d) $\angle MAB = 18°$ (AM bisects $\angle BAC$)

$\sin 18° = \frac{x}{1}$ ✓

$\sin 18° = \frac{\sqrt{5}-1}{4}$