BAULKHAM HILLS HIGH SCHOOL

JULY 2013
YEAR 11 TASK 2

Mathematics

General Instructions

- Working time – 70 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in questions 6 to 13
- Marks may be deducted for careless or badly arranged work

Total marks – 55
Exam consists of 4 pages.

This paper consists of TWO sections.

Section 1 – Page 2
Questions 1-5 (5 marks)

- Attempt Question 1-5

Section II – Pages 3-4 (50 marks)

- Attempt questions 6 to 13

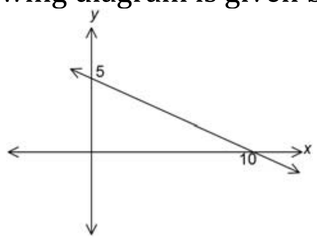
Section I - 5 marks
Attempt questions 1-5

Use the multiple choice answer sheet for question 1-5

1. Evaluate $\lim_{x \to 5} \dfrac{2x^2 - 11x + 5}{x^2 - 25}$

(A) 0 (B) $\frac{9}{25}$ (C) $\frac{9}{10}$ (D) not defined

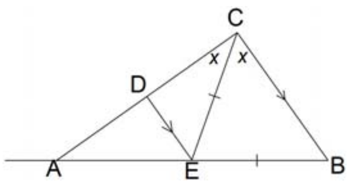
2. Equation of the line in the following diagram is given by



(A) $y = \frac{1}{2}x + 5$ (B) $y = x + 5$

(C) $y = -\frac{1}{2}x + 5$ (D) $y = -\frac{1}{2}x + 10$

3.

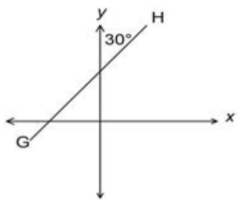


In the diagram, $EB = EC, \angle BCE = \angle DCE$ and $BC \parallel ED. \Delta CDE$

(A) is equilateral (B) is isosceles

(C) is scalene (D) cannot be determined

4.

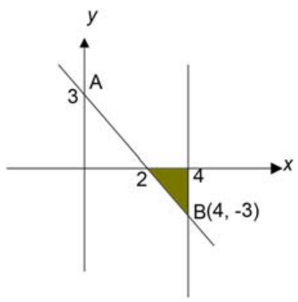


The line GH makes an angle of 30° with the y-axis, as shown in the diagram. What is the gradient of GH

(A) $\sqrt{3}$ (B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{2}}$ (D) $\frac{\sqrt{3}}{2}$

5.



The three inequalities to describe the shaded region given above are

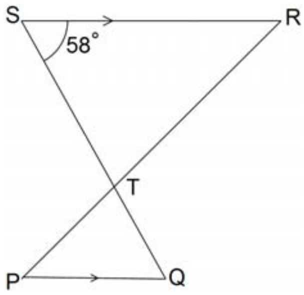
(A) $y \le 0, x \ge 2, 3x - 2y + 6 \le 0$ (B) $y \le 0, x \le 4, 3x - 2y - 6 \le 0$

(C) $y \le 0, x \ge 2, 3x - 2y - 6 \ge 0$ (D) $y \le 0, x \le 4, 3x - 2y + 6 \ge 0$

Section II – Extended Response

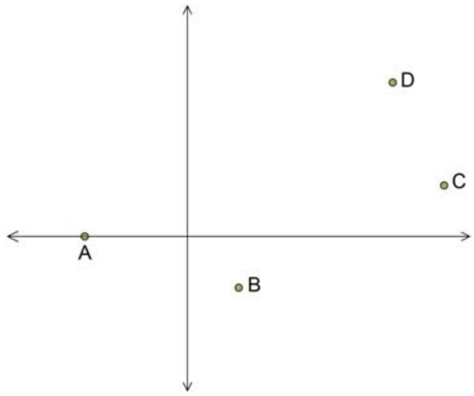
Attempt questions 6-13.
Answer each question on a SEPARATE PAGE. Clearly indicate question number.
Each piece of paper must show your name.
All necessary working should be shown in every question.

6.



Given $SR \parallel PQ$ and that SQ, RP are straight lines, and $\angle RST = 58°$, $\angle PTQ = 53°$, find the size of $\angle TPQ$.(without giving reason) 1

7.

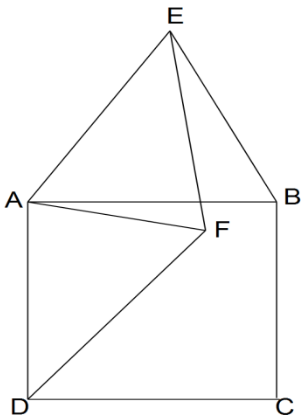
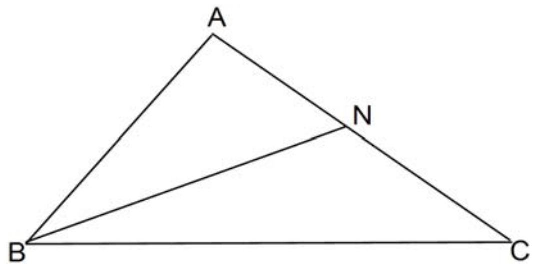


Consider the quadrilateral $ABCD$ with A(-2, 0) , B(1, -1), C(5, 1) and D(4, 3) .
Copy the diagram with all the information given.

(i) Find the lengths of AD and BC. 3
(ii) Find the gradient of AD. 1
(iii) Show that the equation of the line passing through A and D is given by $x - 2y + 2 = 0$. 2
(iv) Show that $AD \parallel BC$. 2
(v) What type of quadrilateral is $ABCD$? 1
(vi) Calculate the perpendicular distance of B from the line AD. 2
(vii) Calculate the area of $ABCD$. 2

8. Differentiate the following

(i) $y = 5x^3 + \frac{1}{x}$ 2
(ii) $y = \frac{3x^2+10x}{x}$ 2
(iii) $y = (3x+7)(1-x)^3$ 2
(iv) $y = \frac{5x^2-2}{2x+1}$ 2

9.	Find the derivative of the function given by $y = x^2 + 2x$ using first principles.	**3**
10.	Find the equation of the line which is passing through the intersection of $x - 3y + 5 = 0$ and $x + 2y = 0$ and the point $(3, 1)$.	**3**
11.	(i) Show that the equation of the normal to the curve $y = \frac{1}{\sqrt{x^2-3}}$ at $(2,1)$ is given by $x - 2y = 0$. (ii) Find the coordinates of the point Q where the normal meets the curve again.	**2** **3**
12.	 In the diagram $ABCD$ is a square and AEB is an equilateral triangle. EF bisect $\angle AEB$. $\angle EAF = \angle DAF$. Copy the diagram and label the given information. (i) Show that $\angle BAF = 15°$. (ii) Prove that $EF = DF$	 **3** **4**
13.	ABC is a triangle and N is point on AC. $\angle ABN = \angle CBN = \angle BCN = \theta$. $BC = 2a$, $CA = b, AB = c. BN = d$.  (i) Copy the diagram, labelling all of the given information. (ii) Explain why $CN = d$ (iii) Prove that $\angle ANB = \angle ABC$ (iv) Prove that ΔABN is similar to ΔACB. (v) Show that $2ac = bd$ (vi) Hence show that $c^2 = b^2 - 2ac$	 **1** **1** **2** **2** **2** **2**

~ End of Exam ~

P2 2C 2013 — Solutions Yr 11 Mathematics 2013

M.C

1) C 2) C 3) B 4) A

Q6 $\angle TPQ = 69$ — (1)

Q7 (i) $AD = \sqrt{(4+2)^2+3^2}$ — (1)

$= \sqrt{36+9} = \sqrt{45}$ or $3\sqrt{5}$ — (1)

$BC = \sqrt{(5-1)^2+2^2}$

$= \sqrt{20} = 2\sqrt{5}$ — (1)

(ii) $m_{AD} = \frac{3-0}{4+2} = \frac{3}{6} = \frac{1}{2}$ — (1)

(iii) $y - 0 = \frac{1}{2}(x+2)$ — (1)

$2y = x + 2$

$x - 2y + 2 = 0$ } — (1)

(iv) $m_{BC} = \frac{1+1}{5-1} = \frac{2}{4} = \frac{1}{2}$ — (1)

$\therefore m_{AD} = m_{BC}$

Hence $AD \parallel BC$ } — (1)

(v) Trapezium — (1)

(vi) $D = \frac{|1\times 1 - 2\times -1 + 2|}{\sqrt{1+2^2}}$ — (1) — (1)

$= \frac{5}{\sqrt{5}} = \sqrt{5}$ — (1)

(vii) $A_r = \frac{1}{2} \times h \times (a+b)$

$= \frac{1}{2} \times \sqrt{5} \times (3\sqrt{5} + 2\sqrt{5})$

$= \frac{1}{2} \times \sqrt{5} + (5\sqrt{5})$

$= \frac{25}{2} = 12.5$ unit^2 — (2) with working & correct ans.

Q8 (i) $y' = 15x^2 - \frac{1}{x^2}$ — (1) (1)

(ii) $y = 3x + 10$

$y' = 3$ — (2)

(iii) $y' = 3(1-x)^3 + (3x-7) \times 3(1-x)^2 \times -1$ — (1) (1)

$= 3(1-x)^3 - 3(3x-7)(1-x)^2$

$= 3(1-x)^2[(1-x) - 3x + 7]$

$= 3(1-x)^2(8-4x) = 12(1-x)^2(2-x)$

(iv) $y' = \frac{(2x+1)(10x) - (5x^2-2)(2)}{(2x+1)^2}$ — (1) — (1)

$= \frac{20x^2 + 10x - 10x^2 + 4}{(2x+1)^2}$

$= \frac{10x^2 + 10x + 4}{(2x+1)^2}$

Q9 $f(x) = y = x^2 + 2x$ — (1)

$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 + 2(x+h) - x^2 - 2x}{h}$

$= \lim_{h \to 0} \frac{x^2 + h^2 + 2xh + 2x + 2h - x^2 - 2x}{h}$ — (1)

$= \lim_{h \to 0} \frac{h^2 + 2xh + 2h}{h}$

$= \lim_{h \to 0} h + 2x + 2$ } (1)

$f'(x) = 2x + 2$

Q10
$$x - 3y + 5 = 0$$
$$x + 2y = 0$$
$$-5y + 5 = 0$$
$$y = 1, \quad x = -2 \quad \text{— (1)}$$

$\therefore$ eq. of the line passing through $(-2,1)$ & $(3,1)$

$$m = \frac{1-1}{3+2} = 0 \quad \text{— (1)}$$

$\therefore$ eq. is

$$y - 1 = 0(x+2) \quad \text{— (1)}$$
$$\boxed{y = 1}$$

OR

$$x - 3y + 5 + k(x+2y) = 0 \quad \text{— (1)}$$
$$3 - 3\times1 + 5 + k(3+2) = 0$$
$$k = -1 \quad \text{— (1)}$$

$\therefore$ eq. is

$$x - 3y + 5 - 1(x+2y) = 0$$
$$x - 3y + 5 - x - 2y = 0$$
$$-5y = -5$$
$$\boxed{y = 1} \quad \text{— (1)}$$

Q11 (i)
$$y = (x^2-3)^{-1/2}$$
$$y' = -\frac{1}{2}(x^2-3)^{-3/2} \times 2x$$
$$= -x(x^2-3)^{-3/2} \quad \text{— (1)}$$

m_T at $x = 2$

$$m_T = -2(4-3)^{-3/2}$$
$$= -2$$
$$m_N = \frac{1}{2} \quad \text{— (1)}$$

eq of Normal $y - 1 = \frac{1}{2}(x-2)$

$$\Rightarrow 2y - 2 = x - 2$$
$$x - 2y = 0$$

ii) Pt of int. of the Normal to the curve $\left(\frac{x}{2}\right)^2 = \left(\frac{1}{\sqrt{x^2-3}}\right)^2$ — (1)

$$\frac{x^2}{4} = \frac{1}{x^2-3}$$
$$x^4 - 3x^2 = 4$$

Sub $x^2 = t$

$$t^2 - 3t - 4 = 0 \quad \text{— (1)}$$
$$(t-4)(t+1) = 0$$
$$t = 4, \quad t = -1$$
$$x^2 = 4 \qquad x^2 = -1 \text{ not possible} \quad \text{— (1)}$$
$$x = \pm 2$$
$$x = 2, y = 1 \quad \text{or} \quad x = -2, y = 1$$
$$Q(-2,1) \quad \text{— (1)}$$

OR Does not the curve anywhere el.

Q12 In dia ABCD is a sq & AEB is an equilateral Δ

The line EF bisects $\angle AEB$ & $\angle EA$

$$\angle EAF = \angle DAF$$

(i) let $\angle BAF = x$

$\angle EAB = 60°$ (all $\angle$s of equilateral Δ)

$\therefore \angle EAF = 60° + x$ — (1)

$\angle DAB = 90°$ (ABCD is a sq) — (1)

$$\angle DAF = 90 - x$$

(1) $90 - x = 60 + x$ (given $\angle DAF = \angle EAF$)

$$2x = 30°$$
$$x = 15°$$

(ii) In Δ's DAF & EAF

(1) AD = AB (sides of a sq)

AE = AB (sides of equilateral Δ)

$\therefore$ AD = AE — (1)

$\angle DAF = \angle EAF$ given

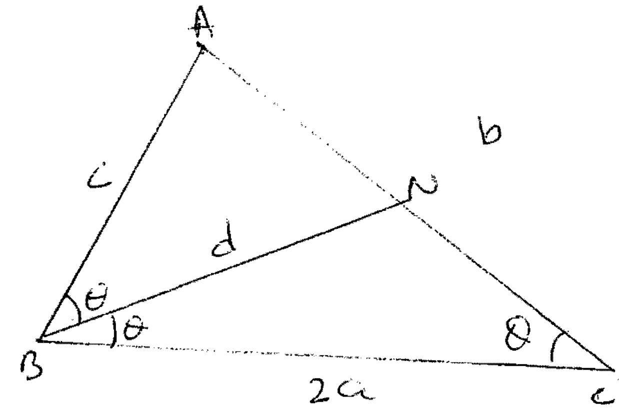
AF is common

$\therefore \Delta AEF \equiv \Delta ADF$ (SAS) — (1)

$\therefore$ EF = DF (as matching sides of $\equiv \Delta$s) — (1)

Q13.

(i)



— (1)

(ii) BN = CN (sides opp equal ∠s)

∴ BN = CN = d. — (1)

(iii) ∠BNC = 180 − 2θ (∠ sum of Δ) — (1)

∴ ∠ANB = 2θ (st line ∠) — (1)

Also ∠ABC = θ + θ = 2θ — given

Hence ∠ANB = ∠ABC.

(iv) In ΔABN & ΔACB

∠BAC common
∠ANB = ∠ABC (proved above) — (1)

∴ ΔABN ||| ΔACB (all matching ∠s are =) — (1)

v) $\frac{BN}{BC} = \frac{AN}{AB} = \frac{AB}{AC}$

(matching sides are in the same ratio in ||| Δs) — (1)

$\frac{d}{2a} = \frac{b-d}{c} = \frac{c}{b}$ — (1)

$\frac{d}{2a} = \frac{c}{b}$

$bd = 2ac.$

(vi) $\frac{b-d}{c} = \frac{c}{b}$ (1)

$c^2 = b^2 - bd$ — (1)

but $bd = 2ac$ (from above)

$\therefore c^2 = b^2 - 2ac$