BAULKHAM HILLS HIGH SCHOOL

JULY 2014
YEAR 11 TASK 2

Mathematics

General Instructions
- Working time – 60 minutes, plus 5 minutes reading time
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in questions 6 to 14
- Marks may be deducted for careless or badly arranged work

Total marks – 47
Exam consists of 5 pages.

This paper consists of TWO sections.

Section 1 – Page 2
Questions 1-5 (5 marks)
- Attempt Question 1-5

Section II – Pages 3-5 (42 marks)
- Attempt questions 6 to 14

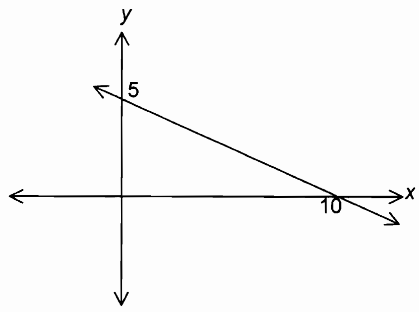
Section I - **5 marks**
Attempt questions 1-5

Use the multiple choice answer sheet (in the booklet) for question 1-5

1. The midpoint of the line joining (0,-5) and (d,0) is:

(A) $\left(\frac{d-5}{2},0\right)$ (B) $\left(0,\frac{5-d}{2}\right)$ (C) $\left(\frac{d}{2},\frac{-5}{2}\right)$ (D) $\left(\frac{d}{2},\frac{5}{2}\right)$

2.



The angle of inclination with the positive x-axis of the line shown above is approximately:

(A) 27° (B) 63° (C) 117° (D) 153°

3. ABCD is a parallelogram with diagonals AC and BD.
Consider the following statements:

I. If these diagonals are perpendicular, then ABCD is a rhombus.

II. If these diagonals are equal, then ABCD is a square.

Which of these statements are correct?

(A) I only (B) II only

(C) Both I and II (D) Neither I nor II

4. The centre of a circle is (-3,4) and the circle passes through (1,2). The length of the diameter is:

(A) $2\sqrt{2}$ (B) $2\sqrt{5}$ (C) $4\sqrt{2}$ (D) $4\sqrt{5}$

5. Evaluate $\lim\limits_{x\to\infty} \dfrac{3x^3 - 4x + 5}{x^2 - 3x^3}$

(A) 1 (B) -1 (C) 3 (D) -3

Section II – Extended Response

Attempt questions 6-14.
Answer each question in the booklet provided. Clearly indicate question number.
All necessary working should be shown in every question.

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6. Find the equation of the line which is perpendicular to $y = -2x + 3$ and passes through (3,-4). Answer in general form. **3**

7. Evaluate, clearly showing working to justify your answer: **2**

$$\lim_{x \to 3} \frac{x^3 - 27}{x^2 - 9}$$

8. Differentiate the following

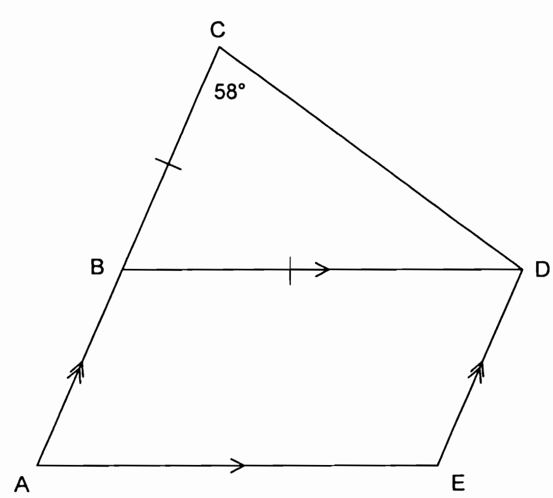
a) $y = 3x^4 + 4x - 5$ **2**

b) $y = \frac{x^4 - 10x^5}{5x^2}$ **2**

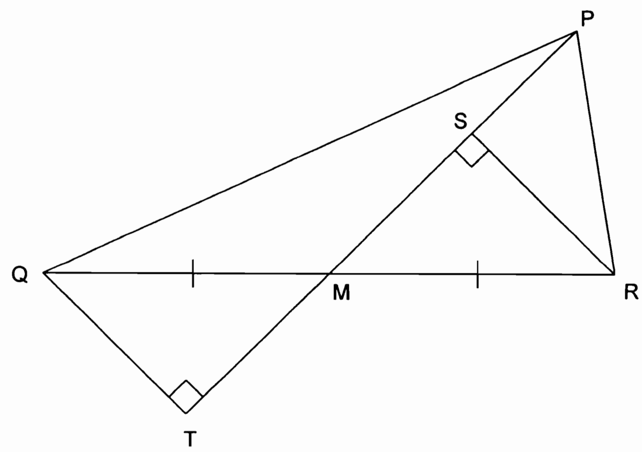
c) $f(x) = (x^2 + 1).\sqrt{x - 1}$ **2**

d) $y = \frac{x+1}{4x-1}$ **2**

9. In the diagram shown, AB||ED, BD||AE, BC=BD and the points A, B and C are collinear. If $\angle BCD = 58°$, find the size of $\angle DEA$, giving all reasons **3**



10. QT and RS are perpendicular to PT. M is the midpoint of QR and TS=15cm.

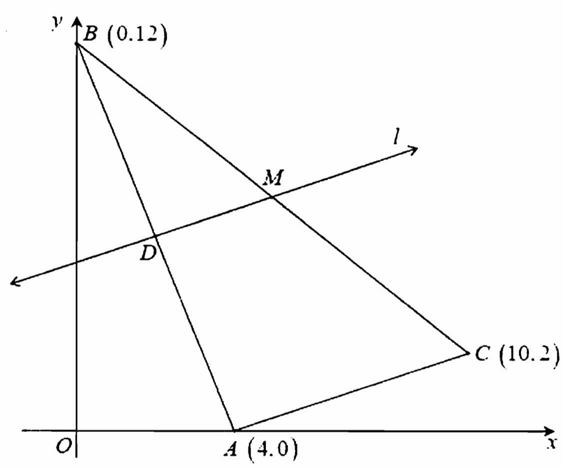


a) Prove that ΔQMT is congruent to ΔRMS. **3**

b) Find the length of MT with reasons. **1**

c) What type of quadrilateral is QSRT? Justify your answer. **1**

11.



In the above diagram, A, B and C are the points (4,0), (0,12) and (10,2) respectively

(i) Find the gradient of AC. **1**

(ii) Find the coordinates of D, the midpoint of AB. **1**

(iii) The line l is parallel to AC and passes through D. Show that the equation of l is $x - 3y + 16 = 0$ **1**

(iv) The line l meets BC at M. Explain why M must be the midpoint of BC. **1**

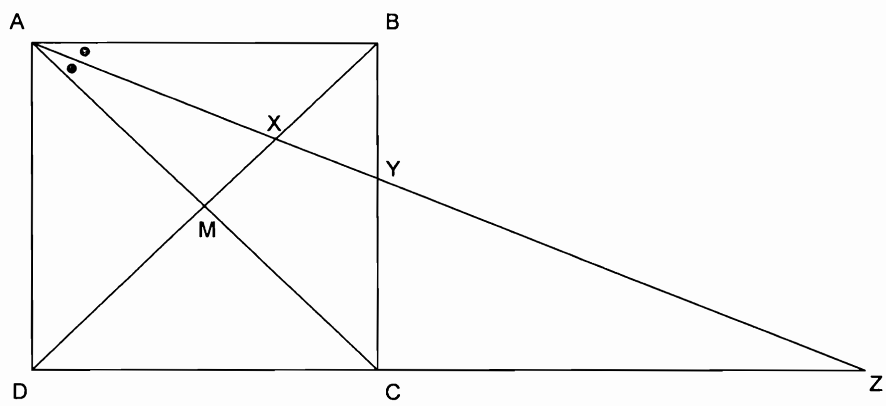
(v) Find the perpendicular distance from C to line l. **2**

(vi) Find the area of ΔDMC. **2**

12. Find the derivative of $f(x) = 4x - x^2$ by first principles. **3**

13. Find the equation of the tangent to the curve $y = (x^2 + 1)^3$ at the point on the curve where $x = 1$. **3**

14.



ABCD is a square whose diagonals intersect at M. The bisector of $\angle BAC$ cuts BM at X and BC at Y. DC and AY are produced to meet at Z.

a) Prove ΔAMX is similar to ΔZCY. **3**

b) Find the ratio of CY:MX, giving reasons. **4**

~ End of Exam ~

Yr 11 Task 2 2014

SOLNS.

1. $\left(\frac{0+d}{2}, \frac{-5+0}{2}\right)$

$= \left(\frac{d}{2}, \frac{-5}{2}\right)$ (C)

2. $m = -\frac{1}{2} = \tan\theta$

$\theta \doteqdot 153°$ (D)

3. I true, II false. (A)

4. $r = \sqrt{(1+3)^2 + (2-4)^2}$

$= \sqrt{20}$

$= 2\sqrt{5}$ $\therefore d = 4\sqrt{5}$ (D)

5. $\lim_{x\to\infty} \dfrac{3 - 4/x^2 + 5/x^3}{1/x - 3}$

$= \dfrac{3-0+0}{0-3} = -1$ (B)

6. $y = -2x+3$ has $m = -2$

Perp. $m = \frac{-1}{-2} = \frac{1}{2}$ 1

$y + 4 = \frac{1}{2}(x-3)$ 1

$2y + 8 = x - 3$

$x - 2y - 11 = 0$ 1

7. $\lim_{x\to 3} \dfrac{(x-3)(x^2+3x+9)}{(x-3)(x+3)}$ 1

$= \dfrac{9+9+9}{3+3}$

$= \dfrac{27}{6}$ or 4.5 1

8. a) $y = 3x^4 + 4x - 5$

$y' = 12x^3 + 4$ 1, 1

b) $y = \dfrac{x^4 - 10x^5}{5x^2} = \frac{1}{5}(x^2 - 10x^3)$ ← 1

$y' = \frac{1}{5}(2x - 30x^2)$ ← 1

or equivalent.

c) $f(x) = (x^2+1).\sqrt{x-1}$

$u = x^2+1$ $u' = 2x$

$v = \sqrt{x-1}$ $v' = \frac{1}{2}(x-1)^{-1/2}.1$ 1

$= (x-1)^{1/2}$ $= \dfrac{1}{2\sqrt{x-1}}$

$f'(x) = uv' + vu'$ 1

$= (x^2+1).\dfrac{1}{2\sqrt{x-1}} + \sqrt{x-1}\cdot 2x$

(or $\frac{1}{2}(x^2+1)(x-1)^{-1/2} + (x-1)^{1/2}.2x$)

d) $y = \dfrac{x+1}{4x-1}$

$u = x+1$ $u' = 1$

$v = 4x-1$ $v' = 4$

$y' = \dfrac{vu' - uv'}{v^2}$

$= \dfrac{(4x-1).1 - (x+1).4}{(4x-1)^2}$ 1

$= \dfrac{4x - 1 - 4x - 4}{(4x-1)^2}$

$= \dfrac{-5}{(4x-1)^2}$ 1

9. $\angle BDC = 58°$ (Equal $\angle$s opposite equal sides of Δ) 1

$\angle ABD = 2 \times 58$ (Exterior $\angle$ of ΔBCD = Sum of interior opposite $\angle$s) 1

$= 116°$

ABCD is a parallelogram (Opposite sides parallel)

$\therefore \angle DEA = 116°$ (opposite $\angle$s of parallelogram equal) 1

10. a) In ΔQMT, ΔRMS:

$\angle QTM = \angle SRM$ (both 90° or given) 1

$\angle QMT = \angle SMR$ (vertically opposite $\angle$s) 1

$QM = MR$ (given)

$\therefore \Delta QMT \equiv \Delta RMS$ (AAS) 1

b) $MT = \frac{1}{2} \times 15 = 7.5$ cm

(M is the midpoint of TS) 1

c) QRST is a parallelogram (diagonals bisect each other). 1

11. i) $m_{AC} = \dfrac{2-0}{10-4} = \dfrac{2}{6} = \dfrac{1}{3}$ 1

ii) $D = \left(\dfrac{0+4}{2}, \dfrac{12+0}{2}\right) = (2, 6)$ 1

iii) Through $(2,6)$ with $m = 1/3$:

$y - 6 = \frac{1}{3}(x-2)$ 1

$3y - 18 = x - 2$

$x - 3y + 16 = 0$

iv) Intercepts on parallel lines are proportional

OR Parallel lines cut all transversals in the same ratio.

OR Line from midpoint of one side of a Δ, parallel to another side, meets the third side at its midpoint 1

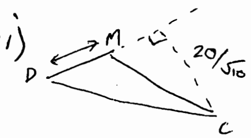
v) $C(10,2)$ Line $l: x - 3y + 16 = 0$

$$d = \left| \frac{(10) - 3(2) + 16}{\sqrt{1^2 + (-3)^2}} \right| \quad \leftarrow 1$$

$$= \left| \frac{20}{\sqrt{10}} \right|$$

$$= \frac{20}{\sqrt{10}} \quad \leftarrow 1$$

vi)



$D(2,6)$ $M = (5,7)$ ... midpt of BC

$$DM = \sqrt{(5-2)^2 + (7-6)^2}$$

$$= \sqrt{9+1}$$

$$= \sqrt{10} \quad \leftarrow 1$$

$$\therefore \text{Area} = \frac{1}{2}bh = \frac{1}{2} \cdot \sqrt{10} \cdot \frac{20}{\sqrt{10}}$$

$$= 10 \text{ units}^2 \quad \leftarrow 1$$

12. $y = f(x) = 4x - x^2$

$$f(x+h) = 4(x+h) - (x+h)^2$$

$$= 4x + 4h - (x^2 + 2xh + h^2)$$

$$= 4x + 4h - x^2 - 2xh - h^2 \quad \leftarrow 1$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{4x + 4h - x^2 - 2xh - h^2 - 4x + x^2}{h} \quad \leftarrow 1$$

$$= \lim_{h \to 0} \frac{4h - 2xh - h^2}{h}$$

* Structure + notation important

$$= \lim_{h \to 0} 4 - 2x - h \quad \leftarrow 1$$

$$= \underline{4 - 2x}$$

13. $y = (x^2+1)^3$

$$y' = 3(x^2+1)^2 \cdot 2x \quad 1$$

$$= 6x(x^2+1)^2$$

$$= 6(1+1)^2 \text{ at } x = 1$$

$$= 24$$

Pt of contact $(1, 8)$ ← $(1^2+1)^3$ 1

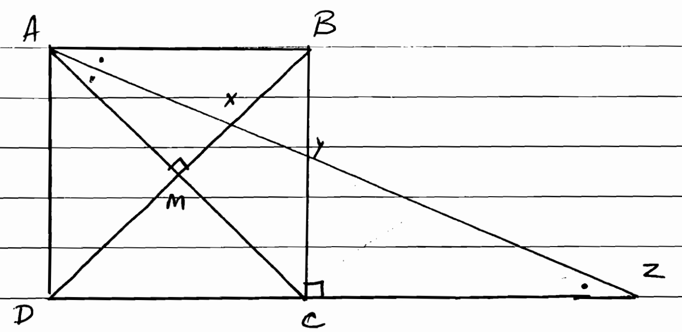
$$\therefore y - 8 = 24(x - 1) \quad 1$$

$$\underline{y = 24x - 16}$$

14.

14.

~~13~~.



a) $\angle AMX = 90^\circ$ (diagonals of square perpendicular)

$\angle BCD = 90^\circ$ ($\angle$s of square are right $\angle$s)

$\therefore \angle ZCY = 90^\circ$ ($BC \perp DC$)

$\therefore$ $\underline{\angle AMX = \angle ZCY}$ (both 90°) 1

$\angle MAX = \angle BAX$ (AX bisects $\angle BAC$)

$AB \parallel DC$ (opposite sides of square parallel)

$\angle BAX = \angle CZY$ (alternate $\angle$s, $AB \parallel DC$)

$\therefore$ $\underline{\angle MAX = \angle CZY}$ (both $= \angle BAX$) 1

$\therefore \Delta AMX \,|||\, \Delta ZCY$ (matching $\angle$s equal) 1

b) ΔCZA is isosceles (2 equal $\angle$s from (a))

$\therefore$ $CA = CZ$ (Equal sides opposite equal $\angle$s of ΔCZA) 1

~~$\frac{AM}{CZ}$~~

$\frac{CY}{MX} = \frac{CZ}{AM}$ (matching sides of similar Δs proportional) 1

$= \frac{CA}{AM}$ ($CA = CZ$ from above)

$= \frac{2.AM}{AM}$ (diagonals of square bisect each other) 1

(*) logical reasoning leading to (1)*