BAULKHAM HILLS HIGH SCHOOL

2015

YEAR 11

TASK 2 JULY ASSESSMENT TASK

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 60 minutes
- Write using black or blue pen
- Black pen is preferred
- Board-approved calculators may be used
- All relevant mathematical reasoning and/or calculations must be shown

Total marks – 45

(Pages 2-5)

Attempt Questions 1-4

Total Marks – 45
Attempt Questions 1 – 4

Answer each question on the appropriate page of your answer booklet.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 1 (12 marks) Start on the appropriate page of your answer booklet

a) A line makes an angle of 60° with the positive x-axis. What is the equation of the line if it passes through the point (-1,4)? **2**

b) Evaluate:

(i) $\lim_{x \to 3} \dfrac{x^2 - 8x + 15}{x - 3}$ **2**

(ii) $\lim_{x \to \infty} \dfrac{x^2 + 2}{2x^2 - 3x + 1}$ **2**

c) Differentiate

(i) $\dfrac{1}{4x + 3}$ **2**

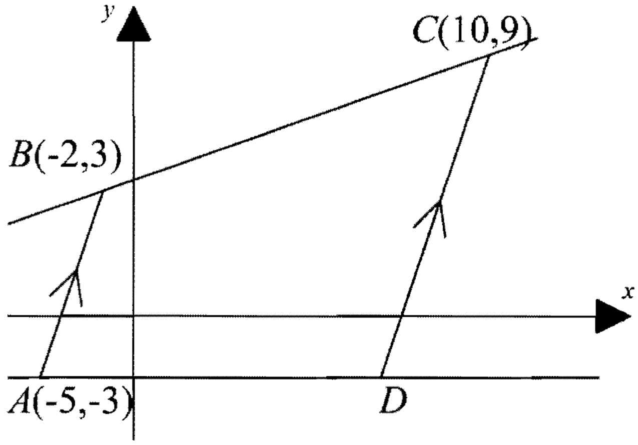
(ii) $\dfrac{2x}{x^2 + 3}$ **2**

(iii) $\sqrt{2x^4 - 7x^3 + 1}$ **2**

END OF QUESTION 1

Question 2 (12 marks) Start on the appropriate page of your answer booklet

a) On a number plane the points A(-5,-3), B(-2,3), C(10,9) and D form a trapezium in which AD is parallel to the x axis and $AB \parallel CD$.



(i) Show that the equation of the line DC is $2x - y - 11 = 0$ **2**

(ii) Find the coordinates of D. **1**

(iii) Find the perpendicular distance from B to DC **2**

(iv) Find the coordinates of the midpoint of BD. **1**

(v) The point E lies on the line BA produced such that BCDE is a rhombus. Find the coordinates of E. **1**

b) Joel is trying to find the equation of a line perpendicular to $x - 2y + 13 = 0$ which passes through the point of intersection of the lines $3x + 2y - 1 = 0$ and $7x - 11y + 3 = 0$.

As part of his working Joel has correctly set up the following equation:

$$3x + 2y - 1 + k(7x - 11y + 3) = 0$$

(i) Show that the gradient of the line is $\dfrac{3 + 7k}{11k - 2}$ **2**

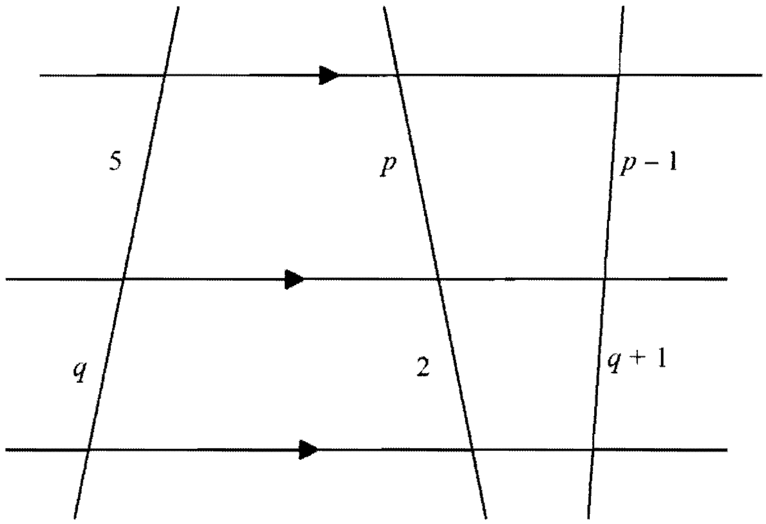
(ii) Find the value of k **2**

(iii) Hence determine the required equation of the line. **1**

END OF QUESTION 2

Question 3 (12 marks) Start on the appropriate page of your answer booklet

a) The diagram below comprises three parallel lines.

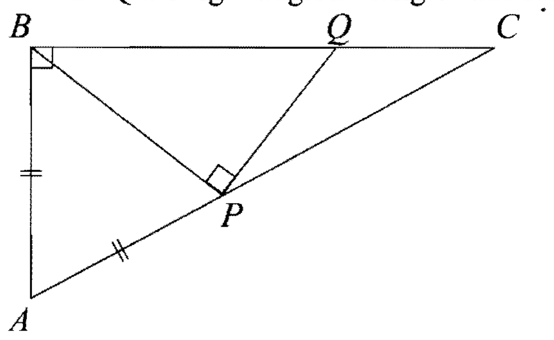


i) Explain why $\frac{5}{q} = \frac{p}{2}$ **1**

ii) Find the exact value of p and q **2**

b) Find the equation of the tangent to the curve $y = 3x^3 - 4x^2 + x - 1$ at the point where $x = 1$. **3**

c) Triangles ABC and BPQ are right angled triangles and AB=AP



Copy the diagram into your answer booklet

(i) Prove $\angle CBP = \angle CPQ$ **3**

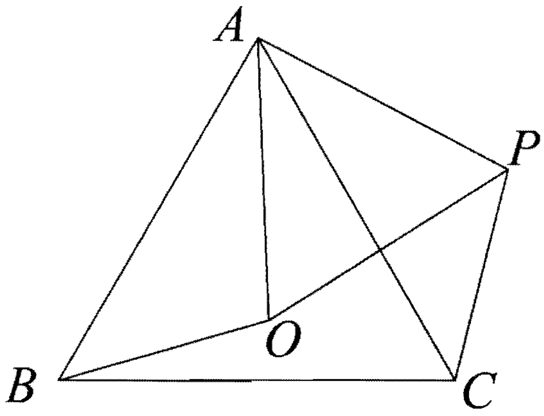
(ii) Hence prove that $PC^2 = BC \times QC$ **3**

END OF QUESTION 3

Question 4 (9 marks) Start on the appropriate page of your answer booklet

a) The parabola $y = ax^2 + bx + c$ passes through the point (0,-2). At the point (2,-3) **3**
on this curve, the equation of the tangent is $2x - y - 7 = 0$.
Find the values of a, b and c.

b) In the figure triangles ACB and APO are equilateral.



NOT TO SCALE

Copy this diagram into your answer booklet.

(i) Prove that $\angle BAO = \angle PAC$ **2**

(ii) Prove Δ AOB $\equiv$ Δ APC **3**

(iii) Hence, prove OB=CP. **1**

END OF EXAMINATION

YR 11 JUNE 2015 2UNIT

Q1 a) $m = \tan 60^\circ$

$= \sqrt{3}$ ✓

$y - y_1 = m(x - x_1)$

$y - 4 = \sqrt{3}(x+1)$

$y = \sqrt{3}x + \sqrt{3} + 4$ ✓

$(\text{OR } \sqrt{3}x - y + \sqrt{3} + 4 = 0)$

(2) correct solution
(1) finding grad
or using incorrect gradient to correctly find line

b) i) $\lim_{x\to 3} \frac{x^2 - 8x + 15}{x-3}$

$= \lim_{x\to 3} \frac{(x-5)\cancel{(x-3)}}{\cancel{x-3}}$

$= \lim_{x\to 3} (x-5)$ ✓

$= 3 - 5$

$= -2$ ✓

(2) correct solution
(1) correct factorisation
(1) and subsequent cancelling

ii) $\lim_{x\to\infty} \frac{x^2+2}{2x^2-3x+1}$

$= \lim_{x\to\infty} \frac{\frac{x^2}{x^2} + \frac{2}{x^2}}{\frac{2x^2}{x^2} - \frac{3x}{x^2} + \frac{1}{x^2}}$ ✓

$= \lim_{x\to\infty} \frac{1 + \frac{2}{x^2}}{2 - \frac{3}{x} + \frac{1}{x^2}}$

$= \frac{1+0}{2-0+0}$

$= \frac{1}{2}$ ✓

(2) correct solution with/without working
(1) attempts to use $\lim_{x\to\infty} \frac{1}{x^2} = 0$
or (1) divides throughout by x^2

1c i) $\frac{d}{dx}(4x+3)^{-1} = -1(4x+3)^{-2}.4$ ✓

$= \frac{-4}{(4x+3)^2}$ ✓

② correct solution
① differentiation without multiplying by 4.

ii) $\frac{d}{dx}\left(\frac{2x}{x^2+3}\right) = \frac{(x^2+3).2 - 2x.2x}{(x^2+3)^2}$ ✓

$= \frac{2x^2+6-4x^2}{(x^2+3)^2}$

$= \frac{6-2x^2}{(x^2+3)^2}$ or $\frac{2(3-x^2)}{(x^2+3)^2}$ ✓

② correct solution
① attempts to use quotient rule

(iii) $\frac{d}{dx}(2x^4-7x^3+1)^{\frac{1}{2}} = \frac{1}{2}(2x^4-7x^3+1)^{-\frac{1}{2}}.(8x^3-21x^2)$ ✓

$= \frac{8x^3-21x^2}{2\sqrt{2x^4-7x^3+1}}$ or $\frac{x^2(8x-21)}{2\sqrt{2x^4-7x^3+1}}$

② correct soln
① differentiates without using chain rule

2 a) i $m_{CD} = m_{AB}$

$= \frac{3-(-3)}{-2-(-5)}$

$= \frac{6}{3}$

$= 2$ ✓

Using $y - y_1 = m(x - x_1)$

$y - 9 = 2(x - 10)$

$y = 2x - 20 + 9$

$y = 2x - 11$ or $2x - y - 11 = 0$ ✓

② correct solution
① correct gradient or finds equation of line using incorrect gradient

ii) when $y = -3$

$2x + 3 - 11 = 0$

$2x = 8$

$x = 4$ ✓

$\therefore D$ is $(4, -3)$

① correct solution

2a (iii) Perp Dist $= \left| \dfrac{ax_1+by_1+c}{\sqrt{a^2+b^2}} \right|$

$= \left| \dfrac{2\times -2 -3 -11}{\sqrt{2^2+1^2}} \right|$ ✓

$= \left| \dfrac{-18}{\sqrt{5}} \right|$

$= \dfrac{18\sqrt{5}}{5}$ ✓

(2) correct solution
(1) substitutes into perpendicular distance formula.

(iv) Midpoint $= \left(\dfrac{-2+4}{2}, \dfrac{3+-3}{2} \right)$

$= (1, 0)$ ✓

(1) correct solution

(v) E $\quad 1 = \dfrac{x+10}{2}$, $0 = \dfrac{y+9}{2}$

(1) correct solution

$2 = x+10$
$x = -8$ $\qquad y+9=0$
$\qquad y = -9$

$\therefore$ E is $(-8, -9)$ ✓

2b (i) $3x+2y-1+7kx-11ky+3k=0$ ✓
$(7k+3)x \quad +3k-1 \quad = (11k-2)y$
$\therefore m = \dfrac{7k+3}{11k-2}$ ✓

(2) correct solution
(1) expresses in general form or $y=mx+b$

(ii) grad$\perp = -2$: $-2 = \dfrac{7k+3}{11k-2}$ ✓

$2y = x+13$
$y = \frac{x}{2} + \frac{13}{2}$
$m_\perp = -2$

(2) correct solution
(1) equating gradient with -2

$-2(11k-2) = 7k+3$

$-22k+4 = 7k+3$
$29k = 1$
$k = 1/29$ ✓

(iii) $3x+2y-1+\frac{1}{29}(7x-11y+3)=0$
$87x+58y-29+7x-11y+3=0$
$94x+47y-26 \quad =0$ ✓

(1) correct solution

3a) (i) $\frac{5}{q} = \frac{p}{2}$ (ratio of intercepts on parallel lines) ① correct reason

(ii) $\frac{5}{q} = \frac{p}{2}$

$10 = pq$ ①

$\frac{p}{2} = \frac{p-7}{q+1}$

$pq + p = 2p - 2$ ②

$10 + p = 2p - 2$

$12 = p$

$\therefore q = \frac{10}{p}$

$= \frac{10}{12}$

$q = \frac{5}{6}$ ✓

② correct solution

① uses ratio of intercept theorem to set up simultaneous equations and attempts to solve

b) $\frac{dy}{dx} = 9x^2 - 8x + 1$

when $x = 1$, $\frac{dy}{dx} = 9 - 8 + 1$

$= 2$ ✓

$f(1) = 3 - 4 + 1 - 1$

$= -1$ ✓

$y + 1 = 2(x - 1)$

$y + 1 = 2x - 2$

$y = 2x - 3$ or $2x - y - 3 = 0$ ✓

③ correct solution

② either gradient OR y value OR equation contains error

① finds gradient or y value or equation.

c) i) let $\angle PBQ = \alpha$

$\angle PBA = 90° - \alpha$ (adjacent complementary $\angle$'s) ✓

$\angle BPA = 90° - \alpha$ (equal $\angle$'s opposite equal sides in isosceles $\triangle ABP$, AB = AP given)

$\angle APB + \angle BPQ + \angle QPC = 180°$ (angle sum of straight line) ✓

$90° - \alpha + 90° + \angle QPC = 180°$

$\angle QPC = 180 - 90 + \alpha - 90$

$= \alpha$

$= \angle PBQ$ as req'd. ✓

③ correct solution

② correct with at least 1 reason

① correct with no reasons or expression for $\angle BPA$ with reason

(3) correct solution
(2) correct with 1 reason
(1) correct with no reasons or proves similar without reasons

c (iii) In ΔBCP and ΔPQC

$\angle C$ is common

$\angle CBP = \angle CPQ$ (proven in (ii)) } ✓

$\therefore \Delta BCP \,|||\, \Delta PQC$ (two pairs of matching angles equal) ✓

$\frac{BC}{PC} = \frac{PC}{QC}$ (matching sides of similar Δ's BCP & PQC in same ratio) ✓

$\therefore PC^2 = BC \cdot QC$

Q4 a) $y = ax^2 + bx + c$

Since curve passes through $(0,-2)$

$-2 = 0 + 0 + c$

$\therefore y = ax^2 + bx - 2$

Since $(2,-3)$ lies on curve

$-3 = 4a + 2b - 2$

$-1 = 4a + 2b$

$4a + 2b = -1$ (1)

$\frac{dy}{dx} = 2ax + b$

When $x = 2$, $\frac{dy}{dx} = 2$ gradient of tangent

$\therefore 4a + b = 2$ (2)

(3) correct
(2) forms simultaneous equations and ← using $y' = 2ax + b$ attempts to solve (with c)
(1) finds c or $4a + 2b = -1$ or $4a + b = 2$

(1) − (2) $b = -3$

sub in (1) $4a - 3 = 2$

$4a = 5$

$a = \frac{5}{4}$

$\therefore y = \frac{5}{4}x^2 - 3x - 2$

4 b (i) let $\angle OAC = \alpha$

$\angle BAC = \angle OAP = 60°$ (angles in equilateral Δ's)

$\angle BAO = 60° - \alpha$

$\angle PAC = 60° - \alpha$

$= \angle BAO$

$\therefore \angle BAO = \angle PAC$

(2) correct solution
(1) attempt to use 60° angle to find an expression

(ii) In ΔAOB and ΔAPC

$AB = AC$ (equal sides of equilateral ΔABC) ✓

$OA = PA$ (equal sides of equilateral ΔOAP) ✓

$\angle BAO = \angle PAC$ (proven in (ii)) ✓

$\therefore \Delta AOB \equiv \Delta APC$ (SAS) ✓

(iii) $OB = CP$ (matching sides in congruent Δ's AOB APC) ✓