BAULKHAM HILLS HIGH SCHOOL

JULY 2016

YEAR 11 TASK 2

Mathematics

General Instructions

- Reading time 5 minutes
- Working time – 60 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in questions 6 to 9
- Marks may be deducted for careless or badly arranged work

Total marks – 53
Exam consists of 6 pages.

This paper consists of TWO sections.

<u>**Section 1**</u> **– Page 2-3**
Questions 1-5 (5 marks)

- Attempt Question 1-5

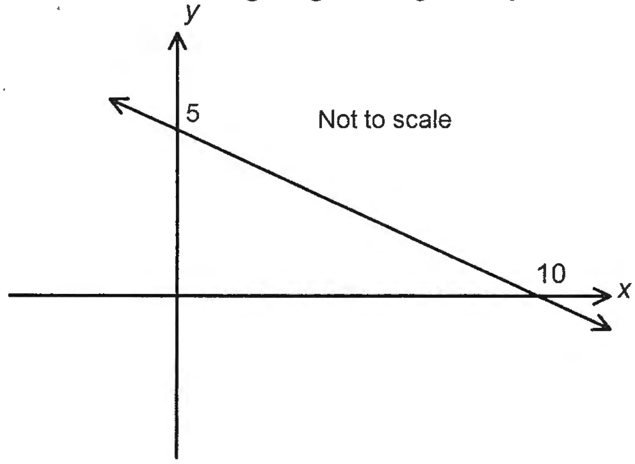
<u>**Section II**</u> **– Pages 3-6 (48 marks)**

- Attempt questions 6 to 9

Section I - 5 marks
Attempt questions 1-5

Use the multiple choice answer page for question 1-5

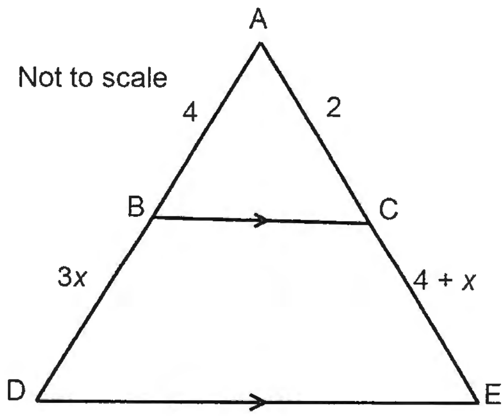
1. The equation of the line in the following diagram is given by



(A) $y = \frac{1}{2}x + 5$ (B) $y = 2x + 5$

(C) $y = -\frac{1}{2}x + 5$ (D) $y = -\frac{1}{2}x + 10$

2. The length of AD in the following is:

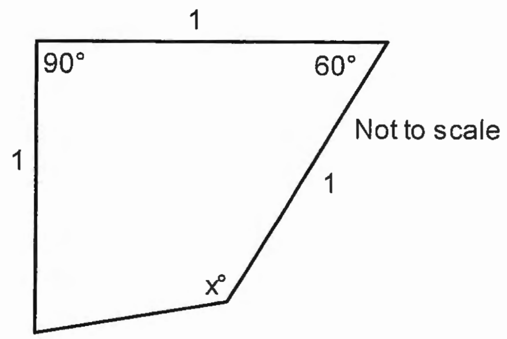


(A) 1.6 (B) 8 (C) 14 (D) 28

3. The derivative of $\frac{1}{(x^2+1)^2}$ is:

(A) $\frac{-2}{(x^2+1)}$ (B) $\frac{-4x}{(x^2+1)}$ (C) $\frac{-2}{(x^2+1)^3}$ (D) $\frac{-4x}{(x^2+1)^3}$

4. In the diagram find the value of x.



(A) 120
(B) 135
(C) 137.5
(D) 140

5. Evaluate $\lim\limits_{x \to \infty} \dfrac{4x^2}{9 - 6x + 7x^2}$

(A) 0

(B) $\frac{4}{9}$

(C) $\frac{4}{7}$

(D) 1

Section 2 Questions 6-9

6. (a) Differentiate :

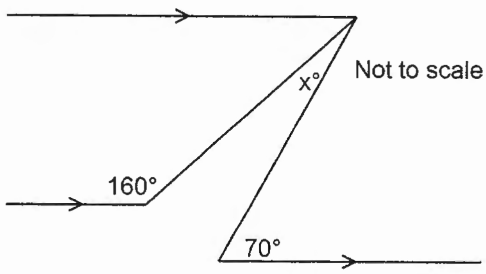
(i) $6x^2 - 4x - 3$ **1**

(ii) $\sqrt[3]{x} - \dfrac{4}{x^3}$ **2**

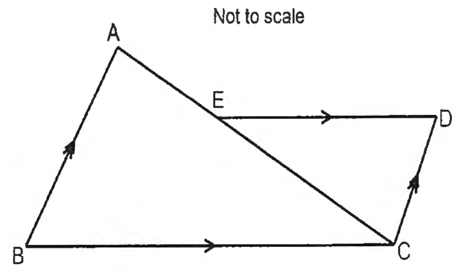
(iii) $\dfrac{2x}{3x^2-4}$ **2**

(iv) $(4x^2 - 3)^6$ **2**

(b) Find the value of the pronumeral (no reasons required) **1**



c)

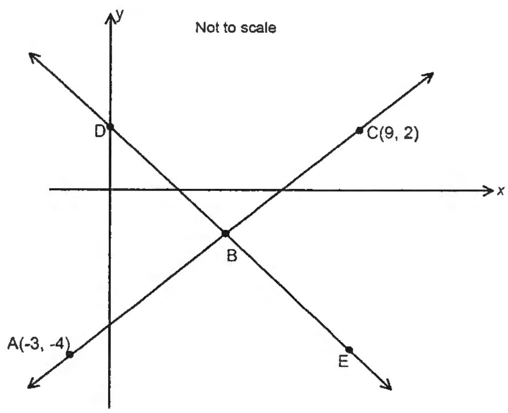


In the above diagram AB || CD and DE || BC.

(i) Prove that ΔABC ||| ΔEDC. **2**

(ii) If 3CD = 2AB and DE + BC = 20 cm, find the length of DE. **3**

7. (a) The points A(-3,-4) and C(9,2) are drawn on the number plane below. B is the midpoint of AC and BD is perpendicular to AC.



(i) Find the exact distance AC. **1**

(ii) Find the co-ordinates of B. **1**

(iii) Find the gradient of AC. **1**

(iv) Show the equation of BD is $2x + y - 5 = 0$. **2**

(v) What are the co-ordinates of the point D where BD cuts the y axis? **1**

(vi) Find the point E on BD such that Δ ABD ≡ Δ BCE. **1**

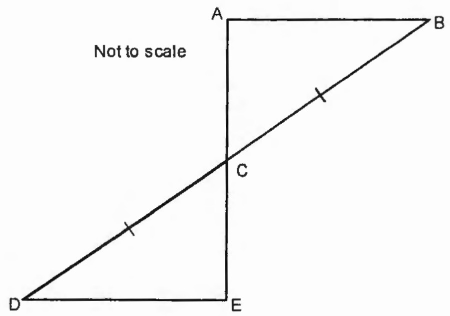
(vii) What type of quadrilateral is AECD? Justify your answer. **2**

(b) Differentiate $x^2(6 - x)^5$. **3**

8. (a) Find the equation of the normal to the curve $y = 2 - 3x^2$ at the point where $x = 2$ **3**
Answer in general form.

(b) The perpendicular distance from the point (3,k) to the line $2x + 4y + 5 = 0$ is $3\sqrt{5}$. Find the value(s) of k. **3**

(c)



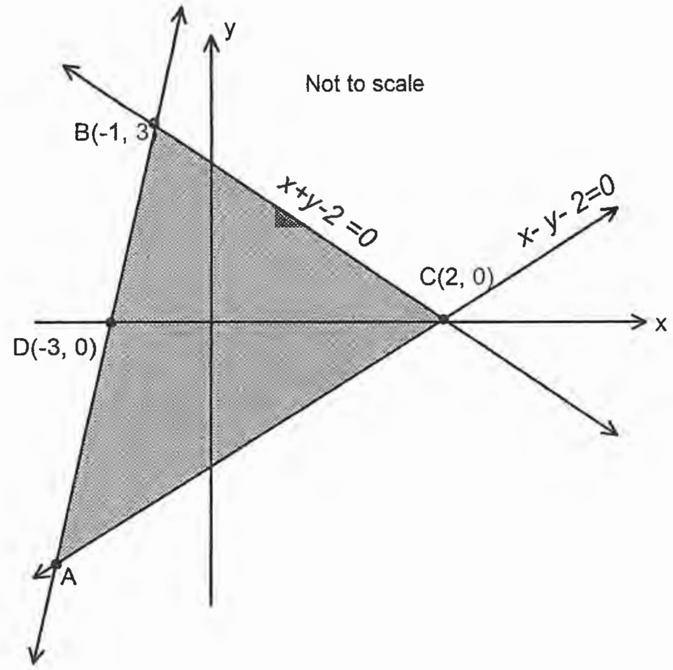
In the above diagram BC = CD, AE⊥AB and AE⊥DE.

(i) Prove $\Delta\, ABC \equiv \Delta$ CDE. **3**

(ii) Hence prove ABED is a parallelogram. **2**

(iii) If AB=6 cm and BD= 18cm find the area of the parallelogram ABED. **2**

9.



(a) The diagram above shows a region enclosed by the lines $x - y - 2 = 0$, $x + y - 2 = 0$ and the line AB.

(i) Show that the equation of the line AB is $3x - 2y + 9 = 0$. **1**

(ii) Show that the co-ordinates of point A are (-13,-15). **2**

(iii) Give a set of inequalities that define the shaded region. **2**

(iv) Find the area of the shaded region **2**

(b) If $y = \frac{2x+1}{\sqrt{4x+2}}$ show that $\frac{dy}{dx} = \frac{1}{2y}$. **3**

~ End of Exam ~

Solutions

Multiple Choice

1. C, 2. D 3. D. 4. B. 5. C

6 a) (i) $y' = 12x - 4$ (1)

(ii) $y = x^{\frac{1}{3}} - 4x^{-3}$ (1)

$y' = \frac{1}{3}x^{-2/3} + 12x^{-4}$ (1)

(iii) $\frac{d}{dx}\left(\frac{2x}{3x^2-4}\right) =$

$\frac{2(3x^2-4) - 6x(2x)}{(3x^2-4)^2}$ (1)

$= \frac{6x^2 - 8 - 12x^2}{(3x^2-4)^2}$

$= \frac{-6x^2 - 8}{(3x^2-4)^2}$ (1)

(iv) $\frac{d}{dx}\left[(4x^2-3)^6\right]$

$= 48x(4x^2-3)^5$ (1) (1)

b) $x = 50°$ (1)

c) (i) $\angle BAC = \angle ACD$ (Alternate ∠'s $AB \parallel CD$) (1)

$\angle ACB = \angle DEC$ (Alternate ∠'s $DE \parallel BC$)

$\therefore \triangle ABC \parallel\!\!\mid \triangle EDC$ (AA) (1)

(ii) $3CD = 2AB$

$\therefore \frac{CD}{AB} = \frac{2}{3} \therefore \frac{DE}{BC} = \frac{2}{3}$

(matching sides in the same ratio in ||| Δ's) (1)

let $DE = x \therefore BC = 20 - x$

$\therefore \frac{2}{3} = \frac{x}{20-x}$ (1)

$2(20-x) = 3x$

$40 - 2x = 3x$

$5x = 40$

$x = 8$

$\therefore DE = 8cm$ (1)

7a) (i) $AC = \sqrt{12^2+6^2}$

$= \sqrt{180}$

$= (6\sqrt{5})$ (1)

(ii) $B\left(\frac{-3+9}{2}, -\frac{4+2}{2}\right)$

$= (3, -1)$ (1)

(iii) $m_{AC} = \frac{2+4}{9+3} = \frac{1}{2}$ (1)

(iv) $m_{BD} = -2$ (1)

$\therefore$ Eq'n $y+1 = -2(x-3)$

$y + 1 = -2x + 6$ (1)

$2x + y - 5 = 0$

(v) when $x = 0$ $y = 5$ $D(0,5)$ (1)

(vi) $\frac{0+x}{2} = 3$ $\frac{5+y}{2} = -1$ (1)

$x = 6$ $y = -7$

$E(6, -7)$

(vii) $DE = \sqrt{12^2+6^2}$

$= 6\sqrt{5} = AC$. (1)

$\therefore$ DAEC is a square diagonals $\Rightarrow \perp$ & bisect each other. (1)

b) $\frac{d}{dx}\left(x^2(6-x)^5\right)$ (1)

$= 2x(6-x)^5 + 5x^2(6-x)^4 \times (-1)$ (1)

$= 2x(6-x)^5 - 5x^2(6-x)^4$ (1)

$= x(6-x)^4[2(6-x) - 5x]$ } not

$= x(6-x)^4(12 - 7x)$ } necessary

(1) mark for applying product rule

(1) " for derivative of $6-x$

(1) " for the final answer.

8a) $y = 2 - 3x^2$

at $x = 2$ $y = -10$ $\therefore (2, -10)$ (1)

$y' = -6x$

at $x = 2$ $y' = -12 \rightarrow m = -12$

$\perp m = \frac{1}{12}$ (1)

$\therefore$ Normal

$y + 10 = \frac{1}{12}(x - 2)$

$12y + 120 = x - 2$ (1)

$x - 12y - 122 = 0$

b) $\frac{|6 + 4k + 5|}{\sqrt{2^2+4^2}} = 3\sqrt{5}$ (1)

$\frac{|11 + 4k|}{\sqrt{20}} = 3\sqrt{5}$

$|11 + 4k| = 30$

$11 + 4k = \pm 30$

$11 + 4k = 30 \Rightarrow k = \frac{19}{4}$

$11 + 4k = -30 \rightarrow k = -\frac{41}{4}$

$\therefore k = \frac{19}{4}, -\frac{41}{4}$ (1) (1)

c) (i) $BC = ED$ (Given)

$\angle DCE = \angle ACB$ (Vertically Opposite ∠'s) (1)

$\angle CAB = 90°$ ($AE \perp AB$)

$\angle CED = 90°$ ($AE \perp DE$)

$\therefore \angle CED = \angle CAB$ (1)

$\triangle ABC = \angle CAB$ (1)

$\triangle ABC \equiv \triangle CDE$ (AAS)

(ii) $AC = CE$ (Matching sides in $\equiv$ Δ's)

$\therefore$ ABED is a parallelogram as Diagonals BD & AE bisect each other.

(iii) $BC = 9cm$

$\therefore AC = \sqrt{9^2 - 6^2}$

$= 3\sqrt{5}$ (1)

Area $= AB \times AE$

$= 6 \times 2(3\sqrt{5})$

$= 36\sqrt{5}$ (1)

$(80.5 cm^2)$

9 (i) $m_{AB} = \frac{3-0}{-1+3}$

$= \frac{3}{2}$

$\therefore$ BD $y - 0 = \frac{3}{2}(x+3)$ (1)

$2y = 3x + 9$

$3x - 2y + 9 = 0$

(ii) can sub $(-13, -15)$ into both lines or solve simult'ly.

$x - y - 2 = 0$ — (1)

$3x - 2y + 9 = 0$ — (2)

from (1) $y = x - 2$

sub in (2)

$3x - 2(x-2) + 9 = 0$ (1)

$3x - 2x + 4 + 9 = 0$

$x = -13$

$y = -13 - 2$

$= -15$

A $(-13, -15)$ (1)

(iii) $x - y - 2 \leq 0$

$x + y - 2 \leq 0$

$3x - 2y + 9 \geq 0$

(1) if 1 inequality is correct

+ (1) – all correct.

(iv) Area = $\Delta BDC + \Delta DAC$

$= \frac{1}{2} \times 5 \times 3$ (1) $+ \frac{1}{2} \times 5 \times 15$

$= 45$ units (1)

Alternatively could do area using $\perp$ distance

b) $y = \frac{2x+1}{\sqrt{4x+2}}$

$y' = \frac{2(\sqrt{4x+2}) - \frac{1}{2}(4x+2)^{-\frac{1}{2}} \cdot 4(2x+1)}{(\sqrt{4x+2})^2}$ (1)

$= \frac{2\sqrt{4x+2} - \frac{2(2x+1)}{\sqrt{4x+2}}}{(4x+2)}$

$= \frac{\frac{2(4x+2) - 2(2x+1)}{\sqrt{4x+2}}}{4x+2}$

$= \frac{8x + 4 - 4x - 2}{(\sqrt{4x+2})(4x+2)}$

$= \frac{4x+2}{(\sqrt{4x+2})(4x+2)}$

$= \frac{1}{\sqrt{4x+2}}$ (1)

now $\frac{1}{2y} = 1 \div \frac{2(2x+1)}{\sqrt{4x+2}}$

$= \frac{1 \times \sqrt{4x+2}}{(4x+2)}$ (1)

$= \frac{1}{\sqrt{4x+2}}$

$= \frac{dy}{dx}$.

$\therefore \frac{dy}{dx} = \frac{1}{2y}$