BAULKHAM HILLS HIGH SCHOOL

2017

YEAR 11 ASSESSMENT TASK 2

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 60 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 6-8
- Marks may be deducted for careless or badly arranged work

Total marks – 51
Exam consists of 5 pages.

This paper consists of TWO sections.

<u>Section 1</u> – **Page 2 (5 marks)**
Questions 1 - 5

- Attempt Questions 1 - 5
  Allow about 5 minutes for this section.

<u>Section II</u> – **Pages 3 – 5 (46 marks)**

- Attempt questions 6 - 8
  Allow about 55 minutes for this section.

Section I - Multiple Choice (5 marks)
Allow about 5 minutes for this section.

Use the multiple choice page for Question 1 - 5

1 If $f(x) = 2x^3$, then $f'(2)$equals

(A) 8 (B) 16 (C) 24 (D) 144

2 A line passes through the origin and makes an angle of 45° with the positive direction of the x axis. The gradient of the line is

(A) 0 (B) -1 (C) 1 (D) 45

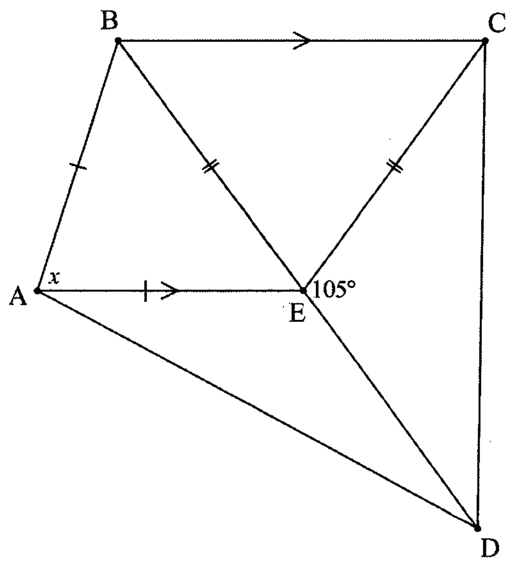
3 Find $\lim_{x\to\infty} \frac{1-x^2}{x^2}$

(A) 1 (B) ∞ (C) 0 (D) -1

4 The line $y = mx + b$ is a tangent to the curve $y = x^3 - 3x + 2$ at the point (-2,0). What are the values of m and b?

(A) $m = 9$ and $b = -18$ (B) $m = 9$ and $b = 18$
(C) $m = 12$ and $b = -18$ (D) $m = 12$ and $b = 18$

5 The vertices of quadrilateral $ABCD$ are joined at E such that $BC \parallel AE$, BE is produced to D. $\angle CED = 105°$, $BE = CE$ and $AB = AE$. Determine the size of x.



Not to Scale

(A) 105°
(B) 85°
(C) 75°
(D) 52.5°

End of Section 1

Section II (46 marks)
Allow about 55 minutes for this section.

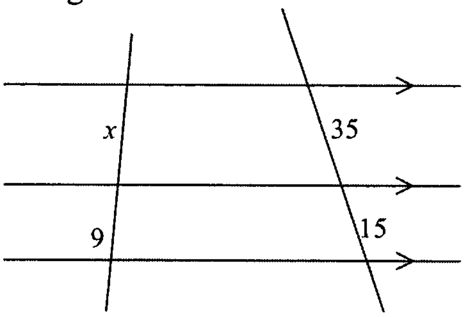
Answer each question on the appropriate page in the writing booklet.

Question 6 (15 marks) **Marks**

a) Find the gradient of the normal to the curve $y = 3 - x^2$ at $x = -1$ **3**

b) Evaluate $\lim_{x \to 3} \dfrac{x^2 - 2x - 3}{x - 3}$ **2**

c) Find the value of x, stating the reason. **2**



Not to Scale

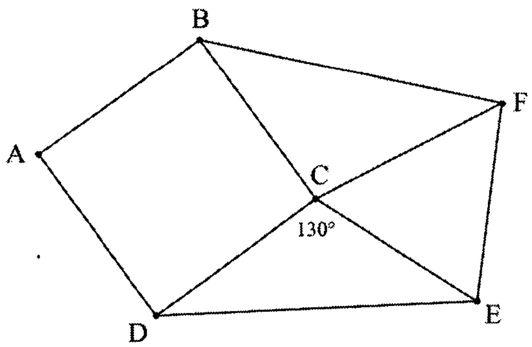
d) Differentiate with respect to x:

(i) $y = \dfrac{6x^3 - 4x^2}{2x}$ **2**

(ii) $f(x) = (3x - 5)(2x + 5)$ **2**

e) Find the equation of the straight line passing through the point (1,5) and through the point of intersection of the lines $3x - 4y + 2 = 0$ and $5x + 2y = 14$. **3**
Give your answer in simplest general form.

f) In the figure below, $ABCD$ is a square, CEF is an equilateral triangle, $\angle DCE = 130°$ and $DC = CE$. **1**



Not to Scale

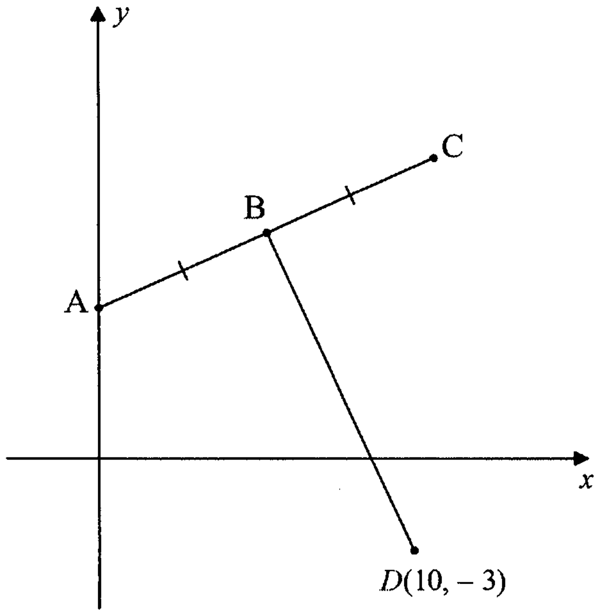
Find the size of $\angle CBF$ (without giving reasons)

End of Question 6

Question 7 (15 marks)

a) Find the derivative of $f(x) = 5x - 2x^2$ by first principles. **3**

b) The diagram shows points A, B and C lying on the line $2y = x + 4$. The point A lies on the y-axis and $AB = BC$. The line from $D(10, -3)$ to B is perpendicular to AC.



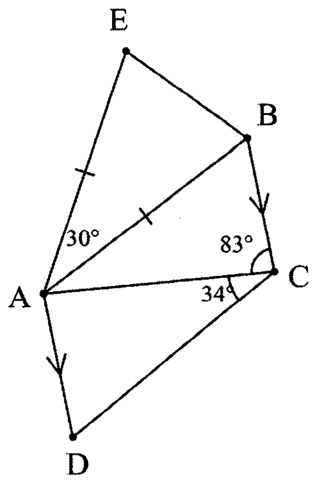
Not to Scale

(i) Find the coordinates of A. **1**

(ii) Find the equation of the line BD. **2**

(iii) Find the coordinates of C. **3**

c) In the diagram below: $AD//BC, AE = AB, \angle BAE = 30°, \angle BCA = 83°, \angle ACD = 34°$ and $\angle EBC = 138°$.



Not to Scale

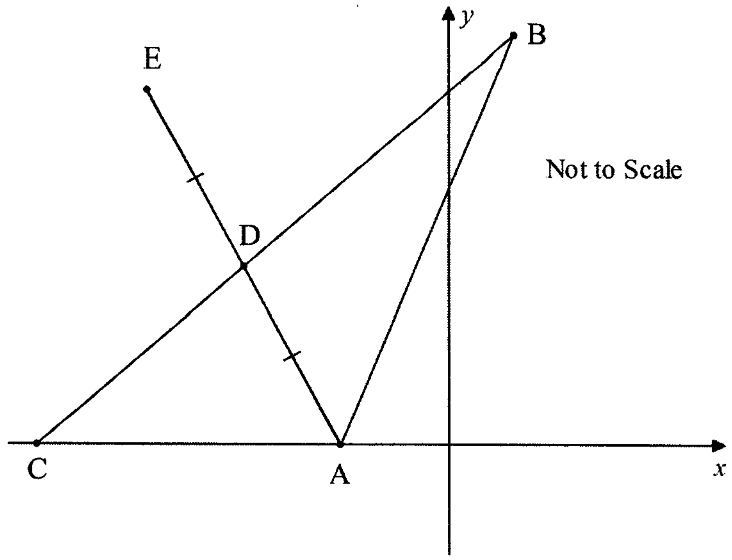
(i) Prove that $AB//DC$ **3**

(ii) Prove that $\Delta ABC \equiv \Delta ACD$ **3**

End of Question 7

Question 8 (16 marks)

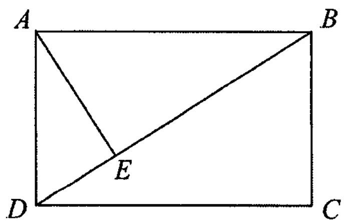
a) In the diagram A, B and C are $(-1,0)$, $(2,4)$ and $(-6,0)$ respectively. D is the midpoint of AE and has coordinates $(-2,2)$.



Copy the diagram into your booklet and include the information above.

(i)	Find the length of the interval AD.	1
(ii)	Show that D is the midpoint of BC.	1
(iii)	Show that the equation of the line BC is $x - 2y + 6 = 0$	2
(iv)	Find the perpendicular distance of A from the line BC.	2
(v)	What type of quadrilateral is $ABEC$? (Give reasons for your answer)	2
(vi)	Find the area of this quadrilateral.	2

b) ABCD is a rectangle and AE $\perp$ BD.
AE =5cm and DE = 2 cm.



(i)	Copy the diagram into your booklet and prove that triangles AED and BCD are similar.	2
(ii)	Hence, show that $AD^2 = BD \times DE$.	2
(iii)	Find the area of ABCD.	2

End of Exam

2unit Yr11 Task 2. 2017.

multi choice.

1. C 2. C 3. D 4. B 5 C

1. $f(x) = 2x^3$

$f'(x) = 6x^2$ $f'(2) = 6 \times 4 = 24.$ (C)

2. $\tan\theta = m$

$\tan 45 = 1$ (C)

3. $\lim_{x\to\infty} \frac{1-x^2}{x^2} = \lim_{x\to\infty} \frac{\frac{1}{x^2} - \frac{x^2}{x^2}}{\frac{x^2}{x^2}}$

$= \lim_{x\to\infty} \frac{\frac{1}{x^2} - 1}{1}$ as $x \to \infty$ $\frac{1}{x^2} \to 0$. (D)

$= -1$

4. $y = x^3 - 3x + 2$

$\frac{dy}{dx} = 3x^2 - 3$ $\therefore y = 9x + b$. (B)

at $x = -2$ $m = 9$ use $(-2, 0)$ $0 = -18 + b$.

$b = 18$ $\therefore y = 9x + 18$.

5. ∠ sum isosceles triangle with BED a straight ∠. (C)

Question 6. a) $y = 3 - x^2$ $\frac{dy}{dx} = -2x$ at $x = -1$ $m = 2$. $\therefore$ normal $m_2 = -\frac{1}{m_1}$ $= -\frac{1}{2}$.	3 marks correct response 2 marks correct m_1 or correct process with one error. 1 mark correct $\frac{dy}{dx}$.
b) $\lim_{x\to 3} \frac{x^2 - 2x - 3}{x - 3} = \lim_{x\to 3} \frac{(x-3)(x+1)}{x-3}$ $= 4$	2 marks. complete solution. 1 mark. error in factorising with substitution
c) $\frac{x}{9} = \frac{35}{15}$ ratio of intercepts on parallel lines $x = 21$ units	2 marks answer + reason 1 mark either part correct.

quest 6 cont.	
d i) $y = \frac{6x^3 - 4x^2}{2x}$ $= 3x^2 - 2x$ or quotient rule $\frac{dy}{dx} = 6x - 2$	2 marks correct answer. 1 mark. 1 mistake.
ii) $f(x) = (3x - 5)(2x + 5)$ $= 6x^2 + 5x - 25$ $f'(x) = 12x + 5$	2 marks correct answer 1 mark. 1 mistake
e) Either method. K method. $3x - 4y + 2 + k(5x + 2y - 14) = 0$ passing through $(1, 5)$ $3 - 20 + 2 + k(5 + 10 - 14) = 0$ $-15 + k(15 - 14) = 0$ $k = 15$ $\therefore 3x - 4y + 2 + 15(5x + 2y - 14) = 0$ $3x - 4y + 2 + 75x + 30y - 210 = 0$ $78x + 26y - 208 = 0$ $3x + y - 8 = 0$ or pt of intersection $3x - 4y = -2$ --- ① $5x + 2y = 14$ --- ② ②×2 $10x + 4y = 28$ --- ③ ③+① $13x = 26$ $x = 2$ sub in to ① $6 - 4y = -2$ $-4y = -8$ $y = 2$. $\therefore (2, 2)$ $\therefore$ gradient for $(2,2)$ and $(1,5)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5-2}{1-2} = -3$ $\therefore$ new line $y - y_1 = m(x - x_1)$ $y - 2 = -3(x - 2)$ $3x + y - 8 = 0$	3 marks a correct method with correct equation 2 marks one error in process and simplified general form or correct process not simplified general form. 1 mark. working towards solution 2 mistakes
f) $\angle CBF = 50°$	1 mark.

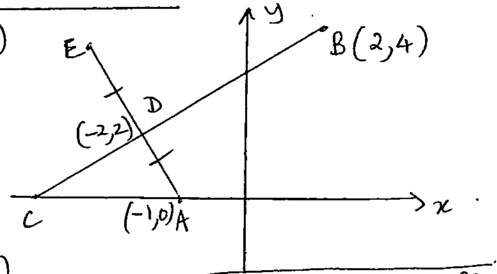
Question 7.

Solution	Marking criteria
a) $f(x) = 5x - 2x^2$ $f(x+h) = 5(x+h) - 2(x+h)^2$ $= 5x + 5h - 2x^2 - 4xh - 2h^2$ $f(x+h) - f(x) = 5h - 4xh - 2h^2$ $\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \to 0} \frac{h(5 - 4x - 2h)}{h}$ $= 5 - 4x$.	**3marks** clear setting out showing the correct answer. **2marks** error with placement of $\lim_{h\to 0}$ or another mistake **1mark**... 2 mistakes
b) i) AC: $2y = x + 4$ at A: $x = 0$ $\therefore$ $2y = 4$, $y = 2$ A is $(0,2)$	**1mark** correct response.
ii) AC: $y = \frac{1}{2}x + 2$ $m_1 = \frac{1}{2}$ BD: $m_2 = -\frac{1}{m_1}$ $= -2$ point D $(10, -3)$	**2marks** correct formula correct answer
$y - y_1 = m(x - x_1)$ $y + 3 = -2(x - 10)$ $y = -2x + 17$ $2x + y - 17 = 0$	**1mark** either correct gradient or correct equation from incorrect gradient
iii) find B BD: $y = -2x + 17$ --- ① AC: $y = \frac{1}{2}x + 2$ --- ② sub ② into ① $\frac{1}{2}x + 2 = -2x + 17$. $x + 4 = -4x + 34$ $5x = 30$ $x = 6$. sub into ② $y = \frac{1}{2} \times 6 + 2$ $= 5$ B $(6,5)$	**3marks** correct point from clear working out **2marks** correct value for B or correct method with one mistake **1mark**. 2 mistakes with appropriate method.

Solution	Marking criteria
quest 7 cont. B is the mid pt of AC $\therefore x = \frac{x_1 + x_2}{2}$ $y = \frac{y_1 + y_2}{2}$ $6 = \frac{0 + x_2}{2}$ $5 = \frac{2 + y_2}{2}$ $x_2 = 12$ $y_2 = 8$ C is $(12, 8)$	
c i) Aim: to prove $AB \parallel DC$. Method: In $\triangle AEB$ $\angle AEB = \angle ABE$ ($\angle$s opposite equal sides are equal) $= x$. $2x + 30° = 180°$ ($\angle$ sum $\triangle$) $x = 75°$ now $\angle EBC = 138°$ (given) $\therefore \angle ABC = 138 - \angle ABE$ (adjacent $\angle$s) $= 63°$ $\therefore \angle ABC + \angle BCD = 63° + 83° + 34°$ $= 180°$ $\therefore AB \parallel DC$ (Co interior $\angle$s supplementary).	**3 Marks** correctly deduces answer. **2 Marks** one error in reasoning **1 mark** two answers in reasoning
ii) Aim: prove $\triangle ABC \equiv \triangle ACD$ Method: In $\triangle ABC$ and $\triangle ACD$ $\angle BAC = \angle DCA$ (alternate $\angle$s, $AB \parallel DC$) $= 34°$ $\angle BCA = \angle DAC$ (alternate $\angle$s, $AD \parallel BC$) $= 83°$ AC is common $\angle ADC = 63°$ ($\angle$ sum $\triangle ACD = 180$) $\therefore$ $= \angle ABC$ (from i) $\therefore \triangle ABC \equiv \triangle ACD$ (S.A.A)	**3 marks** well set out proof. **2marks**. one error in reasoning **1mark**. two errors in reasoning

Question 8.

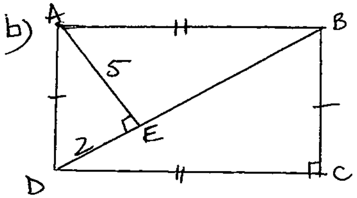
a)



i) AD: $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ $= \sqrt{(-1+2)^2 + (0-2)^2}$ $= \sqrt{5}$ units	1 mark was deducted for no diagram i) 1 Mark correct response
ii) D $(-2,2)$ mid pt BC $x = \frac{x_1+x_2}{2} = \frac{-6+2}{2} = -2$ $y = \frac{y_1+y_2}{2} = \frac{0+4}{2} = 2$ $\therefore$ $(-2,2)$ which is D.	1 Mark correct response but connection should be clear linking midpt to D
iii) BC: $m = \frac{y_2-y_1}{x_2-x_1} = \frac{4-0}{2+6} = \frac{1}{2}$ use pt $(2,4)$ $y - y_1 = m(x-x_1)$ $y - 4 = \frac{1}{2}(x-2)$ $2y - 8 = x - 2$ $x - 2y + 6 = 0$	2 Marks 1 mark m value 1 mark equation in general form.
iv) $A(-1,0)$, perp $d = \frac{\lvert ax_1+by_1+c \rvert}{\sqrt{a^2+b^2}}$ $= \frac{\lvert -1\times 1 - 2\times 0 + 6 \rvert}{\sqrt{1+4}}$ $= \frac{5}{\sqrt{5}}$ or $\sqrt{5}$ units	2 Marks 2 marks clear substitution into formula and correct answer 1 Mark calculator error
v) ABEC is a rhombus - diagonals perpendiculary bisect through D. [- find lengths AB and CA - adjacent sides equal - diagonals bisect at ⊾.]	2 marks ① for Rhombus ① for supporting reasons.
vi) Area $= \frac{1}{2} \times$ diagonals or $2 \times \triangle ACB$. $\triangle ACB = \frac{1}{2} \times 5 \times 4 = 10$ Area ABEC $= 2 \times 10 = 20$ units^2.	2 marks clear method from part (v). answer.

quest 8 Cont

b)



i) Aim: prove $\triangle AED \mathrel{\vert\vert\vert} \triangle BCD$ Method: In $\triangle AED$ and $\triangle BCD$ $AE \perp BD$ (given) $\therefore \angle AED = 90^\circ$ $\angle BCD = 90^\circ$ (∠ in a rectangle) $= \angle AED$ $\angle ADE = \angle CBD$ (alternate ∠s, AD∥BC. $\therefore \triangle AED \mathrel{\vert\vert\vert} \triangle BCD$ (matching ∠s equal or A.A)	2 Marks 2 clear steps with reasons 1 Mark error with reasoning
ii) $\frac{AD}{BD} = \frac{DE}{BC}$ (matching sides in ratio) but $AD = BC$ (equal sides in a rectangle) $\therefore AD \times AD = BD \times DE$. $AD^2 = BD \times DE$.	2 Marks 2 reasons with correct ratio 1 mark working not clearly justified
iii) $AD = \sqrt{AE^2 + DE^2}$ (Pythagoras' theorem) $= \sqrt{25+4}$ $= \sqrt{29}$ from (ii) $AD^2 = BD \times DE$ $= BD \times 2$ $\therefore \frac{29}{2} = BD$ Area ABCD $= 2 \times A_{\triangle}ADB$ $= 2 \times \frac{1}{2} \times 5 \times \frac{29}{2}$ $= 72.5$ cm^2	2 Marks correct answer 1 Mark. BD correct. or equivalent main value.