Girraween High School

Year 11 Mathematics — Task 4 — August 2003

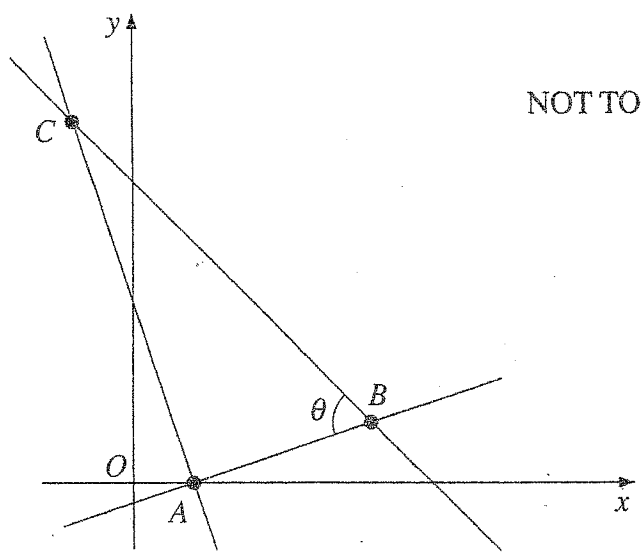
Time : 70 minutes

Instructions :

* Complete the test on your own paper.
* Show all necessary working.
* Marks will be deducted for careless or badly arranged work.

QUESTION 1. (12 marks)

The diagram shows points A(1,0), B(4,1) and C(- 1,6) in the Cartesian plane.
Angle ABC is θ .



Copy or trace this diagram.

(a) Show that A and C lie on the line $3x + y = 3$. **2**

(b) Show that the gradient of AB is $\frac{1}{3}$. **1**

(c) Show that the length of AB is $\sqrt{10}$ units. **1**

(d) Show that AB and AC are perpendicular. **1**

(e) Find $\tan\theta$. **2**

(f) Find the equation of the circle with centre A that passes through B. **2**

(g) The point D is not shown on the diagram. The point D lies on the line $3x + y = 3$ between A and C, and AD = AB. Find the coordinates of D. **2**

(h) On your diagram, shade the region satisfying the inequality $3x + y \leq 3$. . **1**

QUESTION 2 (7 marks)

(a) Find the equation of the line through the point of intersection of $2y - 3 = 5x$ and $x + y + 1 = 0$ which is perpendicular to $4x + 2y - 3 = 0$. **4**

(b) Find the perpendicular distance from the point (1,-2) to the line $3x - 2y - 6 = 0$. **3**

QUESTION 3 (13 marks)

(a) Find : (i) $\lim_{x \to 0} \frac{x^2 + 3x}{x}$ (ii) $\lim_{x \to 2} \frac{x^2 - x - 2}{x^2 - 5x + 6}$ (iii) $\lim_{x \to \infty} \frac{7x^3 - 6x^2 + 3}{3x^3 + 2x^2 - 4}$ **8**

(b) For $f(x) = x^2 + 3$ find $f(x+h) - f(x)$ **2**

(c) Differentiate $x^2 + 3$ from first principles. **3**

QUESTION 4 (20 marks)

(a) Differentiate : (i) $4x^3 - 2x^2 + 3x - 6$ (ii) $(2x+3)(3x^2 - 1)$ (iii) $\frac{5}{\sqrt{4+x}}$ **15**

(iv) $(4x+3)^7$ (v) $\frac{2x+1}{x-3}$ (vi) $\frac{2x^5 + 7x - 1}{2x^2}$

(b) If $f(t) = 5t - 10t^2$, find : **5**

(i) f(2) (ii) $f'(2)$.

QUESTION 5 (12 marks)

(a) Find the equation of the normal to the curve $y = x^3 + 2x^2 - 4x - 1$ at the point (-1,7) . **4**

(b) Find any values of p if the tangent to the curve $y = px^2 - 6x - 3$ is parallel to the line $5x - 3y + 2 = 0$ at the point where $x = -3$. **4**

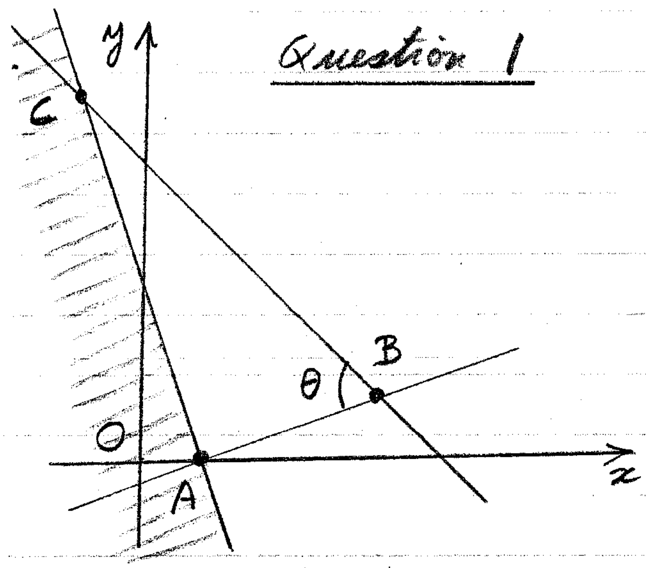
(c) Show that the following three lines are concurrent. **4**

$x - y = 2$

$2x + y = 1$

$3x + 2y = 1$

Solutions to Yr. 11 (2U) Task 4 Aug. 2003



Question 1

(a) $A(1,0)$; $C(-1,6)$; $3x+y=3$

For $A(1,0)$

$LHS = 3\times1+0 = 3 = RHS$

For $C(-1,6)$ ②

$LHS = 3\times -1+6 = 3 = RHS$

$\therefore$ A & C lie on $3x+y=3$

(b) $m_{AB} = \frac{1-0}{4-1} = \frac{1}{3}$ ①

(c) $AB = \sqrt{(1-0)^2+(4-1)^2} = \sqrt{10}$ units ①

(d) $m_{AB} = \frac{1}{3}$, $m_{AC} = \frac{6-0}{-1-1} = \frac{6}{-2} = -3$

$\therefore m_{AB}\times m_{AC} = \frac{1}{3}\times -3 = -1$.

$\therefore AB \perp AC$. ①

(e) $\tan\theta = \frac{AC}{AB}$

$AC = \sqrt{(-1-1)^2+(6-0)^2} = \sqrt{40}$

$= 2\sqrt{10}$.

$\therefore \tan\theta = \frac{2\sqrt{10}}{\sqrt{10}} = 2$ ②

(f) $A(1,0)$; $AB = \sqrt{10}$

$\therefore (x-1)^2+(y-0)^2 = (\sqrt{10})^2$

$\therefore (x-1)^2+y^2 = 10$. ②

(g) D is mid-point of AC

$\therefore D\left(\frac{1-1}{2}, \frac{0+6}{2}\right)$

i.e. $D(0,3)$ ②

(h) see diagram. ①

Question 2

(a) $(5x-2y+3)+k(x+y+1)=0$

$5x+kx-2y+ky+3+k=0$

$x(k+5)+y(k-2)+(k+3)=0$

$y(k-2) = -x(k+5)-(k+3)$

$\therefore m = \frac{-(k+5)}{k-2} = \frac{k+5}{2-k}$

$4x+2y-3=0$

$\therefore 2y = -4x+3$

$\therefore y = \frac{-4x}{2}+\frac{3}{2}$

$= -2x+\frac{3}{2}$

$\therefore m = -2$.

$\therefore \frac{k+5}{2-k}\times -2 = -1$

$2(k+5) = 2-k$

$2k+10 = 2-k$

$2k+k = 2-10$

$3k = -8$

$\therefore k = \frac{-8}{3}$

$\therefore$ eqn. of line is

$5x-2y+3-\frac{8}{3}(x+y+1)=0$

$15x-6y+9-8x-8y-8=0$

$\therefore\ 7x-14y+1=0.$ (4)

(b) $d=\dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$=\dfrac{|3(1)-2(-2)-6|}{\sqrt{3^2+(-2)^2}}$

$=\dfrac{|3+4-6|}{\sqrt{13}}$

$=\dfrac{1}{\sqrt{13}}$ units (3)

Question 3

(a)

(i) $\lim\limits_{x\to 0}\dfrac{x^2+3x}{x}$

$=\lim\limits_{x\to 0}\dfrac{\cancel{x}(x+3)}{\cancel{x}}$ (2)

$=\lim\limits_{x\to 0}(x+3)=3.$

(ii) $\lim\limits_{x\to 2}\dfrac{x^2-x-2}{x^2-5x+6}$

$=\lim\limits_{x\to 2}\dfrac{(x+1)\cancel{(x-2)}}{(x-3)\cancel{(x-2)}}$

$=\lim\limits_{x\to 2}\dfrac{x+1}{x-3}$

$=-3.$ (3)

(iii) $\lim\limits_{x\to\infty}\dfrac{7x^3-6x^2+3}{3x^3+2x^2-4}$

$=\lim\limits_{x\to\infty}\dfrac{7-6/x+3/x^3}{3+2/x-4/x^3}$

$=\frac{7}{3}$ or $2\frac{1}{3}$

since $\lim\limits_{x\to\infty}\frac{k}{x}=0$

and $\lim\limits_{x\to\infty}\frac{k}{x^3}=0.$ (3)

(k a constant).

(b) $f(x)=x^2+3$

$\therefore f(x+h)=(x+h)^2+3$

$=x^2+2hx+h^2+3$

$\therefore f(x+h)-f(x)$

$=2hx+h^2.$ (2)

(c)

$\dfrac{dy}{dx}=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

$=\lim\limits_{h\to 0}\dfrac{2hx+h^2}{h}$

$=\lim\limits_{h\to 0}\dfrac{\cancel{h}(2x+h)}{\cancel{h}}$

$=2x.$ (3)

Question 4

(a)

(i) $y=4x^3-2x^2+3x-6$

$\therefore\dfrac{dy}{dx}=12x^2-4x+3.$ (2)

(ii) $y=(2x+3)(3x^2-1)$

$=6x^3-2x+9x^2-3$

$\therefore\dfrac{dy}{dx}=18x^2+18x-2$

(2)

Question 4 (ctd.)

(iii) $y = \frac{5}{\sqrt{4+x}} = 5(4+x)^{-\frac{1}{2}}$

$\therefore \frac{dy}{dx} = 5 \cdot -\frac{1}{2}(4+x)^{-3/2}$

$= \frac{-5}{2(4+x)^{3/2}}$

$= \frac{-5}{2\sqrt{(4+x)^3}}$ (3)

(iv) $y = (4x+3)^7$

$\therefore \frac{dy}{dx} = 7(4x+3)^6 \cdot 4$

$= 28(4x+3)^6$ (2)

(v) $y = \frac{2x+1}{x-3}$

$\therefore \frac{dy}{dx} = \frac{(x-3)2 - (2x+1)1}{(x-3)^2}$

$= \frac{2x-6-2x-1}{(x-3)^2}$

$= \frac{-7}{(x-3)^2}$ (3)

(vi) $y = \frac{2x^5+7x-1}{2x^2}$

$= x^3 + \frac{7}{2}x^{-1} - \frac{1}{2}x^{-2}$

$\therefore \frac{dy}{dx} = 3x^2 - \frac{7}{2}x^{-2} + x^{-3}$

$= 3x^2 - \frac{7}{2x^2} + \frac{1}{x^3}$ (3)

(b) $f(t) = 5t - 10t^2$

(i) $f(2) = 5(2) - 10(2^2)$

$= 10 - 40$

$= -30.$ (2)

(ii) $f'(t) = 5 - 20t$

$\therefore f'(2) = 5 - 40$

$= -35.$ (3)

Question 5

(a) $y = x^3 + 2x^2 - 4x - 1$

$\frac{dy}{dx} = 3x^2 + 4x - 4$

at $(-1,7)$ $\frac{dy}{dx} = 3(-1)^2 + 4(-1) - 4$

$= -5$

$\therefore$ gradient of normal is $\frac{1}{5}$

$\therefore$ eqn. of normal is

$y - 7 = \frac{1}{5}(x+1)$

$5y - 35 = x + 1$

$\therefore x - 5y + 36 = 0$ (4)

(b) $y = px^2 - 6x - 3$

$\therefore \frac{dy}{dx} = 2px - 6$

at $x = -3$, $\frac{dy}{dx} = -6p - 6$

$5x - 3y + 2 = 0$

$\therefore 3y = 5x + 2$

$\therefore y = \frac{5}{3}x + \frac{2}{3}$

$\therefore m = \frac{5}{3}$

$\therefore \quad \frac{5}{3} = -6p - 6$

$5 = -18p - 18$

$18p = -23$

$\therefore p = \frac{-23}{18}$ or $-1\frac{5}{18}$

4

(c) $x - y = 2 \quad \ldots (1)$

$2x + y = 1 \quad \ldots (2)$

$3x + 2y = 1 \quad \ldots (3)$

$(1) + (2) \Rightarrow \quad 3x = 3$

$\therefore \quad x = 1$

From (1), $\quad 1 - y = 2$

$\therefore \quad y = -1$

Substitute $x = 1$, $y = -1$ into (3)

$\therefore$ LHS $= 3(1) + 2(-1)$

$= 3 - 2$

$= 1$

$=$ RHS.

4

$\therefore$ 3 lines above go through $x = 1$, $y = -1$.

ie. 3 lines are concurrent.