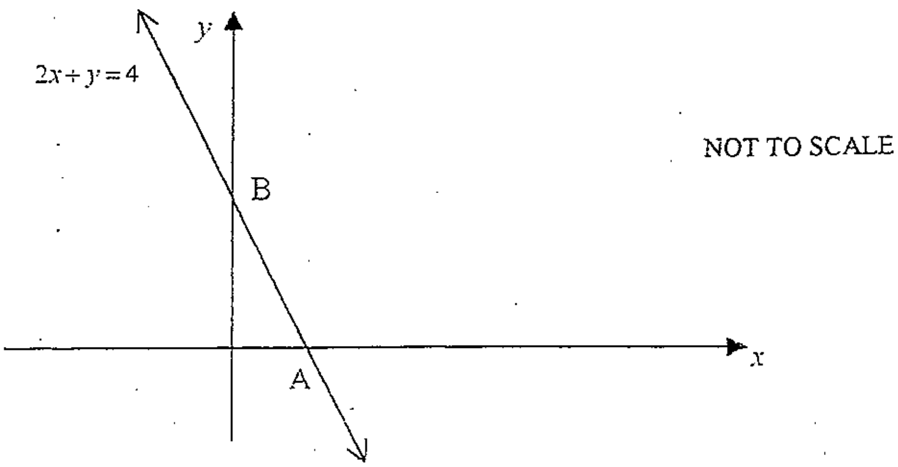
*GIRRAWEEN HIGH SCHOOL*

**MATHEMATICS**

**Year 11** 11th August 2005
Task 4 Time: **90 minutes**

**Instructions**: Attempt all questions.
Write your answers on your own paper.
All necessary working must be shown.
Marks may be deducted for careless or badly arranged work.
Begin each question on a new page.

**Question 1**(13 marks)




In the diagram above, the line $2x + y = 4$ cuts the $x$-axis at A and the y-axis at B.

Copy the diagram into your answer sheet.

a. Find the coordinates of points A and B. **2**

b. Find the perpendicular distance of the point C(5,2) from the line $2x + y = 4$. **3**

c. Show that the gradient of the line AC is $\frac{2}{3}$. **2**

d. Hence or otherwise find the equation of the line AC. **2**

e. Find the distance AB. **2**

f. Find the exact area of $\Delta ABC$. **2**

**Question 2**(12marks)

a. Find the equation of the line that has an angle of inclination of 45° with the $x$-axis and a $y$-intercept of $-1$. **2**

b. Prove that the line $4x + 3y - 20 = 0$ is a tangent to the circle $x^2 + y^2 = 16$. **4**

c. Find the equation of the straight line through (-4,-1) that passes through the intersection of the lines $2x + y - 1 = 0$ and $3x + 5y + 16 = 0$. **4**

d. Prove that the points A(1,2), B(-1,6) and C(2,0) are collinear. **2**

**Question 3**(12marks)

a. Evaluate

(i) $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3}$ (ii) $\lim\limits_{x \to 0} \dfrac{x^2 + 7x}{x}$ **4**

(iii) $\lim\limits_{x \to \infty} \dfrac{x}{x + 3}$ (iv) $\lim\limits_{x \to 5} \dfrac{x^3 - 125}{x - 5}$ **4**

b. If $f(x) = 2x^2 - 5$,

(i) find $f(x+h) - f(x)$. **2**

(ii) differentiate $2x^2 - 5$ from first principles. **2**

**Question 4** (18marks)

a. Differentiate

(i) $y = 3x^2 + 5x + 1$ (ii) $y = 4x^5 - 2x^7$ **4**

(iii) $y = \frac{7}{x}$ (iv) $y = 3\sqrt{x}$. **4**

(v) $y = x^2(x+3)$ (vi) $y = \frac{2x^3 + 5x}{x}$ **4**

b. Find $\frac{dy}{dx}$ when $x = -2$ if $y = 7x^2 - 5x + 6$. **3**

c. If $f(x) = x^3 + 2x^2 - 4x$, find $f'(-3)$. **3**

**Question 5**(16marks)

Differentiate

a. $y = (2x^2 - 7)(3x+1)$ b. $y = \frac{x+5}{3x+2}$ **6**

c. $y = (5x^2 - 8)^6$ d. $y = 2x\sqrt{8x-5}$ **7**

e. $y = (x+3)^2(2x-5)$ **3**

**Question 6**(17marks)

a. Find the equation of the **tangent** and **normal** to the curve $y = 2x^4 + 4x$ at the point where x = 1. **5**

b. Find any x values for which the gradient of the tangent to the curve $y = x^2 - 3x - 1$ is parallel to the line $x - 2y - 1 = 0$. **5**

c. Find the equation of the tangent to the curve $y = x^3 - x^2 + 2x + 6$ at the point P(1,8). Find the coordinates of point Q where this tangent meets the $y$-axis and calculate the exact length of PQ. **7**

Year 11 Task 4 Solutions 2005

## Question 1 (13 marks)

a) $2x + y = 4$

x int $\Rightarrow y = 0$

$2x = 4$

$x = 2$ $\therefore A(2,0)$

y int $\Rightarrow x = 0$

$y = 4$ $\therefore B(0,4)$ (2)

b) $d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$  $C(5,2)$, $2x+y-4=0$

$= \frac{|2(5)+1(2)-4|}{\sqrt{2^2+1^2}}$

$= \frac{8}{\sqrt5} \times \frac{\sqrt5}{\sqrt5} = \frac{8\sqrt5}{5}$ units (3)

c) $m_{AC} = \frac{2-0}{5-2} = \frac{2}{3}$ (2)

d) $y - y_1 = m(x - x_1)$  $m = 2/3$, $A(2,0)$

$y - 0 = \frac{2}{3}(x-2)$

$3y = 2x - 4$

$2x - 3y - 4 = 0$ (2)

e) $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-2)^2 + (4)^2}$

$= \sqrt{20}$

$= 2\sqrt5$ units (2)

f) Area of $\Delta ABC$

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 2\sqrt5 \times \frac{8}{\sqrt5}$

$= 8\ u^2$. (2)

## Question 2 (12 marks)

a) $m = \tan 45° = 1$  $b = -1$

Eq : $y = x - 1$ (2)

b) distance of tangent from origin  $4x+3y-20=0$, $(0,0)$

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \frac{|-20|}{\sqrt{4^2+3^2}}$

$= \frac{20}{5} = 4$ = radius of circle

$\therefore 4x+3y-20=0$ is a tangent to the circle $x^2+y^2=16$. (4)

c) pt $(-4,-1)$

$(2x+y-1)+k(3x+5y+16)=0$

$(2(-4)-1-1)+k(3(-4)+5(-1)+16)=0$

$-10 - k = 0$

$k = -10$

$\therefore (2x+y-1) - 10(3x+5y+16) = 0$

$2x+y-1-30x-50y-160=0$

$-28x-49y-161=0$

$4x+7y+23=0$ (4)

d) $m_{AB} = \frac{6-2}{-1-1} = \frac{4}{-2} = -2$

$m_{BC} = \frac{0-6}{2-(-1)} = \frac{-6}{3} = -2$ (2)

$\therefore$ A, B and C are collinear

## Question 3 (12 marks)

a) i) $\lim_{x\to3} \frac{x^2-9}{x-3}$

$= \lim_{x\to3} \frac{(x-3)(x+3)}{(x-3)}$

$= 6$ (2)

ii) $\lim_{x\to0} \frac{x^2+7x}{x}$

$= \lim_{x\to0} \frac{x(x+7)}{x}$

$= 7$ (2)

iii) $\lim_{x\to\infty} \frac{x}{x+3}$

$= \lim_{x\to\infty} \frac{1}{1+3/x}$

$= 1$ (2)

iv) $\lim_{x\to5} \frac{x^3-125}{x-5}$

$= \lim_{x\to5} \frac{(x-5)(x^2+5x+25)}{x-5}$

$= 75$ (2)

b) $f(x) = 2x^2 - 5$

i) $f(x+h) - f(x)$

$= 2(x+h)^2 - 5 - (2x^2-5)$

$= 2(x^2+2xh+h^2) - 5 - 2x^2 + 5$

$= 2x^2 + 4xh + 2h^2 - 5 - 2x^2 + 5$

$= 4xh + 2h^2$ (2)

ii) $f'(x) = \lim_{h\to0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to0} \frac{4xh+2h^2}{h}$

$= \lim_{h\to0} \frac{h(4x+2h)}{h}$

$= 4x$ (2)

## Question 4 (18 marks)

a) (i) $y = 3x^2 + 5x + 1$

$\frac{dy}{dx} = 6x + 5$ (2)

(ii) $y = 4x^5 - 2x^7$

$\frac{dy}{dx} = 20x^4 - 14x^6$ (2)

(iii) $y = \frac{7}{x} = 7x^{-1}$

$\frac{dy}{dx} = -7x^{-2}$

$= \frac{-7}{x^2}$ (2)

iv) $y = 3\sqrt{x} = 3x^{1/2}$

$\frac{dy}{dx} = \frac{3}{2}x^{-1/2}$

$= \frac{3}{2\sqrt{x}}$ (2)

v) $y = x^2(x+3) = x^3 + 3x^2$

$\frac{dy}{dx} = 3x^2 + 6x$ (2)

vi) $y = \frac{2x^3+5x}{x}$

$y = 2x^2 + 5$

$\frac{dy}{dx} = 4x$ (2)

b) $y = 7x^2 - 5x + 6$

$\frac{dy}{dx} = 14x - 5$

when $x = -2$, $\frac{dy}{dx} = 14(-2) - 5$

$= -33$ (3)

c) $f(x) = x^3 + 2x^2 - 4x$

$f'(x) = 3x^2 + 4x - 4$

$f'(-3) = 3(-3)^2 + 4(-3) - 4$

$= 27 - 12 - 4$

$= 11$ (3)

Question 5 (16 marks)

a) $y = (2x^2-7)(3x+1)$

$\frac{dy}{dx} = vu' + uv'$

$= (3x+1)4x + (2x^2-7)3$

$= 12x^2 + 4x + 6x^2 - 21$

$= 18x^2 + 4x - 21$ (3)

b) $y = \frac{x+5}{3x+2}$

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

$= \frac{(3x+2)1 - (x+5)3}{(3x+2)^2}$

$= \frac{3x+2-3x-15}{(3x+2)^2}$

$= \frac{-13}{(3x+2)^2}$ (3)

c) $y = (5x^2-8)^6$

$\frac{dy}{dx} = 6(5x^2-8)^5 . 10x$

$= 60x(5x^2-8)^5$ (3)

d) $y = 2x\sqrt{8x-5}$

$\frac{dy}{dx} = vu' + uv'$

$u = 2x$
$u' = 2$
$v = (8x-5)^{1/2}$
$v' = \frac{1}{2}(8x-5)^{-1/2} \cdot 8$
$= 4(8x-5)^{-1/2}$

$= (8x-5)^{1/2} . 2 + 2x[4(8x-5)^{-1/2}]$

$= 2\sqrt{8x-5} + \frac{8x}{\sqrt{8x-5}}$

$= \frac{2(8x-5)+8x}{\sqrt{8x-5}}$

$= \frac{16x-10+8x}{\sqrt{8x-5}}$

$= \frac{24x-10}{\sqrt{8x-5}}$ (4)

e) $y = (x+3)^2(2x-5)$

$\frac{dy}{dx} = vu' + uv'$

$= (2x-5)[2(x+3)] + (x+3)^2 . 2$

$= 2(x+3)(2x-5+x+3)$

$= 2(x+3)(3x-2)$ (3)

Question 6 (17 marks)

a) $y = 2x^4 + 4x$ at $x = 1$, pt $(1,6)$

Eq of tangent

$\frac{dy}{dx} = 8x^3 + 4$

$m = 8(1)^3 + 4$

$= 12$

$y - 6 = 12(x-1)$

$y - 6 = 12x - 12$

$\therefore 12x - y - 6 = 0$ (3)

Equation of normal

$m = -\frac{1}{12}$ pt $(1,6)$

$y - 6 = \frac{-1}{12}(x-1)$

$12y - 72 = -x + 1$

$x + 12y - 73 = 0$ (2)

b) $y = x^2 - 3x - 1$

$\frac{dy}{dx} = 2x - 3$

$x - 2y - 1 = 0$

$2y = x - 1$

$y = \frac{x}{2} - \frac{1}{2}$

$m = \frac{1}{2}$

$2x - 3 = \frac{1}{2}$

$2x = \frac{7}{2}$

$x = 1\frac{3}{4}$ (5)

c) $\frac{dy}{dx} = 3x^2 - 2x + 2$ (1)

at $(1,8)$ $m_{tangent} = 3(1)^2 - 2(1) + 2$

$m = 3$ (1)

Eq: $y - 8 = 3(x-1)$

$3x - y + 5 = 0$ (2)

$y$ intercept $= 5$

$\therefore Q(0,5)$ (1)

$PQ = \sqrt{(8-5)^2 + (0-1)^2}$

$= \sqrt{10}$ units. (2) (7)