GIRRAWEEN HIGH SCHOOL

YEAR 11 - TASK 4

2007

MATHEMATICS

2 UNIT

Time allowed – 90 minutes

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Start each question on a *new* sheet of paper.

Question 1 (11 marks)

Marks

(a) Find the distance between the points A(3, 1) and B(2, –1), leave answer in surd form. **2**

(b) Find the equation of the line joining the points (4, 8) and (2, 16). **3**

(c) If (2, 3) is the midpoint of the two points A(x, y) and B(4, 4). Find the coordinates of A. **3**

(d) A line is represented by the equation $\frac{1}{2}x + y - 3 = 0$. Find the equation of the line which is perpendicular to this line and that passes through the origin. **3**

Question 2 (11 marks)

(a) Find the perpendicular distance from the point (4, 5) to the line $x + 2y - 4 = 0$. **2**

(b) Find the angle of the line $y = -x + 2$ makes with the x-axis. **2**

(c) Find the equation of the line that passes through the point of intersection of the two lines $y = 2x + 1$ and $y = -2x + 5$, that has a gradient of 4. **4**

(d) Find the equation of the line that passes through the points (0,3) and (s, t), in gradient-intercept form. **3**

Question 3 (21 marks)

(a) Differentiate

(i) $y = \frac{1}{x^3} + \sqrt{x}$ **3**

(ii) $y = (3x^2 + 1)(5x^3 + 2)$ **3**

(iii) $y = \frac{2x+5}{x+2}$ **3**

(iv) $y = \frac{x^2}{4-x}$ **3**

(b) Evaluate

(i) $\lim_{x \to 0}\left(\frac{4x^2}{x^4 + x^2}\right)$ **3**

(ii) $\lim_{x \to 3}\left(\frac{x^2 - 9}{x - 3}\right)$ **3**

(iii) $\lim_{x \to \infty}\left(\frac{3x^2 - 15x - 19}{5x^2 + 13x + 1}\right)$ **3**

Question 4 (20 marks)

(a) Find $\frac{dy}{dx}$.

(i) $y = \sqrt[3]{x^2} - \frac{1}{x}$ **3**

(ii) $y = \sqrt{3x+5}$ **3**

(iii) $y = \frac{2}{(1-7x)^3}$ **3**

(iv) $y = (5x^3 + 4x)^{\frac{3}{2}}$ **3**

(v) $y = x^3(6x^2 + 1)^5$ **4**

(vi) $y = \frac{2x}{\sqrt[3]{x^2 + 1}}$ **4**

Question 5 (11 marks)

(a) Find the gradient of the curve $y = 15x^4 + 7x^2$ at the point where $x = 9$. **3**

(b) Find the equation of the tangent to the curve $y = x^2$ at the point (3, 9). **3**

(c) Find the points where $f'(x) = 0$, for the function $f(x) = \frac{1}{3}x^3 - 5x^2 + 16x + 5$ **5**

Question 6 (11 marks)

(a) Given $y = \frac{1}{\sqrt{x^2 + 9}}$, find the equation of the normal of the curve at $x = 4$. **7**

(b) Using first principles, find the derivative of $y = x^2 - 4x + 4$. **4**

* Mathematics (2007)
Year 11 - Task 4

Q1

a) $d_{AB} = \sqrt{(3-2)^2 + (1-(-1))^2}$

$= \sqrt{1+4}$

$d = \sqrt{5}$

b) $m = \frac{16-8}{2-4} = \frac{8}{-2} = -4$

$(4,8), m = -4$

$y - 8 = -4(x-4)$

$y - 8 = -4x + 16$

$y = -4x + 24$

or $4x + y - 24 = 0$

c) $2 = \frac{x+4}{2}$; $3 = \frac{y+4}{2}$

$4 = x + 4$ $\quad$ $6 = y + 4$

$x = 0$ $\quad$ $y = 2$

d) $\frac{1}{2}x + y - 3 = 0$

$y = -\frac{1}{2}x + 3$

$\therefore m_1 = -\frac{1}{2}$ $\therefore m_2 = +2$

$\therefore m = 2, (0,0)$

$y = 2x$

Q2

a) $d_b = \frac{|4 + 2(5) - 4|}{\sqrt{1^2 + 2^2}}$

$= \frac{10}{\sqrt{5}} = 2\sqrt{5}$

b) $m = -1$

$\tan\theta = m$

$\tan\theta = -1$

$\theta = -45°$ or $135°$

Q2 a) $y = 2x + 1$ ①; $y = -2x + 5$ ②

$\therefore$ Sub ① into ②

$2x + 1 = -2x + 5$

$4x = 4$

$x = 1$

$\therefore y = 2(1) + 1$

$y = 3$ $\quad$ $(1,3)$ intersection point.

$m = 4$.

$\therefore y - 3 = 4(x-1)$

$y - 3 = 4x - 4$

$y = 4x - 1$

or $4x - y - 1 = 0$

d) $(0,3)$ (s,t)

$m = \frac{t-3}{s-0} = \frac{t-3}{s}$

$y - 3 = \frac{t-3}{s}(x-0)$

$y = \frac{t-3}{s}x + 3$

Q3

(a) (i) $y = x^{-3} + x^{\frac{1}{2}}$

$y' = -3x^{-4} + \frac{1}{2}x^{-\frac{1}{2}}$

$y' = \frac{-3}{x^4} + \frac{1}{2\sqrt{x}}$

(ii) $y = 15x^5 + 5x^3 + 6x^2 + 2$

$y' = 75x^4 + 15x^2 + 12x$

$y' = 3x(25x^3 + 5x + 4)$

(iii) $y = \frac{2x+5}{x+2}$

$y' = \frac{(x+2)(2) - (2x+5)(1)}{(x+2)^2}$

$= \frac{2x+4-2x-5}{(x+2)^2}$

$y' = \frac{-1}{(x+2)^2}$

(iv) $y = \frac{x^2}{4-x}$

$y' = \frac{(4-x)(2x) - x^2(-1)}{(4-x)^2}$

$= \frac{8x - 2x^2 + x^2}{(4-x)^2}$

$y' = \frac{x(8-x)}{(4-x)^2}$

(b) (i) $\lim_{x \to 0} \frac{4x^2}{x^2(x^2+1)}$

$= \lim_{x \to 0} \frac{4}{x^2+1}$

$= \frac{4}{0^2+1}$

$= 4$

(ii) $\lim_{x \to 3} \frac{(x+3)(x-3)}{(x-3)}$

$= \lim_{x \to 3} (x+3)$

$= 3+3$

$= 6$

(iii) $\lim_{x \to \infty} \frac{3 - \frac{15}{x} - \frac{19}{x^2}}{5 + \frac{13}{x} + \frac{1}{x^2}}$

$= \frac{3-0-0}{5+0+0}$

$= 3/5$

Q4

(a) (i) $y = x^{2/3} - x^{-1}$

$\frac{dy}{dx} = \frac{2}{3}x^{-\frac{1}{3}} + x^{-2}$

$\frac{dy}{dx} = \frac{2}{3\sqrt[3]{x}} + \frac{1}{x^2}$

(ii) $y = (3x+5)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(3x+5)^{-\frac{1}{2}} \times 3$

$\frac{dy}{dx} = \frac{3}{2\sqrt{3x+5}}$

(iii) $y = 2(1-7x)^{-3}$

$\frac{dy}{dx} = -6(1-7x)^{-4} \times (-7)$

$\frac{dy}{dx} = \frac{42}{(1-7x)^4}$

(iv) $y = (5x^3+4x)^{3/2}$

$\frac{dy}{dx} = \frac{3}{2}(5x^3+4x)^{\frac{1}{2}} \times (15x^2+4)$

$\frac{dy}{dx} = \frac{3}{2}(15x^2+4)(5x^3+4x)^{\frac{1}{2}}$

(v) $y = x^3(6x^2+1)^5$

$\frac{dy}{dx} = (6x^2+1)^5(3x^2) + (x^3)5(6x^2+1)^4 \times 12$

$= 3x^2(6x^2+1)^5 + 60x^4(6x^2+1)^4$

$= 3x^2(6x^2+1)^4[6x^2+1+20x^2]$

$= 3x^2(26x^2+1)(6x^2+1)^4$

(vi) $y = \frac{2x}{(x^2+1)^{\frac{1}{3}}}$

$\frac{dy}{dx} = \frac{(x^2+1)^{\frac{1}{3}}(2) - (2x)\frac{1}{3}(x^2+1)^{-\frac{2}{3}} \times 2x}{\left((x^2+1)^{\frac{1}{3}}\right)^2}$

$\frac{dy}{dx} = \frac{2(x^2+1)^{\frac{1}{3}} - \frac{4}{3}x^2(x^2+1)^{-\frac{2}{3}}}{(x^2+1)^{2/3}}$

Q5 (a) $y = 15x^4 + 7x^2$

$y' = 60x^3 + 14x$

when $x=9$: $y' = 60(9)^3 + 14(9)$

$y' = 43740 + 126$

$y' = 43866$

(b) $y = x^2$

$\frac{dy}{dx} = 2x$

when $x=3$: $\frac{dy}{dx} = 2(3)$

$(3,9)$ $m = 6$

$y - 9 = 6(x-3)$

$y - 9 = 6x - 18$

$y = 6x - 9$.

(c) $f(x) = \frac{1}{3}x^3 - 5x^2 + 16x + 5$

$f'(x) = x^2 - 10x + 16$

$f'(x) = (x-8)(x-2)$

when $f'(x)=0$: $0 = (x-8)(x-2)$

$\therefore x = 8$, or 2

$\therefore f(8) = \frac{1}{3}(8)^3 - 5(8)^2 + 16(8) + 5$

$= -16\frac{1}{3}$ $\therefore \underline{(8, -16\frac{1}{3})}$

$f(2) = \frac{1}{3}(2)^3 - 5(2)^2 + 16(2) + 5$

$= 19\frac{2}{3}$ & $\underline{(2, 19\frac{2}{3})}$

Q6 $y = (x^2+9)^{-\frac{1}{2}}$

$y' = -\frac{1}{2}(x^2+9)^{-3/2} \times 2x$

$y' = \frac{-x}{(x^2+9)^{3/2}}$

when $x=4$: $y' = \frac{-4}{(4^2+9)^{3/2}}$ & $y = (4^2+9)^{-\frac{1}{2}}$, $y = 1/5$

$y' = -\frac{4}{125}$ perpendicular line

$\therefore m_1 = -\frac{4}{125}$ $\therefore \underline{m_2 = 125/4}$.

$(4, \frac{1}{5})$

$y - \frac{1}{5} = \frac{125}{4}(x-4)$

$y - \frac{1}{5} = \frac{125}{4}x - 125$

$y = \frac{125}{4}x - \frac{624}{5}$

or

$20y = 625x - 2496$

$\underline{625x - 20y - 2496 = 0}$.

b) $f(x) = x^2 - 4x + 4$

$f(x+h) = (x+h)^2 - 4(x+h) + 4$

$\frac{dy}{dx} = \lim_{h\to 0} \frac{(x+h)^2 - 4(x+h) + 4 - (x^2 - 4x + 4)}{h}$

$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 4x - 4h + 4 - x^2 + 4x - 4}{h}$

$= \lim_{h\to 0} \frac{h(2x - 4 + h)}{h}$

$= 2x - 4 + 0$

$\underline{\frac{dy}{dx} = 2x - 4}$