GIRRAWEEN HIGH SCHOOL

YEAR 11 - TASK 4

2008

MATHEMATICS
2 UNIT

Time allowed – 90 minutes

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Start each question on a *new* sheet of paper.

Question 1 (10 marks) **Marks**

(a) Evaluate

(i) $\lim_{x \to 4} \frac{2x-3}{x}$ **1**

(ii) $\lim_{x \to -3} \frac{x+3}{x^2-9}$ **2**

(iii) $\lim_{x \to \infty} \frac{x^3+5x^2+3}{4x^4-x^2+x}$ **2**

(iv) $\lim_{x \to \infty} \frac{x^3+5x^2+3}{4x^3-x^2+x}$ **2**

(b) If $f(x) = x^2 - 3x + 2$, differentiate $f(x)$ from first principles **3**

Question 2 (15 marks)

(a) Differentiate

(i) $y = 2x^5 - 4x^2 + 21x + 7$ **1**

(ii) $y = \frac{15x^5 + 6x^3 - 12x}{3x}$ **2**

(iii) $y = 12x^3 - \sqrt{2}\,x^2 + \pi$ **2**

(iv) $y = \sqrt{5}\,x + \frac{1}{x^4}$ **2**

(v) $y = x^{\frac{3}{5}} - 7x^{\frac{1}{4}}$ **3**

(vi) $y = \sqrt[4]{x^3}$ **2**

(b) Differentiate $f(x) = x^3 - 7x^2 - 5x + 6$ and hence find the two points on $y = x^3 - 7x^2 - 5x + 6$ where the tangent is horizontal. **3**

Question 3 (7 marks)

For the function $y = x^3 - 2x^2$

(a) Find $\dfrac{dy}{dx}$ **1**

(b) Find the equations of the tangent and normal lines to the curve at $(2,0)$. Give your answer in general form. **3**

(c) Find the gradient at $(a, a^3 - 2a^2)$ **1**

(d) Find the equation of the tangent at $(a, a^3 - 2a^2)$. Give your answer in gradient / intercept form. **2**

Question 4 (7 marks)

(a) Differentiate $y = (x^2 + 1)(2x^3 - 3x)$ using product rule. **3**

(b) Differentiate $y = (7x^5 - 13x^2 + 4)^7$ **2**

(c) Differentiate $y = \dfrac{1}{\sqrt{5x+2}}$ **2**

Question 5 (13 marks)

(a) Differentiate $y = 8x^3(4x^2 - 9)^5$ **4**

(b) Differentiate $y = \sqrt{5-x}(10-x)^3$ **5**

(c) Find $\dfrac{dy}{dx}$ for $y = (2x+5)^2(x+6)^2$ and find where $\dfrac{dy}{dx} = 0$ **4**

Question 6 (12 marks)

(a) Differentiate the following

(i) $f(x)=\dfrac{x}{4x-7}$ **3**

(ii) $y=\dfrac{(2x+3)^3}{6x+5}$ **4**

(b) Find the point on $y=\dfrac{1}{\sqrt{x+1}}$ where the tangent is parallel to $x+128y+256=0$ **5**

Question 7 (10 marks)

(a) Find the acute angle (to the nearest minute) at which the tangent of $y=5x^4-6x^2+4$ at $\left(\dfrac{1}{2},\dfrac{45}{16}\right)$ will intersect the x-axis **3**

(b) For $f(x)=\dfrac{(3x-2)^2}{4x^5}$, find

(i) $f'(x)$ **4**

(ii) $f'(1)$ **1**

(iii) $f'(x+2)$ **2**

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Q1

a) i) $\lim_{x\to 4} \frac{2x-3}{4}$

$= \frac{2(4)-3}{4}$

$= 5/4$

1

ii) $\lim_{x\to -3} \frac{x+3}{x^2-9}$

$= \lim_{x\to -3} \frac{x+3}{(x+3)(x-3)}$

$= \lim_{x\to -3} \frac{1}{x-3}$

2

$= -1/6$

iii) $\lim_{x\to\infty} \frac{x^3+5x^2+3}{4x^4-x^2+x} \cdot \frac{1/x^4}{1/x^4}$

$= \lim_{x\to\infty} \frac{1/x + 5/x^2 + 3/x^4}{4 - 1/x^2 + 1/x^3}$

$= \frac{0+0+0}{4-0+0}$

2

$= 0$

iv) $\lim_{x\to\infty} \frac{x^3+5x^2+3}{4x^3-x^2+x} \cdot \frac{1/x^3}{1/x^3}$

$= \lim_{x\to\infty} \frac{1 + 5/x + 3/x^3}{4 - 1/x + 1/x^2}$

2

$= \frac{1+0+0}{4+0+0}$

$= 1/4$

b) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{(x+h)^2-3(x+h)+2-(x^2-3x+2)}{h}$

$= \lim_{h\to 0} \frac{x^2+2hx+h^2-3x-3h+2-x^2+3x-2}{h}$

$= \lim_{h\to 0} \frac{2hx+h^2-3h}{h}$

3

$= \lim_{h\to 0} 2x+h-3$

$= 2x-3$

Q2 i) $y' = 10x^4 - 8x + 21$ (1)

a) ii) $y = 5x^4 + 2x^2 - 4$

$y' = 20x^3 + 4x$ (2)

iii) $y' = 36x^2 - 2\sqrt{2}\,x$ (2)

iv) $y = \sqrt{5}x + x^{-4}$

$y' = \sqrt{5} - 4x^{-5}$ (2)

$= \sqrt{5} - \frac{4}{x^5}$

v) $y' = \frac{3}{5}x^{-2/5} - \frac{7}{4}x^{-3/4}$

$= \frac{3}{5x^{2/5}} - \frac{7}{4x^{3/4}}$ (3)

vi) $y = x^{3/4}$

$y' = \frac{3}{4}x^{-1/4}$

$= \frac{3}{4\sqrt[4]{x}}$ (2)

b) $f'(x) = 3x^2 - 14x - 5$

$= (3x+1)(x-5)$

$0 = (3x+1)(x-5)$ (3)

$\therefore x = -1/3 \quad x = 5$

$f(\frac{-1}{3}) = (\frac{-1}{3})^3 - 7(\frac{-1}{3})^2 - 5(\frac{-1}{3}) + 6$

$= \frac{-1}{27} - \frac{7}{9} + \frac{5}{3} + 6$

$= \frac{-1}{27} - \frac{21}{27} + \frac{45}{27} + \frac{162}{27}$

$= \frac{185}{27} \quad \therefore \left(\frac{-1}{3}, \frac{185}{27}\right)$

$f(5) = 5^3 - 7(5)^2 - 5(5) + 6$

$= 125 - 175 - 25 + 6$

$= -69 \quad \therefore (5, -69)$

Q3 a) $\frac{dy}{dx} = 3x^2 - 4x$ (1)

b) $\left.\frac{dy}{dx}\right|_{x=2} = 3(2)^2 - 4(2)$

$= 12 - 8$

$= 4$

TANGENT:

$y - y_1 = m(x - x_1)$

$y = 4(x-2)$ (3)

$0 = 4x - y - 8$

NORMAL

$y - y_1 = m(x - x_1)$

$y = -\frac{1}{4}(x-2)$

$x + 4y - 2 = 0$

c) $\left.\frac{dy}{dx}\right|_{x=a} = 3a^2 - 4a$ (1)

d) TANGENT:

$y - (a^3 - 2a^2) = (3a^2 - 4a)(x - a)$

$y = (3a^2 - 4a)x - 3a^3 + 4a^2 + a^3 - 2a^2$ (2)

$y = (3a^2 - 4a)x + (-2a^3 + 2a^2)$

Q4 $u = x^2 + 1 \qquad v = 2x^3 - 3x$

$u' = 2x \qquad v' = 6x^2 - 3$

a)

$y' = vu' + uv'$

$= (2x^3 - 3x)(2x) + (x^2+1)(6x^2-3)$ (3)

$= 4x^4 - 6x^2 + 6x^4 - 3x^2 + 6x^2 - 3$

$= 10x^4 - 3x^2 - 3$

b) $y' = 7(7x^5 - 13x^2 + 4)^6(35x^4 - 26x)$ (2)

c) $y = (5x+2)^{-1/2}$

$y' = -\frac{1}{2}(5x+2)^{-3/2} \cdot 5$ (2)

$= \frac{-5}{2\sqrt{5x+2}^{\,3}}$

Q5 a) $u = 8x^3$ $\quad v = (4x^2-9)^5$

$u' = 24x^2$ $\quad v' = 5(4x^2-9)^4(8x)$

$= 40x(4x^2-9)^4$

$$y' = vu' + uv'$$

$$= (4x^2-9)^5(24x^2) + 8x^3 \cdot 40x(4x^2-9)^4$$

$$= 8x^2(4x^2-9)^4\left[3(4x^2-9) + 40x^2\right]$$

$$= 8x^2(2x+3)^4(2x-3)^4\left[12x^2 - 27 + 40x^2\right]$$

$$= 8x^2(2x+3)^4(2x-3)^4(52x^2-27)$$

4

b) $u = \sqrt{5-x} = (5-x)^{1/2}$ $\quad v = (10-x)^3$

$u' = \frac{-1}{2\sqrt{5-x}}$ $\quad v' = -3(10-x)^2$

$$y' = vu' + uv'$$

$$= (10-x)^3\frac{(-1)}{2\sqrt{5-x}} + \sqrt{5-x}\left(-3(10-x)^2\right)$$

$$= \frac{-(10-x)^3 + 2\sqrt{5-x}\sqrt{5-x}\left(-3(10-x)^2\right)}{2\sqrt{5-x}}$$

$$= \frac{(10-x)^2\left[-(10-x) - 6(5-x)\right]}{2\sqrt{5-x}}$$

5

$$= \frac{(10-x)^2(7x-40)}{2\sqrt{5-x}}$$

c) $u = (2x+5)^2$ $\quad v = (x+6)^2$

$\frac{du}{dx} = 4(2x+5)$ $\quad \frac{dv}{dx} = 2(x+6)$

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$= (x+6)^2 \cdot 4(2x+5) + (2x+5)^2\, 2(x+6)$$

$$= 2(x+6)(2x+5)\left[2(x+6) + (2x+5)\right]$$

$$= 2(x+6)(2x+5)(4x+17)$$

$\therefore \frac{dy}{dx} = 0$ WHEN $x = -6$ $\quad y = 0$

$x = \frac{-5}{2}$ $\quad y = 0$

$x = \frac{-17}{4}$ $\quad y = \frac{2401}{64}$

Q6 a) i)

$u = x \qquad v = 4x-7$

$u' = 1 \qquad v' = 4$

$$f'(x) = \frac{vu' - uv'}{v^2}$$

$$= \frac{4x-7 - 4x}{(4x-7)^2}$$

3

$$= \frac{-7}{(4x-7)^2}$$

ii)

$u = (2x+3)^3 \qquad v = 6x+5$

$u' = 3(2x+3)^2 \cdot 2 \qquad v' = 6$

$= 6(2x+3)^2$

$$y' = \frac{vu' - uv'}{v^2}$$

$$= \frac{(6x+5)\,6(2x+3)^2 - (2x+3)^3 \cdot 6}{(6x+5)^2}$$

$$= \frac{6(2x+3)^2\left[6x+5 - (2x+3)\right]}{(6x+5)^2}$$

4

$$= \frac{6(2x+3)^2(4x+2)}{(6x+5)^2}$$

$$= \frac{12(2x+3)^2(2x+1)}{(6x+5)^2}$$

b)

$y = (x+1)^{-1/2}$

$y' = \frac{-1}{2}(x+1)^{-3/2}$

$$= \frac{-1}{2\sqrt{x+1}^{\,3}}$$

$x + 128y + 256 = 0$

$128y = -x - 256$

$y = \frac{-1}{128}x - 2$

$\therefore m = \frac{-1}{128}$

$$\therefore \frac{-1}{128} = \frac{-1}{2\sqrt{x+1}^{\,3}}$$

$-128 = -2\sqrt{x+1}^{\,3}$

$64 = (x+1)^{3/2}$

$64^{2/3} = x+1$

5

$16 - 1 = x$

$15 = x$

$\therefore y = \frac{1}{\sqrt{15+1}}$

$= \frac{1}{4}$

So $\left(15, \frac{1}{4}\right)$

Q7 $y' = 20x^3 - 12x$

a) $y'\Big|_{x=\frac{1}{2}} = 20\left(\frac{1}{2}\right)^3 - 12\left(\frac{1}{2}\right)$

$= \frac{20}{8} - \frac{12}{2}$

$= \frac{-7}{2}$ 3

$\tan^{-1}(7/2) = 74°3'$

b) $u = (3x-2)^2$ $v = 4x^5$

$u' = 2(3x-2)3$ $v' = 20x^4$

$= 6(3x-2)$

$f'(x) = \frac{vu' - uv'}{v^2}$

$= \frac{4x^5(6(3x-2)) - (3x-2)^2 20x^4}{(4x^5)^2}$

$= \frac{4x^4(3x-2)[6x - 5(3x-2)]}{16x^{10}}$

$= \frac{(3x-2)(-9x+10)}{4x^6}$

$= \frac{(3x-2)(10-9x)}{4x^6}$

$f'(1) = (3(1)-2)(10-9(1)) / 4(1)^6$

$= 1/4$

$f'(x+2) = \frac{(3(x+2)-2)(10-9(x+2))}{4(x+2)^6}$

$= \frac{(3x+4)(-9x-8)}{4(x+2)^6}$

$= \frac{-(3x+4)(9x+8)}{4(x+2)^6}$