GIRRAWEEN HIGH SCHOOL

YEAR 11 - TASK 4

2010

MATHEMATICS

Part A

Time allowed –45 minutes

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Start each question on a <u>*new*</u> sheet of paper.

Question 1 (10 marks)

Marks

(a) Evaluate

(i) $\lim_{x \to -1}(x^2 + 4x)$ 1

(ii) $\lim_{x \to 2} \frac{x-2}{x^2 - 4}$ 2

(iii) $\lim_{x \to -4}(\frac{x^2 + 2x - 8}{x + 4})$ 2

(iv) $\lim_{x \to \infty}(\frac{2x^3 - 3x - 6}{3x^3 + 1})$ 2

(b) If $f(x) = x^2 - 4x - 3$, differentiate $f(x)$ from first principles 3

Question 2 (16 marks)

(a) Differentiate

(i) $y = x^3 + 2x^2 - 7x - 3$ 1

(ii) $y = x\sqrt{x}$ 3

(iii) $y = \frac{x^3 + 2x^2 + 5x - 4}{x^2}$ 3

(iv) $y = \sqrt[3]{x^2}$ 3

(v) $y = 2x^{\frac{3}{2}} - 5x^{\frac{5}{3}}$ 3

(vi) $y = \frac{x-2}{\sqrt{x}}$ 3

Question 3 (9 marks)

Differentiate the following:

(i) Use the product rule to differentiate $y=(x^3+1)(x^2-3x)$ **3**

(ii) $y=(2x^3+3x^4)^4$ **3**

(iii) $y=\dfrac{x+5}{3x+1}$ **3**

Question 4 (10marks)

(i) Find the equations of the *tangent* and *normal* to the curve

$y=x^3+2x^2-4x-1$ at the point (-1,7) **6**

(ii) Find the coordinates of the point on the curve $y=2x(x-3)$ where the

tangent is parallel to the x-axis. **4**

Question 5 (8 marks)

(i) Find any point on the curve $y=x^2+\frac{1}{3}x^3$ where the tangent

has the angle of inclination 135^0. **3**

(ii) The tangent to the curve x^2-3x+1, at the point P(2,-1) cuts the y-axis at Q. The normal to the curve at the same point, cuts the y-axis at R. Find the area of ΔPQR **5**

GIRRAWEEN HIGH SCHOOL

YEAR 11 - TASK 4

2010

MATHEMATICS
2 UNIT

Time allowed – 45 minutes

Part B

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Start each question on a *new* sheet of paper.

Question 1 (10 marks)

Marks

(a) Find the quadratic equation with roots $2+\sqrt{3}$ and $2-\sqrt{3}$. **2**

(b) Write down the equation of a monic quadratic for which one of the zeroes is $x=1$ and the axis of symmetry is $x=-7$ **2**

(c) Show that the quadratic equation $x^2+(2k+3)x+6k=0$ has rational roots for all values of k. **3**

(d) For what value(s) of m does the equation $(m-1)x^2+2(m+1)x+2m-1=0$ have equal roots? **3**

Question 2 (15 marks)

(a) Solve the equation $3^{2x}+2\times 3^x-15=0$ **4**

(b) Find the maximum value of the function $y=17+4x-x^2$ **3**

(c) For what values of k is the expression $kx^2-(3k-1)x+k$ positive definite? **3**

(d) For the equation $x^2+(m-3)x+m=0$

(i) Find an expression for the discriminant b^2-4ac in terms of m **1**

(ii) Hence find the values of m for which this equation has:

(α) exactly one root. **2**

(β) two roots. **2**

Question 3.(15 marks)

(a) Find the values of A, B and C if

$$6x^2 - 11 \equiv A(x+2)^2 + Bx + C$$ 4

(b) Given that α and β are the roots of the quadratic equation $3x^2 + 4x - 3 = 0$,
Evaluate

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ 2

(iv) $\alpha^2 + \beta^2$ 2

(v) $\alpha^3\beta + \alpha\beta^3$ 2

(c) Find the value of k in the quadratic equation $(2k-1)x^2 - 5kx + (k+1) = 0$,
if the roots are reciprocal. 3

Year 11 Mathematics 2010

Task 4 Part A solutions.

Question 1.

(i) $\lim_{x \to -1} (x^2 + 4x) = 1 + 4(-1)$

$= -3$ (1)

(ii) $\lim_{x \to 2} \frac{x-2}{x^2-4} = \lim_{x \to 2} \frac{(x-2)}{(x-2)(x+2)}$

$= \lim_{x \to 2} \frac{1}{x+2}$

$= \frac{1}{4}$ (2)

(iii) $\lim_{x \to -4} \frac{x^2+2x-8}{x+4}$

$= \lim_{x \to -4} \frac{(x+4)(x-2)}{x+4}$

$= \lim_{x \to -4} x-2$

$= -6$ (2)

(iv) $\lim_{x \to \infty} \frac{2x^3 - 3x - 6}{3x^3+1}$

$= \lim_{x \to \infty} \frac{2 - \frac{3}{x^2} - \frac{6}{x^3}}{3 + \frac{1}{x^3}}$

$= \frac{2}{3}$ (2)

(v) $f(x) = x^2 - 4x - 3$

$f(x+h) = (x+h)^2 - 4(x+h) - 3$

$= x^2 + 2xh + h^2 - 4x - 4h - 3$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 4x - 4h - 3 - (x^2 - 4x - 3)}{h}$

$= \lim_{h \to 0} \frac{2xh + h^2 - 4h}{h}$

$= \lim_{h \to 0} \frac{h(2x + h - 4)}{h}$

$= 2x - 4$ (3)

Question 2

(i) $y = x^3 + 2x^2 - 7x - 3$

$\frac{dy}{dx} = 3x^2 + 4x - 7$ (1)

(ii) $y = x\sqrt{x} = x^{3/2}$

$\frac{dy}{dx} = \frac{3}{2} x^{1/2} = \frac{3\sqrt{x}}{2}$ (3)

(iii) $y = \frac{x^3 + 2x^2 + 5x - 4}{x^2}$

$= x + 2 + \frac{5}{x} - \frac{4}{x^2} = x + 2 + 5x^{-1} - 4x^{-2}$

$\frac{dy}{dx} = 1 - 5x^{-2} + 8x^{-3}$

$= 1 - \frac{5}{x^2} + \frac{8}{x^3}$ (3)

(iv) $y = \sqrt[3]{x^2} = x^{2/3}$

$\frac{dy}{dx} = \frac{2}{3} \cdot x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$ (3)

(v) $y = 2x^{3/2} - 5x^{5/3}$

$\frac{dy}{dx} = 2 \cdot \frac{3}{2} x^{1/2} - 5 \cdot \frac{5}{3} x^{\frac{2}{3}}$

$= 3x^{1/2} - \frac{25}{3} x^{2/3}$ (3)

Question 2

(vi) $y = \frac{x-2}{\sqrt{x}} = \sqrt{x} - \frac{2}{\sqrt{x}}$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 2.\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$

$= \frac{1}{2\sqrt{x}} + \frac{1}{x^{\frac{3}{2}}} = \frac{1}{2\sqrt{x}} + \frac{1}{\sqrt{x^3}}$

(3)

Question 3

(i) $y = (x^3+1)(x^2-3x)$

$\frac{dy}{dx} = vu' + uv'$

$v = x^3+1$
$v' = 3x^2$
$u = x^2-3x$
$u' = 2x-3$

$= (x^3+1)(2x-3) + (x^2-3x)3x^2$

$= 2x^4 + 2x - 3x^3 - 3 + 3x^4 - 9x^3$

$= 5x^4 - 12x^3 + 2x - 3$

(3)

(ii) $y = (2x^3+3x^4)^4$

$\frac{dy}{dx} = 4\cdot(2x^3+3x^4)(6x^2+12x^3)$

~~$= 4x^3(2x^3+3x^4)\,6x^2(1+2x)$~~

$= 4x^3(2+3x)\cdot 6x^2(1+2x)$

$= 24x^5(2+3x)(1+2x)$

(3)

(iii) $y = 4x(3x-2)^5$

$v = 4x$
$v' = 4$
$u = (3x-$
$u' = 15(3x-$

$\frac{dy}{dx} = vu' + uv'$

$= 4x.15(3x-2)^4 + (3x-2)^5\cdot 4$

$= 60x(3x-2)^4 + 4(3x-2)^5$

$= 4(3x-2)^4(15x+3x-2)$

$= 4(3x-2)^4(18x-2)$

$= 8(3x-2)^4(9x-1)$

(3)

(iv) $y = \frac{x+5}{3x+1}$

$u = x+5$
$u' = 1$
$v = 3x+1$
$v' = 3$

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

$= \frac{(3x+1).1 - (x+5).3}{(3x+1)^2}$

$= \frac{3x+1-3x-15}{(3x+1)^2} = \frac{-14}{(3x+1)^2}$

(3)

Question 4

(i) $y = x^3 + 2x^2 - 4x - 1$

$\frac{dy}{dx} = 3x^2 + 4x - 4$

at $x = -1$, gradient $m =$

$= 3(-1)^2 + 4(-1) - 4$

$= 3 - 4 - 4 = -5$

equation of tangent Pt $(-1, 7)$

$y - 7 = -5(x+1) = -5x - 5$

ie $5x + y - 2 = 0$ (3)

gradient of normal $= \frac{1}{5}$

equation of normal

$y - 7 = \frac{1}{5}(x+1)$

$5y - 35 = x + 1$

ie $x - 5y + 36 = 0$ (3)

(ii) $y = 2x(x-3) = 2x^2 - 6x$ — (1)

$\frac{dy}{dx} = 4x - 6$

Parallel to x-axis $\Rightarrow$ gradient $= 0$

$\therefore 4x - 6 = 0 \Rightarrow x = \frac{3}{2}$

sub $x = \frac{3}{2}$ in (1)

$y = 2 \cdot \frac{3}{2}\left(\frac{3}{2} - 3\right) = 3 \cdot \left(\frac{-3}{2}\right)$

$= \frac{-9}{2}$

$\therefore$ Co-ordinates are $\left(\frac{3}{2}, \frac{-9}{2}\right)$.

Question 5

(i)

$y = x^2 + \frac{1}{3}x^3$

$\frac{dy}{dx} = 2x + x^2$

gradient $= \tan 135^\circ = -1$

$\therefore 2x + x^2 = -1$

$x^2 + 2x + 1 = 0 \Rightarrow (x+1)^2 = 0$

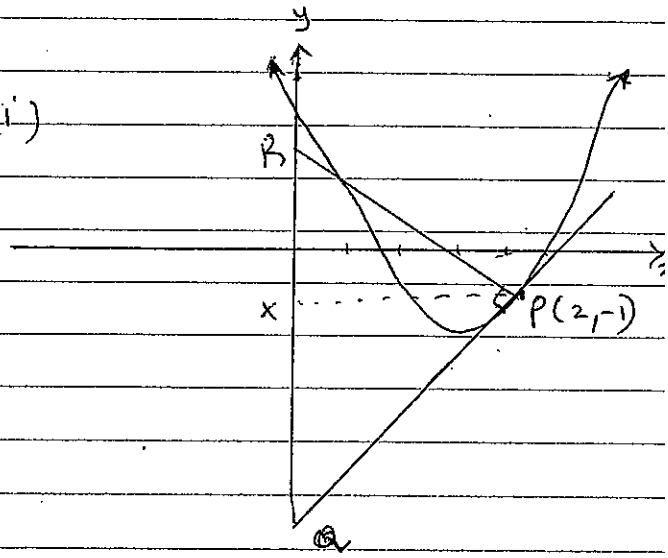
$\therefore x = -1$

$\therefore y = (-1)^2 + \frac{1}{3}(-1)^3 = 1 - \frac{1}{3}$

$= \frac{2}{3}$

$\therefore$ the point is $\left(-1, \frac{2}{3}\right)$

(ii)



$y = x^2 - 3x + 1$ $P = (2, -1)$

$\frac{dy}{dx} = 2x - 3$.

at P, gradient $= 4 - 3 = 1$

$\therefore$ equation of tangent.

$y + 1 = 1(x - 2) \Rightarrow y = x - 3$.

$\therefore$ Co-ordinates of $Q = (0, -3)$.

eq^n of normal at P is

$y + 1 = -1(x-2) \Rightarrow y = -x + 1$

Co-ordinates of $R = (0, 1)$.

Area of ΔPQR = area($\Delta PRX + \Delta PQX$) $= \frac{1}{2} \cdot 2 \cdot 2 + \frac{1}{2} \cdot 2 \cdot 2$

$= 4u^2$

Year 11 Mathematics.
Task 4 2010 Solutions.
Part B

Question 1.

a) roots: $2+\sqrt{3}$, $2-\sqrt{3}$

$\alpha+\beta = 4$; $\alpha\beta = 4-3 = 1$

$\therefore$ equation: $x^2 - 4x + 1 = 0$ (2)

b) $y = (x+7)^2 + k$

one zero is at $x = 1$.

sub in $x=1$

$\therefore 0 = 8^2 + k \Rightarrow k = -64$

$\therefore y = (x+7)^2 - 64$

$y = x^2 + 14x - 15$ (2)

c) $x^2 + (2k+3)x + 6k = 0$.

$\Delta = (2k+3)^2 - 4 \cdot 1 \cdot 6k$

$= 4k^2 + 12k + 9 - 24k$

$= 4k^2 - 12k + 9$

$= (2k-3)^2$ is a perfect square.

$\therefore$ has rational roots for all k

d) $(m-1)x^2 + 2(m+1)x + 2m-1 = 0$

equal roots $\Rightarrow b^2 - 4ac = 0$.

$\therefore 4(m+1)^2 - 4(m-1)(2m-1) = 0$.

$\Rightarrow 4[m^2 + 2m + 1 - 2m^2 + 3m - 1] = 0$

$\Rightarrow 4[-m^2 + 5m] = 0$, ...

Question 2.

a) $3^{2x} + 2 \times 3^x - 15 = 0$.

Let $u = 3^x$

then $u^2 + 2u - 15 = 0$.

$(u+5)(u-3) = 0$

$u = -5$, 3

ie $3^x = 3$ or $3^x = -5$

$3^x = 3^1$ or no solution

$\therefore x = 1$ (4)

b) $y = 17 + 4x - x^2$

$= -(x^2 - 4x - 17)$

$= -[(x-2)^2 - 4 - 17]$

$= -(x-2)^2 + 21$

$\therefore$ maximum value $= 21$ (3)

c) $kx^2 - (3k-1)x + k$

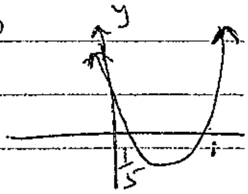
positive definite $\Rightarrow a > 0$, $\Delta < 0$

$\therefore k > 0$ and $\Delta = (3k-1)^2 - 4k^2 < 0$

$9k^2 - 6k + 1 - 4k^2 < 0$

$5k^2 - 6k + 1 < 0$

$(5k-1)(k-1) < 0$



$\frac{1}{5} < k < 1$

(3)

Question 2

a) $x^2 + (m-3)x + m = 0$

(i) $D = b^2 - 4ac = (m-3)^2 - 4m$

$= m^2 - 10m + 9$ (1)

(ii) (α) exactly one root $\Rightarrow D = 0$

$\therefore m^2 - 10m + 9 = 0$

$(m-9)(m-1) = 0$

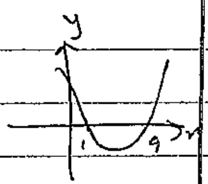
$\therefore m = 1$ or 9

β) two roots $\Rightarrow D > 0$

$m^2 - 10m + 9 > 0$

$(m-9)(m-1) > 0$

$\therefore m > 9$ or $m < 1$



Question 3

a) $6x^2 - 11 = A(x+2)^2 + Bx + C$

$= A(x^2 + 4x + 4) + Bx + C$

$= Ax^2 + x(4A + B) + 4A + C$

Equating the coefficients of same powers of x,

$A = 6$; $4A + B = 0 \Rightarrow B = -24$

$4A + C = -11 \Rightarrow C = -35$

$\therefore A = 6$; $B = -24$; $C = -35$

(4)

b) $3x^2 + 4x - 3 = 0$

(i) $\alpha + \beta = \frac{-b}{a} = \frac{-4}{3}$ (1)

(ii) $\alpha\beta = \frac{c}{a} = \frac{-3}{3} = -1$ (1)

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-\frac{4}{3}}{-1} = \frac{4}{3}$

(2)

(iv) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \frac{16}{9} + 2 = \frac{34}{9} = 3\frac{7}{9}$

(2)

(v) $\alpha^3\beta + \alpha\beta^3 = \alpha\beta(\alpha^2 + \beta^2)$

$= -1\left(\frac{34}{9}\right) = \frac{-34}{9}$

$= -3\frac{7}{9}$

(2)

c) $(2k-1)x^2 - 5kx + k + 1 = 0$

Let the roots be α and $\frac{1}{\alpha}$

$\alpha\beta = \frac{c}{a}$

$\therefore \alpha \times \frac{1}{\alpha} = 1 = \frac{k+1}{2k-1}$

$\therefore 2k - 1 = k + 1$

$\therefore k = 2$ (3)