Name: ..... Maths Class: .....

SYDNEY TECHNICAL HIGH SCHOOL

Year 11

Mathematics

Term 3 Examination

July 2004

TIME ALLOWED: 70 minutes

Instructions:

- Write your name and class at the top of this page.
- At the end of the examination this examination paper must be attached to the front of your answers.
- All questions may be attempted.
- All necessary working must be shown. Marks will be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- START EACH QUESTION ON A NEW PAGE

Q 1	Q 2	Q 3	Q 4	Q 5	Q 6	Q 7	Q 8	TOTAL
								/70

QUESTION 1: (9 marks)

(a) Fully factorise:

1 (i) $8x^3 - 1$

2 (ii) $y^4 - 16$

1 (b) Fully simplify $3\sqrt{2} \times 2\sqrt{6}$

3 (c) If $(2+3\sqrt{2})^2 = a + \sqrt{b}$ find the values of a and b

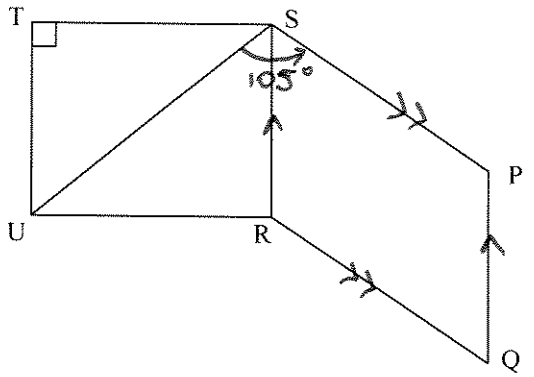
2 (d) Fully simplify

$$\frac{x^2 - 64}{4x - 32}$$

QUESTION 2: (8 marks)

3 (a) STUR is a square, and $\angle USP = 105°$.

Showing all reasoning, find $\angle PQR$



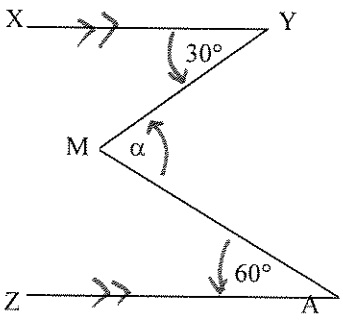
2 (b) Solve the equation

$$x^2 - 6x - 16 = 0$$

3 (c) (i) Copy the diagram at right onto your answer page

In the diagram, $\angle XYM = 30°$, $\angle MAZ = 60°$, and $XY \parallel ZA$

(ii) By drawing a suitable line LN through M (or otherwise) and by labelling it appropriately, find the value of α, showing all reasoning



QUESTION 3: (9 marks)

2 (a) Solve

$$8 - x \geq -4$$

and graph the solution on a number line

2 (b) Find exact values for:

(i) $\sin 240°$
(ii) $\sec(-45°)$

2 (c) Rationalize the denominator of and simplify:

$$\frac{5}{\sqrt{6}-1}$$

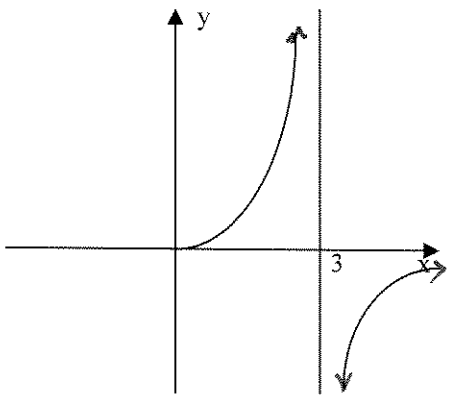
3 (d) Solve the following equations simultaneously:

$$\begin{cases} 5x - y - 4 = 0 \\ y = x^2 \end{cases}$$

QUESTION 4: (9 marks)

3 (a) Part of the graph of $y=g(x)$ is shown

Copy the diagram onto your answer sheet and complete the graph of $y=g(x)$ given that $g(x)$ is an EVEN function



(b) $H(x) = \sqrt{25 - x^2}$

1 (i) Sketch $y = H(x)$

2 (ii) What is the Domain and Range of the function $y = H(x)$?

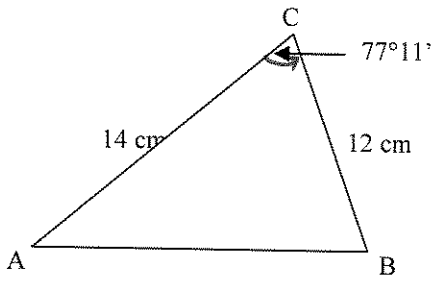
(c) If $f(x) = x^2 + 3$ and $h(x) = 2x - 1$

1 find (i) $f(-1)$

2 (ii) $h(f(y))$

QUESTION 5: (9 marks)

2 (a) Find the area of Δ ABC, correct to 2 decimal places:



2 (b) Solve, for $0^0 \leq \theta \leq 360^0$, the equation $\sin\theta = -\frac{1}{2}$

(c) A ship sails from port P on a bearing of 15°, and after travelling 10km to Q, steers a new course of 105°, until it is due east of P (at the point R).

1 (i) Draw a diagram showing all the above information and labelling all points

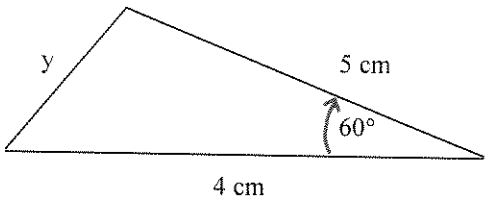
2 (ii) Find, and label, all internal angles on your diagram

2 (iii) Find the distance from R to P, to the nearest 0.1 km

QUESTION 6: (9 marks)

2 (a) If $\tan\theta = -\frac{5}{12}$ and $90^0 \le \theta \le 270^0$, find the exact value of $\cos\theta$ as a fraction.

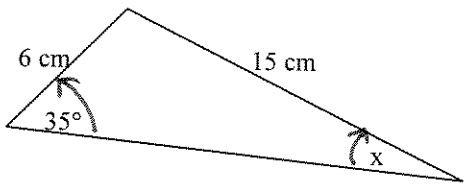
3 (c) Find the exact value of y in the following:



4 (b) Solve $2\sin^2 x - 1 = 0$ for $0^0 \le x \le 360^0$

QUESTION 7: (9 marks)

2 (a) In the diagram below, find the value of x correct to the nearest degree:



3 (b) Shade the region, in the first quadrant only, enclosed by the simultaneous inequalities:

$$x^2 + y^2 \le 9$$
$$y < x$$

4 (c) Prove the following, showing all necessary lines of working:

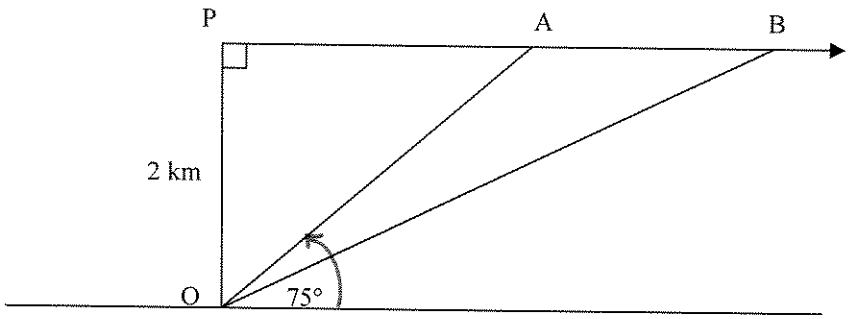
$$\frac{\sin\theta}{1-\cos\theta} + \frac{1-\cos\theta}{\sin\theta} = 2\text{cosec}\theta$$

QUESTION 8: (8 Marks)

3 (a) If $\cos(x + 30) = \sin 2x$, find x

(b)

5



A plane flying horizontally at a constant speed over level ground at a height of 2 Km above the earth passes directly above a viewer on the ground at O. 1 minute later the observer finds the angle of elevation of the plane (at A) is 75° as shown.
What will be the angle of elevation of the plane 1 minute later (at B)?

End of Examination

① (a) i. $(2x-1)(4x^2+2x+1)$ (1)

ii. $(y^2-4)(y^2+4)$ (1)

$= (y+2)(y-2)(y^2+4)$ (1)

(b) $6\sqrt{12} = 12\sqrt{3}$ (1)

(c) $4 + 12\sqrt{2} + 18$ (1) $= 22 + \sqrt{288}$

$a = 22$ (1) & $b = 288$ (1)

(d) $\frac{(x+8)(x-8)}{4(x-8)}$ (1) $= \frac{x+8}{4}$ (1)

② (a) $\angle USR = 45°$ } (1) for both

(diagonals of a square bisect angles through which they pass)

$\angle RSP = 60°$ (1)

$\therefore \angle PQR = 60°$ } (1) for both

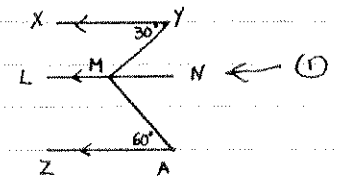
(opposite angles of a parallelogram)

(b) $x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$\therefore x = 8, -2$ (1) and (1)

(c)



(1)

(1) for each with both parts

$\angle YMN = 30°$ (alternate angles $XY \parallel LN$)

$\angle NMA = 60°$ (alternate angles $ZA \parallel LN$)

$\therefore \alpha = 90$

Student's Name/N°:

③ (a) $8 - \square \geqslant -4$

$-x \geqslant -12$

$x \leqslant 12$ (1)

(1) must be completely correct.

(b) i. $\sin 240° = -\sin 60°$

$= -\frac{\sqrt{3}}{2}$ (1)

ii. $\sec(-45)° = \sec 315°$

$= \frac{1}{\cos 45°}$

$= \sqrt{2}$ (1)

(c) $\frac{5}{\sqrt{6}-1} \times \frac{\sqrt{6}+1}{\sqrt{6}+1} = \frac{5\sqrt{6}+5}{6-1}$ (1)

$= \sqrt{6}+1$ (1)

(d) $5x - y - 4 = 0$

$y = x^2$

$5x - x^2 - 4 = 0$

$x^2 - 5x + 4 = 0$ (1)

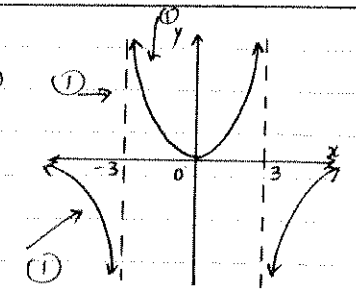
$(x-4)(x-1) = 0$

$\therefore x = 4, y = 16$ or (2)

$x = 1, y = 1$

④

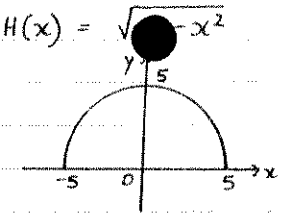
(a)



Teacher's Name:

(b) $H(x) = \sqrt{\square - x^2}$

i.



(1)

ii. $D: -5 \leq x \leq 5$ (1)

$R: 0 \leq y \leq 5$ (1)

(c) $f(x) = x^2 + 3$

$h(x) = 2x - 1$

i. $f(-1) = (-1)^2 + 3$

$= 4$ (1)

ii. $f(y) = y^2 + 3$ (1)

$h(f(y)) = 2(y^2+3) - 1$

$= 2y^2 + 6 - 1$

$= 2y^2 + 5$ (1)

⑤

(a) $A = \frac{1}{2} \times 14 \times 12 \times \sin 77° 11'$ (1 ∠)

$= 81.91$ (1)

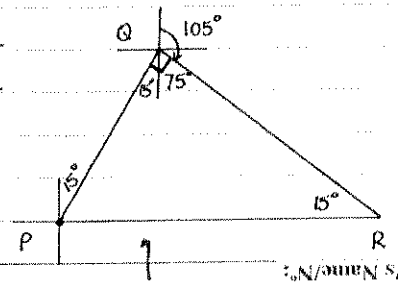
(b) $\sin\theta = -\frac{1}{2}$

$\theta_{acute} = 30°$

$\therefore \theta = 210°, 330°$ (2) 1 each.

(c) i.

ii.



(1) for diagram

(1) for the 15°, (1) for the 75°

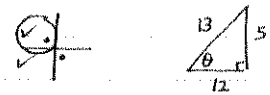
iii. $\cos 75° = \frac{10}{PR}$ (1)

$PR = \frac{10}{\cos 75°}$

$\therefore PR = 38.6$ km (1)

⑥

(a) $\tan\theta = -\frac{5}{12}$



$\cos\theta = -\frac{12}{13}$ (1) for sign (1) for 12/13

(b) $2\sin^2 x - 1 = 0$

$\sin^2 x = \frac{1}{2}$ (1)

$\sin x = \pm\frac{1}{\sqrt{2}}$ (1)

$x_{acute} = 45°$ (1)

$\therefore x = 45, 135, 225, 315$ (1)

(1)

(c) $y^2 = 4^2 + 5^2 - 2 \times 4 \times 5 \times \cos 60°$

$y^2 = 41 - 20$ (1)

$\therefore y = \sqrt{21}$ (1)

⑦

(a) $\frac{6}{\sin x} = \frac{15}{\sin 35°}$ (1)

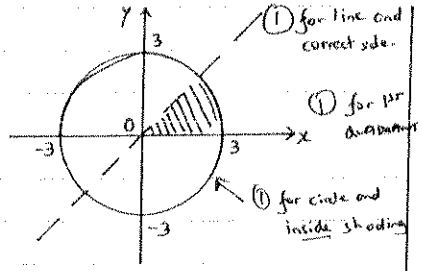
$\sin x = \frac{6 \sin 35°}{15}$

$\therefore x = 13°$ (1)

Student's Name/N°:

Teacher's Name:

(b)

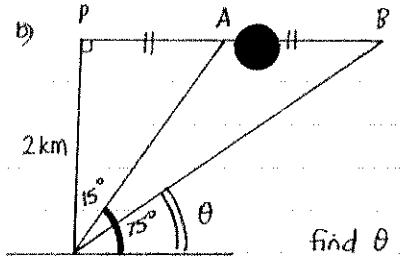


① for line and correct side.

① for 1st quadrant

① for circle and inside shading

b)



find θ

c) $\dfrac{\sin\theta}{1-\cos\theta} + \dfrac{1-\cos\theta}{\sin\theta} = 2\operatorname{cosec}\theta$

LHS $= \dfrac{\sin^2\theta + (1-\cos\theta)^2}{\sin\theta(1-\cos\theta)}$ ① for comm. den.

$= \dfrac{\sin^2\theta + 1 - 2\cos\theta + \cos^2\theta}{\sin\theta(1-\cos\theta)}$ ① for Pythagoras Theorem

$= \dfrac{1 + 1 - 2\cos\theta}{\sin\theta(1-\cos\theta)}$

$= \dfrac{2-2\cos\theta}{\sin\theta(1-\cos\theta)}$

$= \dfrac{2(1-\cos\theta)}{\sin\theta(1-\cos\theta)}$ ① for cancellation

$= \dfrac{2}{\sin\theta}$ ①

$= 2\operatorname{cosec}\theta$

$= $ RHS

$\tan 15^\circ = \dfrac{PA}{2}$ ①

$PA = 2\tan 15^\circ$ ①

$PB = 2 \times 2\tan 15^\circ$ ①

$= 4\tan 15^\circ$

$\tan\theta = \dfrac{2}{4\tan 15^\circ}$ ①

$\therefore \theta = 61^\circ 49'$ ①

OR/

$AP = x$

$\frac{2}{x} = \tan 75^\circ$ ①

$\therefore \frac{x}{2} = \frac{1}{\tan 75}$

$\therefore x \approx 0.5359$ ①

$\therefore PB = 1.0718$ ①

$\tan\theta = \dfrac{2}{1.0718}$ ①

$\therefore \theta = 61^\circ 49'$ ①

(8)

(a) $x + 30 + 2x = 90$ ① ← ①

$3x = 60$

$x = 20$ ①

OR/

$\cos(x+30) = \cos(90-2x)$ ①

$x + 30 = 90 - 2x$ ①

$3x = 60$

$x = 20$ ①

Teacher's Name: Student's Name/No.: