SYDNEY TECHNICAL HIGH SCHOOL

Mathematics

PRELIMINARY HSC ASSESSMENT TASK
JULY 2006

General Instructions

- Working time allowed – 70 minutes.
- Write using black or blue pen.
- Approved calculators may be used.
- All necessary working should be shown.
- Start each question on a new page.
- Attempt all questions.
- Questions are not of equal value.
- Full marks may not be awarded if working is poorly set out or difficult to read.

NAME :_____

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Question 7	TOTAL

Question 1 (7 marks)

Marks

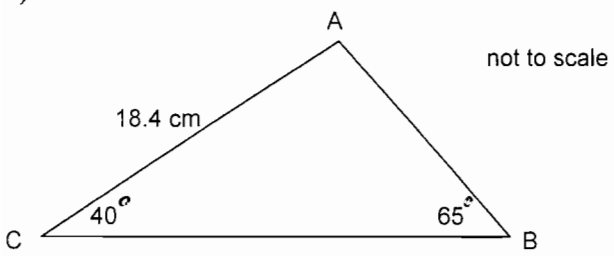
a) The line $ax + 2y - 10 = 0$ passes through the point $(4,-1)$. 2

Find the value of a.

b) Find the exact value of i) $\tan 30°$ 1

ii) $\sec 150°$ 1

c)



i) Find the length of side AB correct to the nearest millimetre. 2

ii) Find the area of triangle ABC correct to the nearest square centimetre. 1

Question 2 (7 marks) (Start a new page)

a) Sketch the graph of $y = \cos x$ for $0° \le x \le 360°$. 2

b) Find the equation of the line perpendicular to $y = 2x - 9$ which passes 3

through the point $(0,3)$. Express your answer in general form.

c) Find the acute angle that the line $3x - y - 5 = 0$ makes with the x axis. 2

(nearest degree)

Question 3 (7 marks) (Start a new page)

a) Simplify $\cos\theta \tan\theta$. 1

b) Solve $3\tan\theta = 5$ for $0° \le \theta \le 360°$ 2
Give answers correct to the nearest degree.

c) Find the point of intersection of the lines 2

$x - 2y - 7 = 0$ and $3x - 4y - 19 = 0$.

d) Sketch the region $x + 2y + 2 \ge 0$. 2

Question 4 (8 marks) (Start a new page)

The points A , B and C have coordinates $(-3,0)$, $(3,4)$ and $(5,1)$ respectively.

a) Draw a neat number plane showing this information. 1

b) Find the coordinates of the midpoint of AB. 1

c) Find the gradient of AB. 1

d) Find the equation of the line AB. 1

e) Find the length of AB. (exact form) 1

f) Find the equation of the circle with diameter AB. 2

g) Find the coordinates of the point D such that ABCD is a parallelogram. 1

Question 5 (8 marks) (Start a new page)

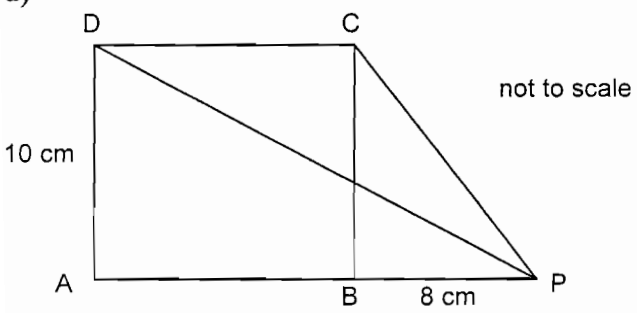
a) Given $\sin\theta = \frac{3}{4}$ and $\cos\theta < 0$ find the exact value of $\tan\theta$. 2

b) Find the perpendicular distance from the point $(-1, 6)$ 2

to the line $x + 2y - 3 = 0$.

c) Find a value of θ so that $\sin(\theta + 30)° = \cos 50°$. 1

d)



ABCD is a square with sides 10cm.

AB is extended to P such that BP=8cm.

Find $\angle DPC$ (correct to the nearest degree). 3

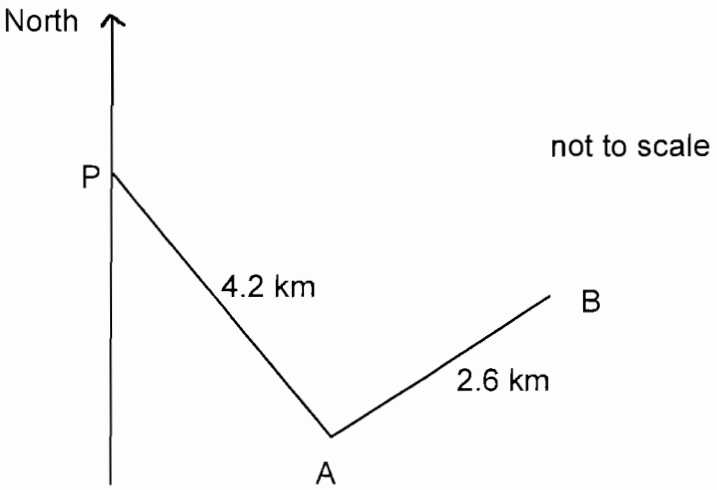
Question 6 **(7 marks)** (Start a new page)

a) If the lines $5x - 2y + 4 = 0$ and $y = mx - 2$ are parallel, 2

find the value of m

.

b)



Jay and Rob sail a boat from port P on a bearing of 165° for 4.2 km to position A.

The boat then sails on a bearing of 070° for 2.6 km to point B.

i) Find the size of angle PAB. 1

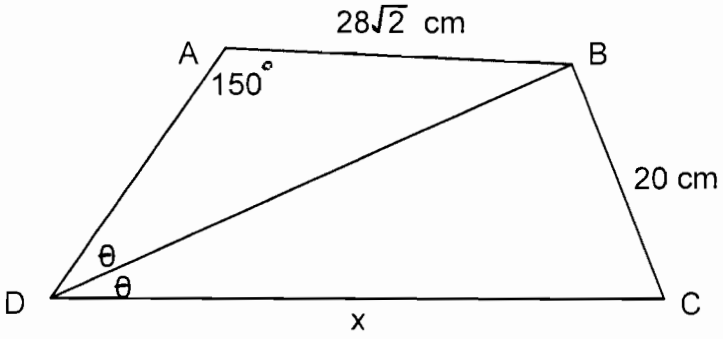
ii) Use the cosine rule to find the distance of position B from P. 2
Give answer in kilometres correct to 1 decimal place.

iii) Find the bearing of the port P from point B. 2
Give answer correct to the nearest degree.

Question 7 **(8 marks)** (Start a new page)

a) Show that $(1-\cos A)(1+\sec A) = \sin A \tan A$ 3

b)



In the above quadrilateral ABCD the diagonal BD bisects angle ADC and BD=DC.

$AB = 28\sqrt{2}\,cm$, $BC = 20\,cm$ and $\angle DAB = 150°$.

Let $\angle BDC = \theta$ and $DC = x\,cm$

i) Using the sine rule in triangle ABD, show that $\sin\theta = \dfrac{14\sqrt{2}}{x}$. 2

ii) By considering triangle BDC, use the cosine rule to show that 1

$$\cos\theta = \frac{x^2 - 200}{x^2}$$

iii) Use the results in parts i) and ii) to find the exact value of x. 2

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Question 1

a. $4a - 2 - 10 = 0$

$4a = 12$

$a = 3$

b. i) $\frac{1}{\sqrt{3}}$

ii) $-\frac{2}{\sqrt{3}}$

c. i) $\frac{AB}{\sin 40^\circ} = \frac{18.4}{\sin 65^\circ}$

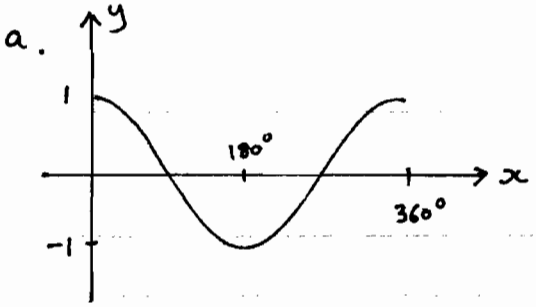
$AB = 13.0 \text{ cm}$

ii) $\text{Area} = \frac{1}{2} \times 18.4 \times 13.0 \times \sin 75^\circ$

$= 116 \text{ cm}^2$

Question 2

a.



b. $m = -\frac{1}{2} \quad (0, 3)$

$\therefore y = -\frac{1}{2}x + 3$

$2y = -x + 6$

$x + 2y - 6 = 0$

c. $m = 3$

$\therefore \tan\theta = 3$

$\theta = 72^\circ$

Question 3

a. $\cos\theta \tan\theta$

$= \cos\theta \frac{\sin\theta}{\cos\theta}$

$= \sin\theta$

b. $3\tan\theta = 5$

$\tan\theta = \frac{5}{3}$

(1st, 3rd quad)

$\theta = 59^\circ, 239^\circ$

c. $x - 2y - 7 = 0$

$3x - 4y - 19 = 0$

$2x - 4y - 14 = 0$

$3x - 4y - 19 = 0$

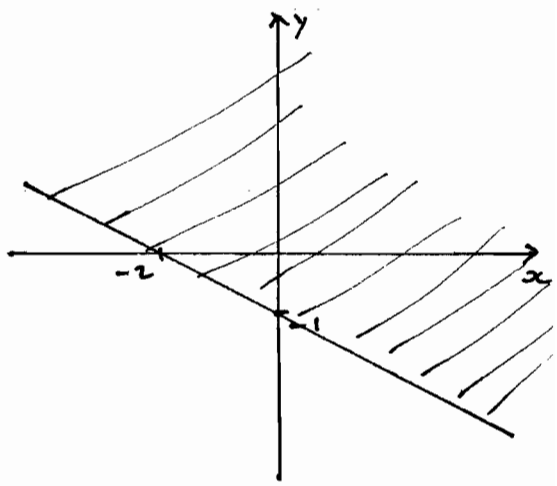
$x - 5 = 0$

$x = 5$

$\therefore y = -1$

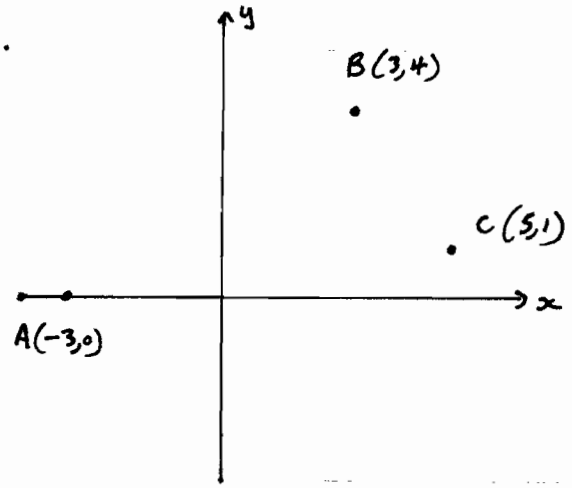
$\therefore$ pt. of intersection $(5, -1)$

d.



Question 4

a.



b. midpoint $= (0,2)$

c. $m = \frac{4-0}{3--3}$

$= \frac{4}{6}$

$= \frac{2}{3}$

d. $y - 4 = \frac{2}{3}(x-3)$

$3y - 12 = 2x - 6$

$2x - 3y + 6 = 0$

e. $d = \sqrt{(3--3)^2 + (4-0)^2}$

$= \sqrt{52}$ units

f. radius $= \frac{\sqrt{52}}{2}$

$= \sqrt{13}$

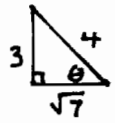
centre $= (0,2)$

$\therefore$ equation $x^2 + (y-2)^2 = 13$

g. $D(-1,-3)$

Question 5

a. 2nd quadrant



$\therefore \tan\theta = -\frac{3}{\sqrt{7}}$

b. $d = \frac{|-1 + 12 - 3|}{\sqrt{1^2 + 2^2}}$

$= \frac{8}{\sqrt{5}}$ units

c. $\theta + 30 + 50 = 90$

$\theta = 10$

d. $\tan \angle DPA = \frac{10}{18}$

$\therefore \angle DPA = 29.05°$

$\tan \angle CPA = \frac{10}{8}$

$\therefore \angle CPA = 51.34°$

$\therefore \angle DPC = 22°$

Question 6

a. $5x - 2y + 4 = 0$

has gradient $\frac{5}{2}$

$\therefore \quad m = \frac{5}{2}$

b. i) $\angle PAB = 85^\circ$

ii) $d^2 = 4.2^2 + 2.6^2 - 2 \times 4.2 \times 2.6 \times \cos 85^\circ$

$\therefore \quad d = 4.7 \text{ km}$

iii) $\frac{\sin \angle PBA}{4.2} = \frac{\sin 85^\circ}{4.7}$

$\therefore \quad \angle PBA = 63^\circ$

$\therefore$ bearing $= 180^\circ + 70^\circ + 63^\circ$

$= 313^\circ$

Question 7

a. $LHS = (1 - \cos A)(1 + \sec A)$

$= 1 + \sec A - \cos A - \cos A \sec A$

$= \sec A - \cos A$

$= \frac{1 - \cos^2 A}{\cos A}$

$= \frac{\sin^2 A}{\cos A}$

$= \sin A \tan A$

$= RHS$

b. i)

$\frac{\sin \theta}{28\sqrt{2}} = \frac{\sin 150^\circ}{x}$

$\sin \theta = \frac{28\sqrt{2} \sin 150^\circ}{x}$

$= \frac{28\sqrt{2} \times \frac{1}{2}}{x}$

$= \frac{14\sqrt{2}}{x}$

ii) $\cos \theta = \frac{x^2 + x^2 - 20^2}{2x^2}$

$= \frac{2x^2 - 400}{2x^2}$

$= \frac{x^2 - 200}{x^2}$

iii) $\sin^2 \theta + \cos^2 \theta = 1$

$\therefore \quad \left(\frac{14\sqrt{2}}{x}\right)^2 + \left(\frac{x^2 - 200}{x^2}\right)^2 = 1$

$\frac{392}{x^2} + \frac{x^4 - 400x^2 + 40000}{x^4} = 1$

$392x^2 + x^4 - 400x^2 + 40000 = x^4$

$8x^2 = 40000$

$x^2 = 5000$

$x = 50\sqrt{2}$