SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

PRELIMINARY HSC ASSESSMENT TASK

JULY 2008

General Instructions

- Working time allowed – 70 minutes
- Write using black or blue pen
- Approved calculators may be used
- All necessary working should be shown
- Start each question on a new page
- Attempt all questions
- Questions are not of equal value
- Full marks may not be awarded if working is poorly set out or difficult to read.

NAME:_____

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Question 7	Question 8	Total

QUESTION 1

Marks

a) Factorise $x^3 + 64$ 2

b) Solve $|3x + 1| \leq 10$ 2

c) Find integers a and b such that $(3 - \sqrt{2})^2 = a - b\sqrt{2}$ 2

d) Solve $\frac{x-5}{3} - \frac{x+1}{4} = 5$ 2

QUESTION 2

a) Sketch the graph of $y = \tan x$ for $0^\circ \leq x \leq 360^o$ 2

b) Find the equation of the line parallel to $3x - y + 4 = 0$ and also having a y intercept of -2. Express your answer in general form. 3

c) Find the angle that $2x + y - 3 = 0$ makes with the positive direction of the $x-$axis. **(correct to nearest minute)** 2

QUESTION 3

a) Simplify $(\text{cosec}\theta - 1)(\text{cosec}\theta + 1)$

b) i) If A is acute and $\sin A = \frac{4}{7}$, find the exact values of cosA and tanA 2

ii) Hence show that $\tan A = \frac{\sin A}{\cos A}$ 1

c) Prove the identity $\frac{\cos\theta}{1-\sin\theta} = \sec\theta\,(1 + \sin\theta)$ 3

QUESTION 4

Marks

a) Find the exact value of $\sin(-300)^\circ$ 2

b) Find the shortest distance between the parallel lines 3

$2x - 5y + 10 = 0$ and $2x - 5y - 3 = 0$

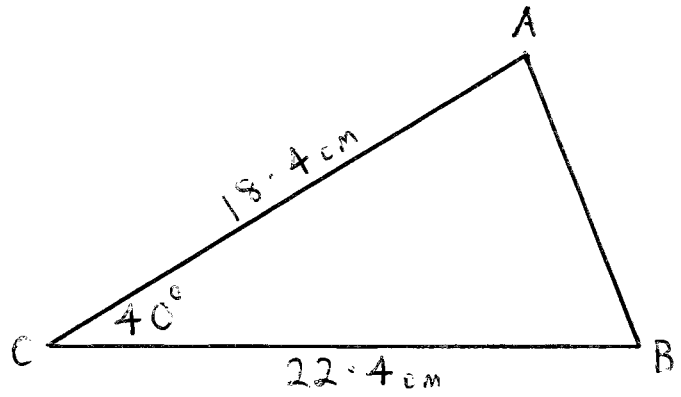
(Hint: Find a point on either line).

c) Solve for $0^\circ \leq x \leq 90^\circ$, 2

$\sin 50^\circ = \cos(2x - 10)^\circ$

QUESTION 5

a)



i) Find the length of side AB correct to one decimal place. 2

ii) Find the area of triangle ABC correct to the nearest square centimetre 1

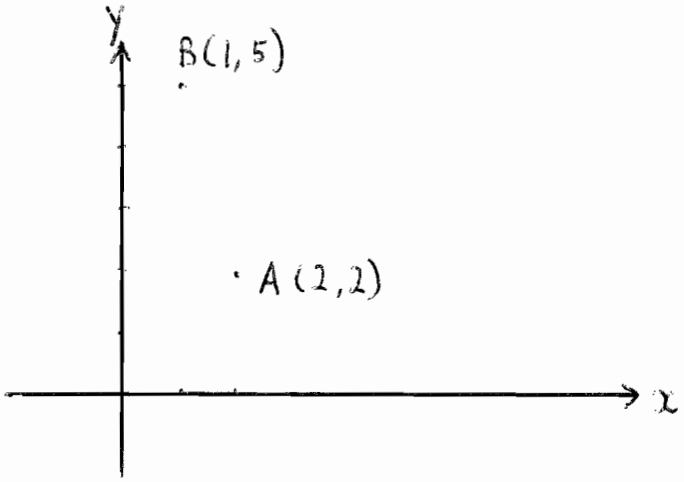
b) Solve $4\cos\theta = -3$ for $0^\circ \leq \theta \leq 360^\circ$.
Give answers correct to the nearest degree. 2

c) Sketch the region $2x - y + 3 < 0$ 2

QUESTION 6

Marks

a)



Copy the diagram onto your answer sheet

i) Find the co-ordinates of M, the midpoint of AB 1

ii) Find the gradient of AB 1

iii) Show the equation of the perpendicular bisector of AB is 2

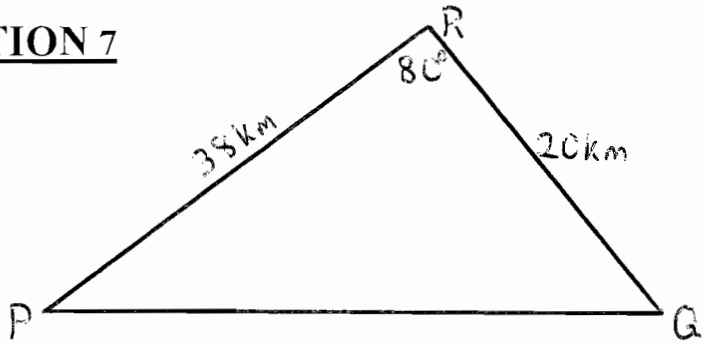
$$x - 3y + 9 = 0$$

iv) Find the co-ordinates of C, which lies on the $y-$ axis and is equidistant from A and B. 1

v) The point D lies on the intersection of the line $y = 5$ and the perpendicular bisector $x - 3y + 9 = 0$. Find the co-ordinates of D and mark the position of D on your diagram. 1

vi) Find the area of the triangle ABD

QUESTION 7



In the diagram, the point Q is due east of P. The bearing of R from Q is 330°T.

i) What is the size of $< PQR$ 1

ii) Find the distance PQ **(correct to one decimal place)** 2

iii) What is the bearing of R from P? (nearest degree) 2

b) Solve simultaneously: 2

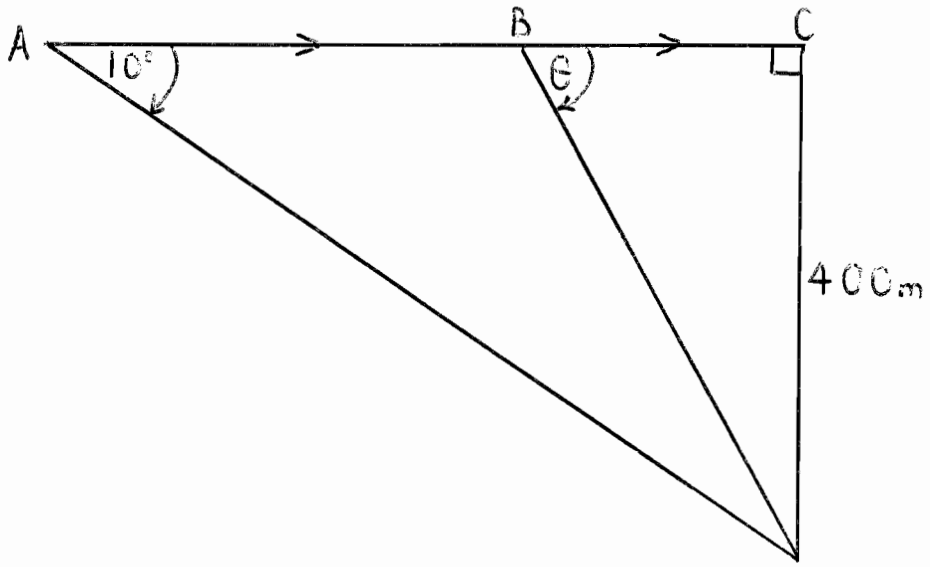
$x - 2y = 8$

$2x + y = 1$

QUESTION 8

a) If the points (-3k,1) (2,3) and (k,4) are collinear, find the value of k 2

b)



A helicopter pilot flying horizontally 400m above the ground at a speed of 60km/h, notices a laser gun shone up at him at an angle of depression of 10°. One minute later he has reached point B.

i) Calculate AB 1

ii) Calculate AC (nearest metre) 2

iii) Calculate θ, the angle of depression from B (nearest degree) 1

iv) From B he radios the police and tells them he will hover directly above the offender at C. How long (correct to the nearest second) does it take to fly from B to C? 2

Teacher's Name: Student's Name/N°:

July 2008 2 Unit Assessment Solutions

Question 1

a) $x^3 + 64$
$= x^3 + 4^3$
$= (x+4)(x^2 - 4x + 16)$ ① ①

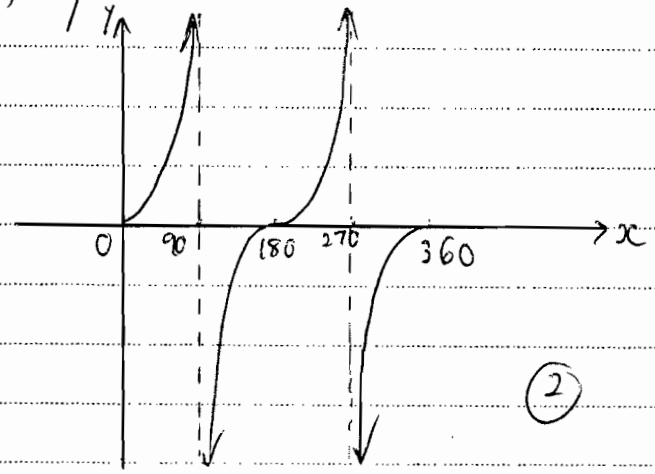
b) $|3x+1| \leq 10$
$-10 \leq 3x+1 \leq 10$ ①
$-11 \leq 3x \leq 9$
$-\frac{11}{3} \leq x \leq 3$ ①

c) $(3-\sqrt{2})^2 = a - b\sqrt{2}$
$9 + 2 - 6\sqrt{2} = a - b\sqrt{2}$
$11 - 6\sqrt{2} = a - b\sqrt{2}$
$a = 11, \quad b = 6$ ① ①

d) $\frac{x-5}{3} - \frac{x+1}{4} = 5$
$4(x-5) - 3(x+1) = 60$
$4x - 20 - 3x - 3 = 60$ ①
$x - 23 = 60$
$x = 83$ ①

Question 2

a) $y = \tan x \quad 0° \leq x \leq 360°$



②

b) $3x - y + 4 = 0$
$y = 3x + 4$
$m = 3 \quad b = -2$ ①
$y = mx + b$ becomes
$y = 3x - 2$ ①
$0 = 3x - y - 2$ ①

c) $2x + y - 3 = 0$
$y = -2x + 3$
$m = -2$
$m = \tan\theta$ ①
$-2 = \tan\theta$
$\theta = 116°33'$ ①

Teacher's Name: **Student's Name/N°:**

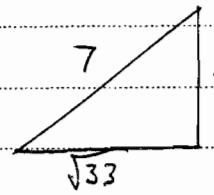
Question 3

a) $(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)$

$\operatorname{cosec}^2\theta - 1$

$= \cot^2\theta$ ①

b) ci)



$\cos A = \frac{\sqrt{33}}{7}$ (

$\tan A = \frac{4}{\sqrt{33}}$ (

cii) $\tan A = \frac{\sin A}{\cos A}$

$= \frac{\frac{4}{7}}{\frac{\sqrt{33}}{7}}$ ①

$= \frac{4}{\sqrt{33}}$

c) $\frac{\cos\theta}{1-\sin\theta} = \sec\theta(1+\sin\theta)$

$\frac{\cos\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta} =$ "

① $\frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta} =$ "

① $\frac{\cos\theta(1+\sin\theta)}{\cos^2\theta} =$ "

$\frac{1+\sin\theta}{\cos\theta} =$ "

① $\sec\theta(1+\sin\theta) = \sec\theta(1+\sin\theta$

Question 4

a) $\sin(-300)$

$= \sin 60$ ①

$= \frac{\sqrt{3}}{2}$ ①

b) $2x - 5y + 10 = 0$

$(0, 2)$ lies on above ①

Now $\perp$ d. from $2x - 5y - 3 = 0$

$d = \frac{|2\times 0 + -5\cdot 2 - 3|}{\sqrt{2^2 + (-5)^2}}$ ①

$d = \frac{13}{\sqrt{29}}$ ①

c) $\sin 50 = \cos(2x - 10)$

① $50 + 2x - 10 = 90$ complementary angles

$40 + 2x = 90$

$2x = 50$

① $x = 25°$

Teacher's Name: Student's Name/N°:

(vi) $A = \frac{1}{2} b h$

$= \frac{1}{2} BC \times 3$ ①

$= 2\frac{1}{2} \times 3$

$A = 7\frac{1}{2}$ units^2 ①

$PQ = 43 \cdot 2 / 30 \cdot 6$

$\frac{PQ}{\sin 80} = \frac{38}{\sin 60}$ or $\frac{20}{\sin 40}$

Question 7

a) (i) $60°$ ①

$70°30'$

(ii) $PQ^2 = 38^2 + 20^2 - 2 \times 38 \times 20 \times \cos 80$ ①

$PQ = 39 \cdot 7$ km ①

(iii) Need $\angle PPQ$

$\frac{\sin P}{20} = \frac{\sin 80}{39 \cdot 7}$ ①

$P = 30°$

∴ Bearing is $\begin{cases} 060°T & ① \\ 050°T \\ 063°T \end{cases}$

b) $x - 2y = 8$ ①

$2x + y = 1$ ②

① + 2 × ②

$\Rightarrow 5x = 10$

$x = 2, y = -3$ ① ①

Question 8

a) $(-3k, 1)$ $(2, 3)$ $(k, 4)$

Collinear if

$\frac{3-1}{2--3k} = \frac{4-3}{k-2}$ ① equal gradients

$2k - 4 = 2 + 3k$

$-6 = k$ ①

b) (i) $AB = 1$ km ①

(ii) $\tan 10 = \frac{400}{AC}$ ①

$AC = 2269$ m ①

(iii) $BC = AC - AB$

$= 1269$ m

$\tan\theta = \frac{400}{1269}$

$\theta = 17°$ ①

(iv) $BC = 1269$

$\text{Time} = \frac{\text{distance}}{\text{speed}}$

$= \frac{1 \cdot 269}{60}$ hours ①

$= 76$ seconds ①

Teacher's Name: Student's Name/N°:

Question 5

a) (i) $AB^2 = 18.4^2 + 22.4^2 - 2 \times 18.4 \times 22.4 \times \cos 40$ ①

$AB = 14.5$ cm ①

(ii) $A = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 18.4 \times 22.4 \times \sin 40$

$A = 132 \text{ cm}^2$ ①

b) $4 \cos\theta = -3$

$\cos\theta = \frac{-3}{4}$

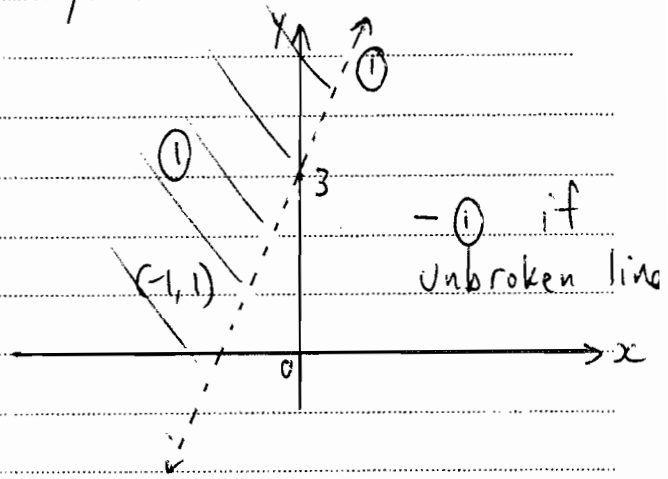
working $\theta = 41°25'$

✓S	A
✓T	C

$\theta = 139°, 221°$ ① ①

c) $2x - y + 3 < 0$

$y > 2x + 3$



① ① ① if unbroken line

Question 6

a) (i) M is $\left(\frac{1+2}{2}, \frac{5+2}{2}\right)$

$\left(\frac{3}{2}, \frac{7}{2}\right)$ ①

(ii) $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{5-2}{1-2}$

$= -3$ ①

(iii) $m = \frac{1}{3}$ $\left(\frac{3}{2}, \frac{7}{2}\right)$

$y - \frac{7}{2} = \frac{1}{3}\left(x - \frac{3}{2}\right)$ ①

$6y - 21 = 2\left(x - \frac{3}{2}\right)$

$6y - 21 = 2x - 3$

$2x - 6y + 18 = 0$ ①

$x - 3y + 9 = 0$

(iv) $x = 0$ in $x - 3y + 9 = 0$

$y = 3$

$\therefore$ C is $(0, 3)$ ①

(v) $x - 3y + 9 = 0$ $y = 5$

$x - 3 \times 5 + 9 = 0$

$x = 6$